Test Code: QR (Short answer type) 2011

M.Tech. in Quality, Reliability and Operations Research

The candidates applying for M.Tech. in Quality, Reliability and Operations Research will have to take two tests : **Test MIII** (objective type) in the forenoon session and **Test QR** (short answer type) in the afternoon session.

For Test **MIII**, see a different Booklet. For Test **QR**, refer to this Booklet **ONLY.**

If you are from **Statistics / Mathematics Stream,** you will be required to **ANSWER PART I.**

If you are from **Engineering Stream**, you will be required to **ANSWER PART II.**

In **PART I**, a **TOTAL of TEN [10]** questions, are divided into **TWO Groups**: **S1**: **Statistics and S2: Probability** – **each group carrying FIVE [5]** questions. You will be required to answer a **TOTAL of SIX [6]** questions, taking **AT LEAST TWO [2]** from **each group**.

In **PART II**, there will be **FIVE Groups: E1-E5**. **E1** will contain **THREE [3]** questions from **Engineering Mathematics** and each other group will contain **TWO [2]** questions from **Engineering and Technology**. You will be required to answer a total of **SIX [6]** questions taking **AT LEAST TWO [2]** from group **E1**.

Syllabus

PART I: STATISTICS / MATHEMATICS STREAM

Statistics (S1)

Descriptive statistics for univariate, bivariate and multivariate data.

Standard univariate probability distributions [Binomial, Poisson, Normal] and their fittings, properties of distributions. Sampling distributions.

Theory of estimation and tests of statistical hypotheses.

Multiple linear regression and linear statistical models, ANOVA.

Principles of experimental designs and basic designs [CRD, RBD & LSD].

Elements of non-parametric inference.

Elements of sequential tests.

Sample surveys – simple random sampling with and without replacement, stratified and cluster sampling.

Probability (S2)

Classical definition of probability and standard results on operations with events, conditional probability and independence.

Distributions of discrete type [Bernoulli, Binomial, Multinomial, Hypergeometric, Poisson, Geometric and Negative Binomial] and continuous type [Uniform, Exponential, Normal, Gamma, Beta] random variables and their moments.

Bivariate distributions (with special emphasis on bivariate normal), marginal and conditional distributions, correlation and regression.

Multivariate distributions, marginal and conditional distributions, regression, independence, partial and multiple correlations.

Order statistics [including distributions of extreme values and of sample range for uniform and exponential distributions].

Distributions of functions of random variables.

Multivariate normal distribution [density, marginal and conditional distributions, regression].

Weak law of large numbers, central limit theorem.
Basics of Markov chains and Poisson processes.

PART II : ENGINEERING STREAM

Mathematics (E1)

Elementary theory of equations, inequalities.
Elementary set theory, functions and relations, matrices, determinants, solutions of linear equations.
Trigonometry [multiple and sub-multiple angles, inverse circular functions, identities, solutions of equations, properties of triangles].
Coordinate geometry (two dimensions) [straight line, circle, parabola, ellipse and hyperbola], plane geometry, Mensuration.
Sequences, series and their convergence and divergence, power series, limit and continuity of functions of one or more variables, differentiation and its applications, maxima and minima, integration, definite integrals areas using integrals, ordinary and partial differential equations (upto second order), complex numbers and De Moivre's theorem.

Engineering Mechanics (E2)

Forces in plane and space, analysis of trusses, beams, columns, friction, principles of strength of materials, work-energy principle, moment of inertia, plane motion of rigid bodies, belt drivers, gearing.

Syllabus

Electrical and Electronics Engineering (E3)

D.C. circuits, AC circuits, energy and power relationships, Transformer, DC and AC machines, concepts of control theory and applications.
Network analysis, 2 port network, transmission lines, elementary electronics (including amplifiers, oscillators, op-amp circuits), analog and digital electronic circuits.

Theromodynamics (E4)

Laws of thermodynamics, internal energy, work and heat changes, reversible changes, adiabatic changes, heat of formation, combustion, reaction, solution and dilution, entropy and free energy and maximum work function, reversible cycle and its efficiency, principles of internal combustion engines, principles of refrigeration.

Engineering Properties of Metals (E5)

Structures of metals, tensile and torsional properties, hardness, impact properties, fatigue, creep, different mechanism of deformation.

SAMPLE QUESTIONS

PART I: STATISTICS / MATHEMATICS STREAM

GROUP S–1: Statistics

1. Let X_1 and X_2 be independent χ^2 variables, each with n degrees of freedom. Show that $\frac{\sqrt{n}\left(X_1 - X_2\right)}{2\sqrt{X_1 X_2}}$ has the t distribution with n degrees of freedom and is independent of $X_1 + X_2$.

2. (a) X, Y and Z are independent random variables with the same variance. If

$$X_1 = \frac{1}{\sqrt{2}}\left(X - Z\right), X_2 = \frac{1}{\sqrt{3}}\left(X + Y + Z\right) \text{ and } X_3 = \frac{1}{\sqrt{6}}\left(X + 2Y + Z\right),$$

show that X_1, X_2 and X_3 have equal variances. Calculate $r_{12..3}$ and $R_{1.23}$.

(b) Let X has the p.m.f.

$$f\left(x,\theta\right) = \begin{cases} \theta^x \left(1-\theta\right)^{1-x}, & x = 0,1 \\ 0, & \text{elsewhere} \end{cases}$$

We test the simple null hypothesis $H_0 : \theta = \frac{1}{4}$ against the alternative composite hypothesis $H_1 : \theta < \frac{1}{4}$ by taking a random sample of size 10 and rejecting $H_0 : \theta = \frac{1}{4}$ if and only if the observed values x_1, x_2, …., x_{10} of the sample observations are such that $\sum_{i=1}^{10} x_i \le 1$. Find the power function $K\left(\theta\right), 0 < \theta \le \frac{1}{4}$ of this test.

3. Consider the linear regression model : $y = \alpha + \beta x + e$ where e's are iid $N(0, \sigma^2)$.

   (a) Based on n pairs of observations on x and y, write down the least squares estimates for α and β.

   (b) Work out exact expression for Cov($\hat{\alpha}$, $\hat{\beta}$).

   (c) For a given y_0 as the "predicted" value, determine the corresponding predictand "x_0" and suggest an estimator "$\hat{x}_0$" for it.

