Test Code: PQB (Short Answer Type) 2014

M.Tech. in Quality, Reliability and Operations Research (Kolkata)

The candidates applying for M. Tech. in Quality, Reliability and Operations Research will have to take two tests: **Test MMA** (objective type) in the forenoon session and **Test PQB** (short answer type) in the afternoon session.

For Test **MMA**, see a different Booklet. For Test **PQB**, refer to this Booklet **ONLY.**

If you are from **Statistics / Mathematics Stream,** you will be required to **ANSWER PART I.**

If you are from **Engineering Stream**, you will be required to **ANSWER PART II.**

In **PART I**, a **TOTAL of TEN [10]** questions, are divided into **TWO Groups**: **S1**: **Statistics and S2: Probability – each group carrying FIVE [5]** questions. You will be required to answer a **TOTAL of SIX [6]** questions, taking **AT LEAST TWO [2]** from **each group**.

In **PART II**, there will be **FIVE Groups: E1 to E5**. **E1** will contain **THREE [3]** questions from **Mathematics** and each other group will contain **TWO [2]** questions from **Engineering and Technology**. You will be required to answer a total of **SIX [6]** questions taking **AT LEAST TWO [2]** from group **E1**.

Syllabus

PART I: STATISTICS / MATHEMATICS STREAM

Statistics (S1)

- Descriptive statistics for univariate, bivariate and multivariate data.
- Standard univariate probability distributions [Binomial, Poisson, Normal] and their fittings, properties of distributions. Sampling distributions.
- Theory of estimation and tests of statistical hypotheses.
- Simple and Multiple linear regression, linear statistical models, ANOVA.
- Principles of experimental designs and basic designs [CRD, RBD & LSD].
- Elements of non-parametric inference.
- Elements of sequential tests.
- Sample surveys – simple random sampling with and without replacement, stratified and cluster sampling.

Probability (S2)

- Classical definition of probability and standard results on operations with events, conditional probability and independence.
- Distributions of discrete type [Bernoulli, Binomial, Multinomial, Hypergeometric, Poisson, Geometric and Negative Binomial] and continuous type [Uniform, Exponential, Normal, Gamma, Beta] random variables and their moments.
- Bivariate distributions (with special emphasis on bivariate normal), marginal and conditional distributions, correlation and regression.
- Multivariate distributions, marginal and conditional distributions, regression, independence, partial and multiple correlations.
- Order statistics [including distributions of extreme values and of sample range for uniform and exponential distributions].
- Distributions of functions of random variables.
- Multivariate normal distribution [density, marginal and conditional distributions, regression].
- Weak law of large numbers, central limit theorem.
- Basics of Markov chains and Poisson processes.

PART II: ENGINEERING STREAM

Mathematics (E1)

- Elementary theory of equations, inequalities, permutation and combination, complex numbers and De Moivre's theorem.
- Elementary set theory, functions and relations, matrices, determinants, solutions of linear equations.
- Trigonometry [multiple and sub-multiple angles, inverse circular functions, identities, solutions of equations, properties of triangles].
- Coordinate geometry (two dimensions) [straight line, circle, parabola, ellipse and hyperbola], plane geometry, Mensuration.
- Sequences, series and their convergence and divergence, power series, limit and continuity of functions of one or more variables, differentiation and its applications, maxima and minima, integration, definite integrals areas using integrals, ordinary and partial differential equations (up to second order)

Engineering Mechanics (E2)

- Forces in plane and space, analysis of trusses, beams, columns, friction, principles of strength of materials, work-energy principle, moment of inertia, plane motion of rigid bodies, belt drivers, gearing.

Electrical and Electronics Engineering (E3)

- DC circuits, AC circuits (1-ϕ), energy and power relationships, Transformer, DC and AC machines, concepts of control theory and applications.
- Network analysis, 2 port network, transmission lines, elementary electronics (including amplifiers, oscillators, op-amp circuits), analog and digital electronic circuits.

Thermodynamics (E4)

- Laws of thermodynamics, internal energy, work and heat changes, reversible changes, adiabatic changes, heat of formation, combustion, reaction, solution and dilution, entropy and free energy and maximum work function, reversible cycle and its efficiency, principles of internal combustion engines. Principles of refrigeration.

Engineering Drawing (E5)

- Concept of projection, point projection, line projection, plan, elevation, sectional view (1st angle / 3rd angle) of simple mechanical objects, isometric view, dimensioning, sketch of machine parts.
  (Use of set square, compass and diagonal scale should suffice).

SAMPLE QUESTIONS

PART I: STATISTICS / MATHEMATICS STREAM

GROUP S1: Statistics

1. Suppose $x_1, x_2, \ldots, x_n$ constitute a random sample from a Bernoulli population with parameter p. Let

$$z_i = \begin{cases} 1 & \text{if } x_i \le x_0 \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2, \ldots\ldots n$.

(a) Find the probability distribution of z_i. Find $E(z_i)$, $V(z_i)$ and $Cov(z_i, z_j)$, $i \neq j$.

(b) Let $y = \sum_{i=1}^{n} z_i$. Find the probability distribution of y.

2. Let $[\{x_i\ ;\ i = 1, 2, \ldots, p\};\ \{y_j\ ;\ j = 1, 2, \ldots, q\};\ \{z_k\ ;\ k = 1, 2, \ldots, r\}]$ represent random samples from $N(\alpha + \beta, \sigma^2)$, $N(\beta + \gamma, \sigma^2)$ and $N(\gamma + \alpha, \sigma^2)$ populations respectively. The populations are to be treated as independent.

(a) Display the set of complete sufficient statistics for the parameters $(\alpha, \beta, \gamma, \sigma^2)$.

(b) Find unbiased estimator for β based on the sample means only. Is it unique?

(c) Show that the estimator in (b) is uncorrelated with all error functions.

(d) Suggest an unbiased estimator for σ^2 with maximum d.f.

(e) Suggest a test for H_0: $\beta = \beta_0$.

3.a) A neighborhood expresses frequent concerns about the dangers of a traffic intersection. Over the last two years there have been 16 accidents at this uncontrolled intersection. The municipality has finally responded and has put up stop signs on each of the four roads that enter the problematic intersection. After one year it was noted that there has been three accidents. From this data would you conclude that the stop signs have really helped in reducing the rate of accidents? Explain stating your assumptions clearly.

b) In a software development organization the project value (X) and productivity (Y) are known to be related. You have heard that the joint density of these two variables could probably be