4. A town has N taxis numbered 1 through N. A person standing on roadside notices the taxi numbers on n taxis that pass by. Let M_n be the largest number observed. Assuming independence of the taxi numbers and sampling with replacement, show that

$$\hat{N} = (n+1)\, M_n / n$$

is an approximately unbiased estimator of N for large N.

5.(a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the rectangular population with density

$$f(x) = \begin{cases} \dfrac{1}{\theta}, & 0 < x < \theta \\ 0, & otherwise \end{cases}$$

Consider the critical region $x_{(n)} > 0.8$ for testing the hypothesis $H_0 : \theta = 1$, where $x_{(n)}$ is the largest of $x_1, x_2, \ldots, x_n$. What is the associated probability of type I error and what is the power function?

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population having p.d.f.

$$f(x,\theta) = \begin{cases} \dfrac{\theta^3}{\Gamma(3)} e^{-\theta x} x^2, & 0 < x < \infty \\ 0, & otherwise \end{cases}$$

Obtain the maximum likelihood estimate of θ and also obtain the Cramer Rao lower bound to the variance of an unbiased estimator of θ.

6.(a) Give an example of a Latin Square Design of order 4 involving 4 rows, 4 columns and 4 treatments. Give the general definition of "treatment connectedness" in the context of a Latin Square Design and show that the Latin Square Design that you have given is indeed treatment connected.

(b) In a CRD set-up involving 5 treatments, the following computations were made:

$n = 105$, Grand Mean = 23.5, SSB = 280.00, SSW = 3055.00

(i) Compute the value of the F-ratio and examine the validity of the null hypothesis.

(ii) It was subsequently pointed out that there was one additional treatment that was somehow missed out! For this treatment, we are given sample size = 20, Sum = 500 and Sum of Squares (corrected) = 560.00. Compute revised value of F-ratio and draw your conclusions.

7. If X_1, X_2, X_3 constitute a random sample from a Bernoulli population with mean p, show why $[X_1 + 2X2 + 3X_3] / 6$ is *not* a sufficient statistic for p.

8. If X and Y follow a trinomial distribution with parameters n, θ_1 and θ_2, show that

(a) $E(Y / X = x) = \dfrac{(n-x)\theta_2}{1-\theta_1}$,

(b) $V(Y / X = x) = \dfrac{(n-x)\theta_2(1-\theta_1-\theta_2)}{(1-\theta_1)^2}$

9. A food products company – Wonderful Bakery products, sells its produce to bakery shops. The sales are made by four sales persons Pratap, Quereshi, Rajinder and Sandip. The products are sold to four bakery shops namely Amit Bakeries (A), Bakeman Products (B), Chocolates and Breads (C) and, Delight Confectioners (D). The owner of Wonderful Bakery Products wanted to assess the effectiveness of the sales persons and carried out an experiment. All the sales persons were sent to the four shops three times each on different days and the quantity sold was noted. The actual sales figures for the different sales persons at different shops on different days are given below:

Pratap			Quereshi			Rajinder			Sandip		
Day	Bakery	Qty	Day	Bakery	Qty	Day	Bakery	Qty	Day	Bakery	Qty
2	A	11	2	B	18	6	D	74	2	C	35
11	C	48	8	D	40	7	C	42	2	D	47
15	B	68	9	A	29	10	A	6	3	A	8
18	D	36	15	C	57	13	D	104	4	C	35
21	A	24	16	A	24	16	D	56	8	B	64
25	C	57	25	B	58	18	B	28	8	C	20
27	C	33	26	D	136	21	C	93	11	A	9
30	D	77	28	A	34	27	B	22	19	B	15
34	D	64	29	C	27	31	A	14	19	D	27
35	B	42	37	D	60	31	B	18	29	B	27
37	A	11	38	B	28	34	C	72	31	D	28
40	B	18	39	C	70	35	A	21	38	A	10
Total Sales		489	Total Sales		581	Total Sales		550	Total Sales		325

While analyzing this data keep in mind that the sales made to a particular shop will depend on the skill of the sales person as well as the level of inventory of the shop.

We also assume here that these bakeries buy their products from these four sales persons only and from no other sources.

How will you analyze this data to compare the performance of the sales persons? Which salesman do you think has performed best? State your assumptions clearly.

GROUP S–2: Probability

1. Ali**,** Bikram and Charlie are three antagonists engaged in a three-way duel. There are two rounds. In the first round each player is given one shot; first Ali, then Bikram and finally Charlie shoots. After the first round the survivors are given a second round again beginning with Ali, then Bikram and then Charlie.

From the point of view of any person fighting the duel, being the sole survivor is the most preferred outcome. Being one of the two survivors is the next best possibility. A case when everyone survives is the third best and obviously not surviving is the worst possible case.

It is known that Ali is a poor shot with a probability of 0.30 of hitting the target. Bikram is a much better shot and hits the target with probability 0.80. Charlie is a crack-shot. He never misses.

Let the optimal strategy for any person be a way of playing the game, i.e., choosing to shoot a particular target or deliberately miss so that the chance of his survival is maximized. What is the optimal strategy of Ali? Assuming that the players follow the optimal strategy, who has the greatest chance of survival and what is the probability?

2. Suppose a young man is waiting for a young lady who is late. To amuse himself while waiting, he decides to take a random walk under the following set of rules:

> He tosses an imperfect coin for which the probability of getting a head is 0.55. For every head turned up, he walks 10 yards to the north and for every tail turned up, he walks 10 yards to the south.
>
> That way he has walked 100 yards.

(a) What is the probability that he will be back to his starting position?

(b) What is the probability that he will be 20 yards away from his starting position?

3. (a) A coin is tossed an odd number of times. If the probability of getting more heads than tails in these tosses is equal to the probability of getting more tails than heads then show that the coin is unbiased.

(b) For successful operation of a machine, we need at least three components (out of five) to be in working phase. Their respective chances of failure are 7%, 4%, 2%, 8% and 12%. To start with, all the components are in working phase and the operation is initiated. Later it is observed that the machine has stopped but the first component is found to be in working phase. What is the likelihood that the second component is also in working phase?

(c) A lot contains 20 items in which there are 2 or 3 defective items with probabilities 0.4 and 0.6 respectively. Items are tested one by one from the lot unless all the defective items are tested. What is the probability that the testing procedure will continue up to the twelfth attempt?

4.(a) Let S and T be distributed independently as exponential with means $1/\lambda$ and $1/\mu$ respectively. Let $U = \min\{S,T\}$ and $V = \max\{S,T\}$. Find $E(U)$ and $E(U+V)$.

(b) Let X be a random variable with $U(0,1)$ distribution. Find the p.d.f. of the random variable $Y = X / (1 + X)$.

5. Let the joint p.d.f. of X and Y be given by

$$f(x,y) = \begin{cases} \dfrac{2}{(1+x+y)^3}, & 0 < x < \infty, 0 < y < \infty \\ 0, & elsewhere \end{cases}$$

a) Compute the marginal p.d.f of X and the conditional p.d.f of Y, given $X = x$.

b) For a fixed $X = x$, compute $E[1+ x + Y \mid x]$ and use the result to compute $E(Y|x)$.

6.(a) Let X be a random variable with density

$$f_X(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

For the minimum $X_{(1)}$ of n iid random observations $X_1, X_2, \ldots, X_n$ from the above distribution, show that $n^{1/4} X_{(n)}$ converges in distribution to a random variable Y with density

$$f_Y(y) = \begin{cases} 4e^{-y^4} y^3, & y > 0 \\ 0, & otherwise \end{cases}$$

(b) A random sample of size n is taken from the exponential distribution having p.d.f.

$$f(x) = \begin{cases} e^{-x}, & 0 \le x < \infty \\ 0, & otherwise. \end{cases}$$

Find the p.d.f. of the sample range.

7. a) *A* and *B* throw alternately a pair of dice. *A* wins if he scores 6 points before *B* gets 7 points, in which case *B* wins. If *A* starts the game, what is his probability of winning?

b) A particle counter is recording two streams of particles, high-energy ones and low-energy ones. Assume that the two streams of particles arrive at the counter according to independent Poisson processes with intensities and respectively.

i) Find the conditional probability mass function of the number of particles registered in (0, t] given that a total of n particles were registered in this time interval.

ii) If a low-energy particle has energy e_L and a high-energy particle has energy e_H, find the expected energy of a particle just registered.

8. Suppose in a big hotel there are *N* rooms with single occupancy and also suppose that there are *N* boarders. In a dinner party to celebrate the marriage anniversary of one of the boarders they start drinking alcohol to their hearts' content and as a consequence they become unable to identify their own rooms. What is the probability that at the end of the dinner party none of the boarders occupies the room originally assigned to them? What is the limiting value of this probability as $N \to \infty$?

9. (a) Consider a Markov Chain with state space I = {1,2,3,4,5,6} and transition probability matrix P given by

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1/8 & 7/8 & 0 & 0 \\ 1/4 & 0 & 1/2 & 0 & 1/8 & 1/8 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \end{bmatrix}$$

Find the various classes of this chain and classify them as recurrent or transient.

(b) Pulses arrive at a Geiger counter according to a Poisson Process with parameter $\lambda > 0$. The counter is held open only a random length of time T (independent of the arrival time of the pulses), where T is exponentially distributed with parameter $\beta > 0$. Find the distribution of N = Total number of pulses registered by the counter.

PART II: ENGINEERING STREAM

GROUP E-1: Engineering Mathematics

1.(a) Let $f(x)$ be a polynomial in x and let a, b be two real numbers where $a \neq b$. Show that if $f(x)$ is divided by $(x - a)(x - b)$ then the remainder is

$$\frac{(x-a)f(b)-(x-b)f(a)}{b-a}.$$

(b) Find $\frac{dy}{dx}$ if $x^{\cos y} + y^{\sin x} = 1$.

2.(a) Let A be the fixed point (0,4) and B be a moving point ($2t$, 0). Let M be the mid-point of AB and let the perpendicular bisector of AB meets the y-axis at R. Find the equation of the locus of the mid-point P of MR.

(b) Inside a square ABCD with sides of length 12 cm, segment AE is drawn where E is the point on DC such that DE = 5 cm. The perpendicular bisector of AE is drawn and it intersects AE, AD and BC at the points M, P and Q respectively. Find the ratio PM: MQ.

3.(a) Evaluate the value of $3.9^{1/2}.27^{1/4}.81^{1/8}\ldots\ldots\infty$.

(b) Let *f* be a twice differentiable function such that

$$f''(x) = -f(x);\ f'(x) = g(x) \text{ and } h(x) = f^2(x) + g^2(x).$$

Given that $h(5)=11$, find $h(10)$.

4.(a) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots\ (upto\,[n/2]\,terms)\right]=\frac{1}{2}.$$

(c) Test the convergence of the series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\cdots\infty$. Assume $x>0$ and examine all possibilities.

5.(a) Find the limit of the following function as $x\to 0$.

$$\frac{|x|}{\sqrt{\left(x^4+4x^2+7\right)}}\sin\left(\frac{1}{3\sqrt{x}}\right).$$

(b) If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then show that $a\cdot b < 0$.

6.(a) If ω is a complex cube root of unity then show that

$$a^3+b^3+c^3-3abc=\left(a+b+c\right)\left(a+b\omega+c\omega^2\right)\left(a+b\omega^2+c\omega\right).$$

(b) Show that $\left[\frac{\sum\limits_{r>s}x^r}{r!}\right]\div\left[\frac{\sum\limits_{r>s}y^r}{r!}\right]>\frac{x^s}{y^s}$, whenever $x > y > 0$.

7.(a) Cable of a suspension bridge hangs in the form of a parabola and is attached to the supporting pillars 200 m apart. The lowest point of the cable is 40 m below the point of suspension. Find the angle between the cable and the supporting pillars. State all the assumptions involved.

(b) Let A, B and C be the angles of a triangle with angle C as the smallest of them.
Show that

(i) $\sin\left(\frac{C}{2}\right) \le \frac{1}{2}$

(ii) Hence, or otherwise, show that $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) < \frac{1}{4}$.

8.(a) Evaluate the following two integrals directly and compare them.

$$\iint\limits_{ax^2+by^2\le 1} dxdy \quad \text{and} \quad \iint\limits_{\sqrt{a}|x|\le 1,\sqrt{b}|y|\le 1} dxdy.$$

(b) Determine *x*, *y* and *z* so that the *3* × *3* matrix with the following row vectors is orthogonal : (1 / √3, 1 / √3, 1 / √3), (1/ √2, -1/ √2, 0), (*x, y, z*).

9.(a) Find the inverse of the following matrix

$$\begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_2 & c_3 & c_0 & c_1 \\ c_3 & -c_2 & c_1 & -c_0 \\ c_1 & -c_0 & c_3 & -c_2 \end{bmatrix}$$

where $c_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}, c_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}, c_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}$ and $c_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}$.

(b) Show that $\int_0^\infty \frac{dx}{(x+1)(x+2)}$ is convergent.

GROUP E-2: Engineering Mechanics

1.(a) The simple planar truss in the given Fig.1 consists of two straight two-force members AB and BC that are pinned together at B. The truss is loaded by a downward force of P=12 KN acting on the pin at B. Determine the internal axial forces F_1 and F_2 in members AB and BC respectively. (Neglect the weight of the truss members).

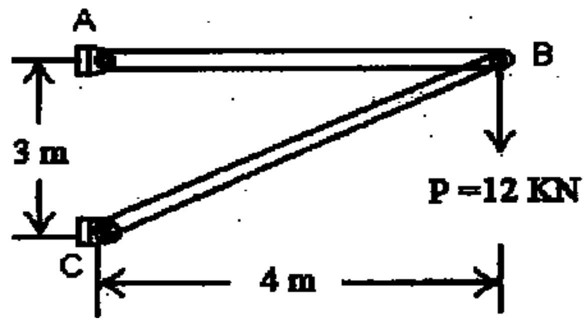


Fig. 1

(b) A particle of weight *W* attached to a fixed point O by a string of length *L* whirls in a horizontal circular path of radius *r* with uniform speed *V* so that the string generates a height *h*. Show that the relation between *V*, *r*, *h* and the tensile force *T* in the string is $T = W\sqrt{1+\left(\frac{r}{h}\right)^2}$.

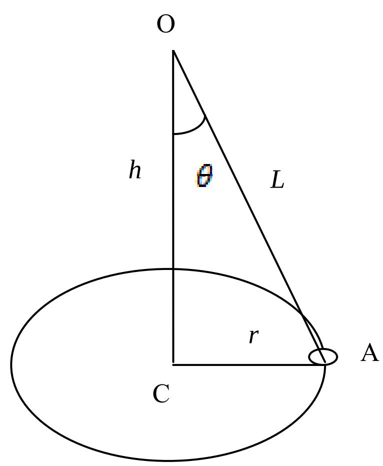


Fig. 2

2.(a) A turbine rotor weighs 20 tonnes and has a radius of gyration of 1.75 meter when running at 200 rpm. It is suddenly relieved of part of its load and its speed rises to 205 rpm in 1 sec. Find the unbalanced uniform turning moment.

(b) An Aluminium thin-walled tube (radius/thickness = 20) is closed at each end and pressurized by 6 MPa to cause plastic deformation. Neglect the elastic strain and find the plastic strain in the circumferential (hoop) direction of the tube. The plastic stress-strain curve is given by $\overline{\sigma} = 170(strain\ rate)^{0.25}$.

3.(a) A uniform ladder 5 m long and 14 kg mass is placed against a vertical wall at an angle 50^0 to the horizontal ground. The co-efficient of friction between ladder and wall is 0.2 and between ladder and ground in 0.5. Calculate how far up the ladder a man of 63 kg. can climb before the ladder shifts.

(b) Determine the diameter of a steel shaft rotating at an angular velocity of 300 rpm transmitting 500 HP. The allowable stress = 800 kg/cm^2. The allowable angle of twist = 0.5^0 per m, G = 8×10^5 kg/cm^2. What would be the savings if a hollow shaft is used to transmit the same power under the same condition, the ratio of diameters being 0.9?

4.(a) For the beam and loading shown in Fig.3, determine the equation defining the shear and bending moment at any point and at point D.

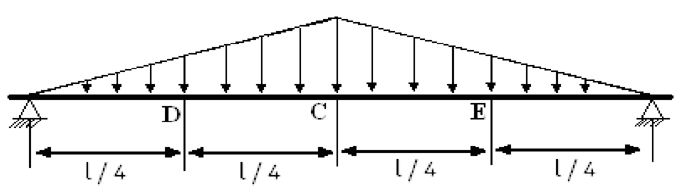


Fig. 3

(b) As illustrated in the given Fig.4 a metal punch (similar in principle to a paper punch) is used to punch holes in thin steel sheet that will be used to make a metal cabinet. To punch a 60 mm diameter disk or "slug" out of the sheet metal that is 10 mm thick requires a punch force of P=500 kg. Determine the average shear stress in the sheet metal resulting from the punching operation.

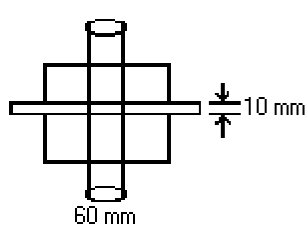


Fig. 4

5. (a) A tie rod in the suspension of a car is to be constructed from a grade of steel, which has 0.1% proof stress equal to 250 MN/m^2. The tie rod is to be constructed as a solid round bar of length 350 mm long. If the tie rod is subjected to a maximum axial force of 10 KN,

i) Determine the minimum diameter of the tie rod
ii) The extension of the tie rod under load (E= 2094 GN/m^2)
iii) The minimum diameter of the tie rod if a factor of safety of 2.5 is applied to the proof stress

(b) Find the width of the belt necessary to transmit 11.25 KW power to a pulley of diameter 300mm when the pulley makes 1600 rpm. Assume the co-efficient of friction between the belt and the pulley is 0.22 and angle of contact is 210^o. Maximum tension in the belt will not exceed 10N/mm width.

GROUP E-3: Electrical and Electronics Engineering

1.(a) A centrifugal pump, which is gear-driven by a DC motor, delivers 810 kg of water per minute to a tank of height 11 meter above the level of the pump. Draw the block diagram of the overall arrangement. Determine input power across the gearing and current taken by the motor operated at 220 volt provided the efficiency of the pump, gearing and motor respectively be 70%, 70% and 90% only. (Take g = 9.8 ms^{-2}).

(b) The rms value of a sinusoidal alternating voltage at a frequency of 50 Hz is 155volt. If at t = 0 it crosses the zero axis in a positive direction, determine the time taken to attain the first instantaneous value of 155 volt. How much time it takes to fall from the maximum peak value to its half? Explain with suitable waveform.

2.(a) On full-load unity power factor test, a meter having specification of 235 V and 5A makes 60 revolutions in 6 minutes, but its normal speed is 520 revolution/KWh. Does the meter have any inaccuracy? If so, find the percentage error.

(b) Write down the transfer function of the given system (as shown in the following figure) and find the values of K for which the system will be stable but under damped.

3(a) By intelligent selection of loop currents write down the mesh equations of the given circuit (as shown in Fig. 5) and determine the current flowing through that branch of the circuit containing capacitor. (All resistance/ reactance are in ohms).

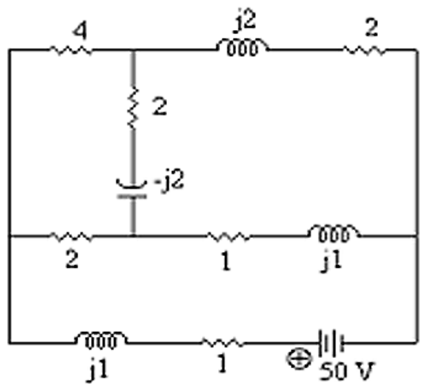


Fig. 5

(b)

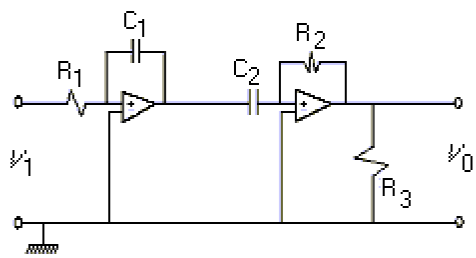


Fig. 6

Refer Fig. 6. Find the expression for V_0. What would be the nature of V_0 when $R_1 = R_2$ and $C_1 = C_2$? (Consider the Op-amps to be identical).

4. (a) A series ac circuit that resonates at 48 Hz consists of a coil (having R = 30 Ω and L = 500 mH) and a capacitor. If the supply voltage is 100 volt determine the value of the capacitor.

(b) Calculate the value of a capacitor which when connected across the circuit (as of Q. 4 (a) above), enhances the resonant frequency to 60 Hz. Compare the value of the source current in both the cases.

5 (a) A 200/400 - V, 10KVA, 50Hz single phase transformer has, at full load, a Cu loss of 120W. If it has an efficiency of 98% at full load unity power factor, determine the iron losses. What would be the efficiency of the transformer at half load 0.8 power factor lagging?

(b) In the 2-port network given below, the parameters at two parts are related by the equations,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

i) Find expressions for A, B, C and D
ii) Show that AD - BC = 1
iii) What are the physical interpretations of the above coefficients?

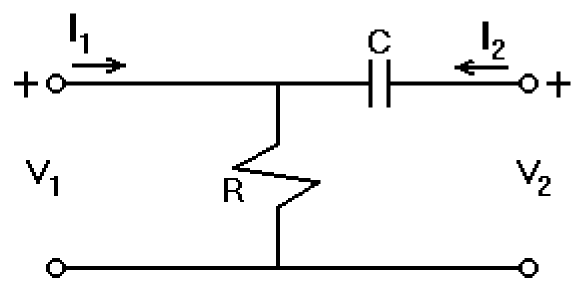


Fig 7

GROUP E-4: Thermodynamics

1 (a) In a thermodynamic system of a perfect gas, let $U = f(V,T)$ where U, V and T refer to internal energy, volume of a gram-molecule of the substance and temperature (in absolute scale) respectively. An amount of heat δQ is added so that the volume expands by δV against a pressure P. Prove that:

$$C_P - C_V = \left[P + \left(\frac{\delta U}{\delta V}\right)_T\right]\left(\frac{\delta V}{\delta T}\right)_P$$

where C_p and C_V stand for specific heat at constant pressure and specific heat at constant volume respectively.

(b) 0.15 cu.m. of air at a pressure of 1.06 kg/cm^2 is compressed to a volume of 0.008 cu.m. at 361 kg/cm^2. Calculate (i) the quantity of heat rejected, (ii) change in internal energy if the process of compression is a) Adiabatic b) Polytropic with $n = 1.3$.

2 (a) A compression ignition engine has a stroke of 28 cm and a cylinder diameter of 18 cm. The clearance volume is 475 cm^3. The fuel injection takes place at constant pressure for 4.5% of the stroke. Find the air standard efficiency of the engine assuming that it works on diesel cycle. If the fuel injection takes place at 10% of the stroke, find the loss in air standard efficiency.

(b) A diesel engine has a compression ratio 14 to 1 and the fuel supply is cut off at 0.08 of the stroke. If the relative efficiency is 0.52, estimate the weight of fuel of a calorific value 10400 k.cal per kg that would be required per horsepower.

3(a) If an ideal gas undergoes a reversible adiabatic process, trace the paths of the process on temperature versus volume diagram, pressure versus volume diagram and temperature versus pressure diagram subsequent to establishing the corresponding thermodynamic relations.

(b) If an ideal gas is changed from P_1, v_1, T_1 to P_2, v_2, T_2, show that the relation to estimate the entropy change without devising a reversible path is $\Delta S = C_P \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$.

4 (a) The approximated equation for adiabatic flow of super heated steam through a nozzle is given by pv^{γ} = constant. Show that

$$\frac{p_2}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma+1}$$

where p_1 = pressure of steam at entry; p_2 = pressure of steam at throat and p_2 / p_1 is the critical pressure ratio.

(b) The dry saturated steam is expanded in a nozzle from pressure of 10 bar to pressure of 4 bar. If the expansion is super saturated, find the degree of under cooling.

GROUP E-5: Engineering Properties of Metals

1. (a) Distinguish between modulus of rigidity and modulus of rupture. Give an expression for the modulus of rigidity in terms of the specimen geometry, torque, and angle of twist. Is the expression valid beyond the yield strength (torsion)?

   (b) A steel bar is subjected to a fluctuating axial load that varies from a maximum of 340 kN to a minimum of 120 kN compression. The mechanical properties of the steel are σ_u = 1090 MPa, σ_0 = 1010 MPa and σ_e = 510 MPa. Determine the bar diameter to give infinite fatigue life based on a safety factor of 2.5

2 (a) A cylindrical bar is subjected to a torsional moment M_T at one end. The twisting moment is resisted by shear stress μ set up in the cross section of the bar. The shear stress is zero at the centre of the bar and increases linearly with the radius. Find the maximum shear stress at the surface of the bar.

Given $J = \frac{\pi D^4}{32}$ (assuming that the torsional deformation is restricted within the zone of elasticity)

where, J : Polar moment of inertia
D : Diameter of cylinder.

(b) Consider a flat plane containing a crack of elliptical cross-section. The length of the crack is $2c$ and stress is perpendicular to the major axis of the ellipse. Show that

$$\sigma = \sqrt{\frac{2\gamma E}{\pi c}}$$

σ : stress
γ : surface energy
E : Young's modulus of elasticity

3. (a) Consider a tension specimen, which is subjected to a total strain ε at an elevated temperature where creep can occur. The total strain remains constant and the elastic strain decreases. Show that

$$\frac{1}{\sigma^{n-1}} = \frac{1}{\sigma_o^{\,n-1}} + BE(n-1)t$$

where,

$\varepsilon = \varepsilon_e + \varepsilon_p$	ε_e : elastic strain
$\varepsilon_e = \sigma / E$	ε_p : plastic strain
$\frac{d\varepsilon_p}{dt} = B\sigma^n$	t : time
$\sigma = \sigma_o$ at $t = 0$.	

(b) Distinguish between slip and twinning with diagrams.

4. (a) Suppose a crystalline material has *fcc* structure with atomic radius of 1.278 Å Determine the density of the crystalline material. Assume number of atoms per unit cell and molecular weight are n and M gm respectively.

   (b) Suppose there is an electron in an electric field of intensity 3200 volts/m. Estimate the force experienced by the electron. If it moves through a potential difference of 100 volts, find the kinetic energy acquired by the electron.

<u>Note</u>: A copy of typical QR Test Question paper is appended to the following pages for enabling the candidate to form an idea about the type of the question paper.

BOOKLET No. TEST CODE: QR

Afternoon

Time: 2 hours

Group	Questions: Total	Questions: To be answered	Maximum marks
Part I (for Statistics/Mathematics Stream)			
S1 (Statistics)	5	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM EACH GROUP.	120
S2 (Probability)	5		
Part II (for Engineering Stream)			
E1 (Mathematics)	3	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM E1	120
E2 (Engineering Mechanics)	2		
E3 (Electrical and Electronics Engineering)	2		
E4 (Thermodynamics)	2		
E5 (Engineering Properties of Metals)	2		

On the answer-booklet write your Name, Registration Number, Test Code, Number of this booklet, etc. in the appropriate places.

There are two parts in this booklet as detailed above. Candidates having Statistics background are required to answer questions from Part I as per instructions given. Those having engineering background are required to answer questions from Part II as per instructions given.

USE OF CALCULATORS IS NOT ALLOWED. SLIDE RULE MAY BE USED

STOP! WAIT FOR THE SIGNAL TO START

PART I (FOR STATISTICS / MATHEMATICS STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS, TAKING AT LEAST TWO [2] FROM EACH GROUP.

GROUP S1
Statistics

1. In a randomized block design (RBD), with b blocks and v treatments, the yields of treatments 1 and 2 in the first block are mixed up and their total yield t is only known. Estimate the mixed up yields. Set up an approximate ANOVA table using these estimates and following the procedure adopted with no missing observation.

[20]

2. If X and Y are standard normal variates with coefficient of correlation ρ,

(a) Obtain the regression equation of Y on X.

Show that

(b) $X+Y$ and $X-Y$ are independently distributed.

(c) $Q = \dfrac{X^2 - 2\rho XY + Y^2}{(1-\rho^2)}$ follows a χ^2 distribution with 2 d.f.

(d) The correlation coefficient between X^2 and Y^2 is ρ^2.

[5+5+5+5=20]

3. Consider a random sample $x_1, x_2, \cdots, x_n$ from the Cauchy distribution $f_\theta(x) = \dfrac{1}{\pi\left[1+(x-\theta)^2\right]}, -\infty < x < \infty$, where $0 < \theta < \infty$.

Show that the sample median $\tilde{X}$ is a consistent estimator of θ and that it has asymptotic efficiency $8/\pi^2$.

[20]

4. After the decision to take a simple random sample (SRSWOR), of size n from a population of size N, had been made, it was realized that y_1 would be unusually low and y_N would be unusually high. Consider the following estimator.

$$\hat{\bar{Y}}_s = \bar{y} + c, \text{ if the sample contains } y_1 \text{ but not } y_N$$
$$= \bar{y} - c, \text{ if the sample contains } y_N \text{ but not } y_1$$
$$= \bar{y}, \text{ for all other samples}$$

where c is a constant and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$.

Prove that

(a) $\hat{\bar{Y}}_s$ is unbiased for $\bar{y}$,

(b) $\text{Var}\left(\hat{\bar{Y}}_s\right) = (1-f)\left[\frac{S^2}{n} - \frac{2c}{(N-1)}\left(y_N - y_1 - nc\right)\right]$,

(c) $\text{Var}\left(\hat{\bar{Y}}_s\right) < \text{Var}\left(\bar{y}\right)$ if $0 < c < \left(y_N - y_1\right)/n$,

where $f = \frac{n}{N}$, $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $S^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$.

[5+10+5=20]

5. An experimenter wants to find the relationship between a dependent variable (Y) and a predictor variable (X). For simplicity, he has coded his predictor variable to lie between [-1, 1]; i.e. $-1 \le X \le 1$. Based on his domain knowledge, he feels a straight-line relationship is most likely between Y and X. However, a quadratic relationship might hold. He does not know σ^2, the variance of X. His budget permits him a maximum of 14 experiments. He consults his friends and comes out with the following alternative strategies:

Strategy A: Conduct 5 experiments at $X = -1$,
Conduct 2 experiments at $X = -1/3$,
Conduct 2 experiments at $X = +1/3$,
Conduct 5 experiments at $X = +1$.

Strategy B: Conduct 5 experiments at $X = -1$,
Conduct 4 experiments at $X = 0$,
Conduct 5 experiments at $X = +1$.

Strategy C: Conduct 6 experiments at $X = -1$,
Conduct 2 experiments at $X = 0$,
Conduct 6 experiments at $X = +1$.

As a statistician, which design would you advise the experimenter to use? Give reasons for your choice.

[20]

GROUP S2
Probability

6. a) A and B are two weak students of Statistics and their chances of solving a problem in Statistics correctly are $1/6$ and $1/8$ respectively. If the probability of their making a common error is $1/525$ and they obtain the same answer, find the probability that their answer is correct.

b) Imagine a game played by 5 people in which each flips a coin at the same time. If all but one of the coins comes up the same, the odd person wins. For example if there is 4 heads and a tail, then the person whose coin came up as 'tail' wins. If such a situation does not occur then the players flip again. What is the probability that the game is settled on the second toss?

[10+10=20]

7. a) A random variable X has a distribution with mean μ and variance σ^2. For c > 0, show that $E\left(\sqrt{X+c}\right) \approx \sqrt{\mu+c}\left(1-\frac{\sigma^2}{8(\mu+c)^2}\right)$, if the variations in X are sufficiently small compared to (μ+c). If X has a χ^2 distribution with a large degree of freedom ν, show that, as a first approximation, $Var\left(\sqrt{X+\nu}\right)=\frac{1}{4}$. [Hint: $(X+c)=(\mu+c)\left(1+\frac{X-\mu}{\mu+c}\right)$].

(b) Let X be a random variable which follows Poisson distribution with parameter m. Prove that,

i) $P\left(X \le \frac{m}{2}\right) \le \frac{4}{m}$

ii) $P(X \ge 2m) \le \frac{1}{m}$

[12+8=20]

8. (a) Two friends Tom and John have agreed to meet at Esplanade between 2 p.m. and 3 p.m. The first one to come waits for 20 minutes and then leaves. What is the probability of meeting between Tom and John if the arrival of each during the indicated hour can occur at random and the times of arrival are independent?

(b) If $X \sim Bin(n, p)$ and Y has negative binomial distribution with parameters r and p, prove that

$$F_X(r-1) = 1 - F_Y(n-r)$$

[10+10=20]

9. The joint p.m.f. of two random variables X and Y is $f(x,y) = e^{-2} \big/ \left[x!(y-x)! \right]$; $y = 0,1,2,\cdots,x$ and $x = 0,1,2,\cdots,y$. Find the moment generating function $M(t_1, t_2)$ of (X,Y) and the correlation coefficient between X and Y. Show that the marginal distributions of X and Y are Poisson.

[20]

10. (a) A couple has planned to celebrate their 25th wedding anniversary in Honeymooner's Paradise, a popular resort in Mussourie. Counting today as the first day, they are supposed to be there on the third and fourth day. They are thinking of buying a vacation insurance which promises to reimburse them for the entire vacation cost of Rs. 25000 if it rains on both days of their stay, and nothing is reimbursed otherwise. The insurance costs Rs. 1000. Suppose that the weather in Mussourie changes according to the following model:

The weather of Mussourie is classified as sunny (1), cloudy (2) and rainy (3). Assume that tomorrow's weather depends on today's weather only.

Let X_n = Weather condition of Mussourie on day n (n = 1, 2, …)

$$X_n = \begin{cases} 1 & \text{if it is sunny on day } n \\ 2 & \text{if it is cloudy on day } n \\ 3 & \text{if it is rainy on day } n \end{cases}$$

Assume that $\{X_n \mid n = 1,2,\cdots\}$ is a Markov Chain with transition probability matrix:

$$P = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.5 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.1 \end{bmatrix}$$

Assume that it is sunny today in Mussourie. Should the couple buy the insurance?

(b) Suppose that the times of successive failures of a machine form a Poisson process on $[0,\infty)$ with parameter $\lambda > 0$. Find the conditional probability of at least one failure by time $t+h$ given that there is no failure by time t.

[12+8=20]

PART II (FOR ENGINEERING STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS TAKING AT LEAST TWO [2] FROM E1.

GROUP E1
Mathematics

1. (a) Let A be a real square matrix of order 5 such that

$$a_{ij} = \begin{matrix} 1+x_i & if & i=j \\ 1 & if & i \neq j \end{matrix} \Big\} \quad \begin{matrix} i = 1 \text{ to } 5 \\ j = 1 \text{ to } 5 \end{matrix}$$

Show that, $\det(A) = \left(\prod_{i=1}^{5} x_i\right)\left(1+\sum_{i=1}^{5}\frac{1}{x_i}\right)$.

.

(b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\log 2$.

[10+10=20]

2. (a) If [x] denotes the integral part of x, then find the value of

$$\lim_{n\to\infty} \frac{[1^2 x]+[2^2 x]+\ldots+[n^2 x]}{n^3}.$$

(b) Let $f : R \to R$ be such that for all $x, y \in R$,

$$|f(x) - f(y)| \leq (x-y)^2.$$

Show that f is constant.

[10+10=20]

3. (a) Show that for $n>1$, $2^n < \binom{2n}{n} < 2^{2n}$.

(b) If a, b, c, d are positive real numbers, show that

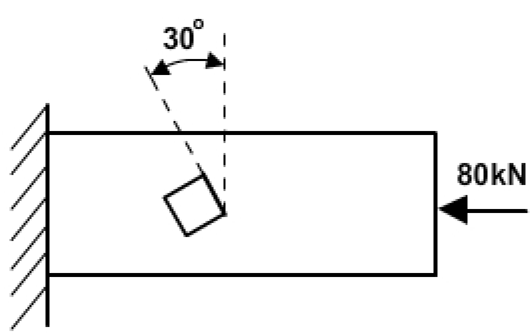
$$\frac{(a^2+1)(b^2+1)(c^2+1)(d^2+1)}{abcd} \geq 16.$$

(c) If α and β are any two irrational numbers then can α^β be rational?

[10+5+5=20]

GROUP E2
Engineering Mechanics

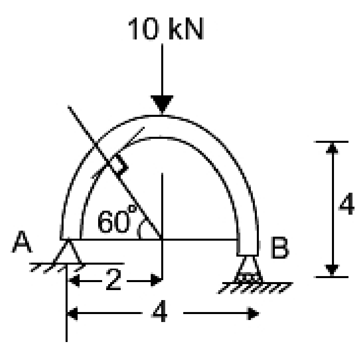
4. (a) A prismatic bar of sides 40 mm $\times$ 30 mm is axially loaded with a compressive force of 80 kN. Determine the stresses acting on an element which makes 30^o inclinations with the vertical plane. Also find the maximum shear stress value in the bar.



(b) Consider a long tube of 25mm outside diameter, d_o, and of 20mm inside diameter d_I, twisted about its longitudinal axis with a torque T of 45 N-m. Determine the shear stresses at the outside and inside of the tube.

[10+10=20]

5. (a) A parabolic beam is subjected to a 10 kN force as shown below. Find axial force, shear force and bending moment at the section, as shown below.



(b) A 10 kg box moving at 5 m/s on a horizontal, frictionless surface runs into a light spring of force constant 100 N/cm. Use the work-energy theorem to find the maximum compression of the spring.

[10+10=20]

GROUP E3
Electrical & Electronics Engineering

6. (a) Write the expression for emf induced in a dc machine, mentioning the names of factors/parameters used therein. State the factors on which electromagnetic torque developed in a dc machine depends.

(b) The following data were obtained when short-circuit (SC) test was performed on a 100 kVA, 2400/240V distribution transformer

$$E_{SC} = 72\text{V}, \quad I_{SC} = 41.6\text{A and } W = 1180 \text{ Watts.}$$

For the transformer, calculate the Cu-loss when the load is (i) 125 kVA and (ii) 85 kW at 0.772 power factor.

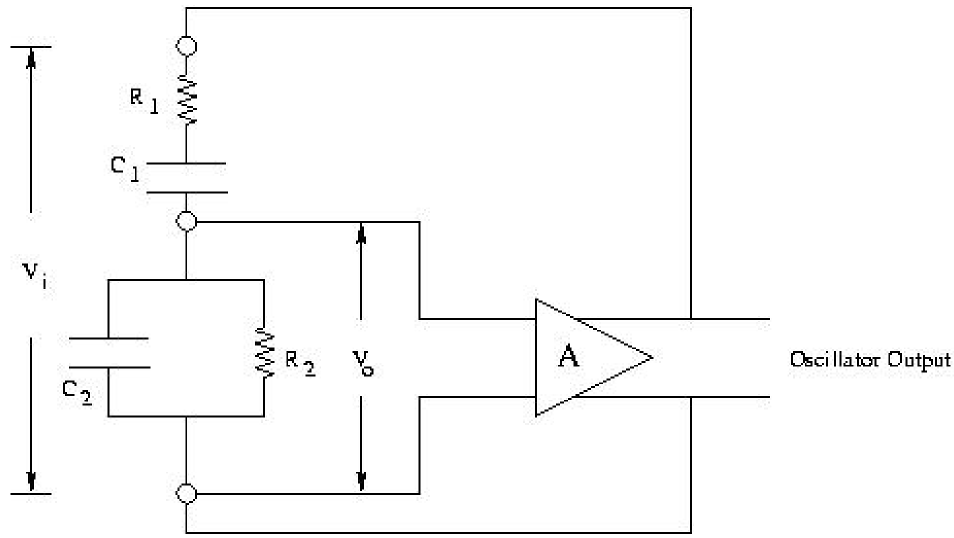
(c) Explain why an induction motor cannot run at synchronous speed. Draw the torque-slip characteristics of a three-phase induction motor. Show the amount of starting torque in the curve.

[6+8+6=20]

7. (a) Determine the necessary and sufficient conditions of variable parameters such that all roots of the cubic $s(s+2)(s+5)+k(s+\alpha)=0$ are in the Left Half of the Complex Plane.

Using Routh's criterion, determine the value of α in terms of k to make the system stable.

(b) The figure below shows the essential features of the Wien Bridge Oscillator Circuit. The amplifiers with high input impedance and low input impedance has a gain of A.



Wien Bridge Oscillator Circuit

Derive the transfer function of this network. Determine the frequency at which maximum gain occurs and find the value of this gain at $R_1 = R_2 = R \text{ and } C_1 = C_2 = C$. Also determine

i) the theoretical value of A to produce constant amplitude sinusoidal oscillation at the amplifier output.

ii) the frequency of oscillation as a function of R-C. Assume 180^o of phase shift in the amplifier.

[7+13=20]

GROUP E4
Thermodynamics

8. (a) The COP of a Carnot refrigerator can be increased either by decreasing the temperature of the high temperature reservoir, while the low temperature reservoir is held at constant temperature or by increasing the temperature of the low temperature reservoir while the high temperature reservoir is held at constant temperature. Determine which of the above two possibilities is more effective.

(b) Two identical blocks of mass *m* are available at temperatures T_1 and T_2. They can be used as source and sink to operate a heat engine. Assuming the specific heat of the blocks as *C*, show that the maximum amount of work that can be obtained is

$$W = mC\left(\sqrt{T_1} - \sqrt{T_2}\right)^2 .$$

[10+10=20]

9. (a) A mass *m* of liquid at temperature T_1 is mixed with an equal mass of the same liquid at temperature T_2 in an insulated container. Show that the total entropy change due to mixing is given by

$$\Delta S = 2mC_p \, ln\left[\frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}\right],$$

where C_p is the specific heat of the liquid.

(b) A four-stroke gas engine has a base of 20 cm and stroke of 30 cm at 300 rpm for every cycle. If the air fuel ratio and volumetric efficiency on NTP basis are 4:1 by volume and 80% respectively, then determine the volume of gas used per minute. If the calorific value of the gas is 8 mJ/m^3 at NTP and the brake thermal efficiency is 25%, determine brake power of the engine.

[10+10=20]

GROUP E5
Engineering Properties of Metals

10. (a) What is strain hardening? Prove that the strain hardening exponent is not the strain hardening rate.

(b) Will the necking behavior be different between a cylindrical tensile specimen and a tensile specimen with a rectangular cross section? What are the two types of tensile flow instability that occur during tensile deformation? In which of the above two types of tensile specimens, these two types of tensile instability will be observed and in what sequence?

(c) What is anelastic behavior in tension? How this is different from anisotropy of tensile properties in material?

(d) If true stress–strain curve is given by $\sigma = 1400\varepsilon^{0.35}$, what is the ultimate tensile strength of the material?

[5+6+4+5=20]

11. (a) What will be the maximum shear stress for a solid cylindrical test specimen put to torsion test and in which part of this specimen, the maximum shear stress will be observed? How this maximum shear stress will change for a tubular specimen?

(b) What is the name of the proportionality constant in the elastic region of deformation in torsion? How this constant is expressed in terms of the twisting moment and the angle of twist applied to the test specimen?

(c) How do you name the progressive deformation of material at constant stress or, at constant load? What are the various stages that this progressive deformation goes through and explain how the stages are different.

(d) How can you find the fatigue crack propagation rate? Can this rate be expressed in terms of stress and also in term of strain?

[5+4+6+5=20]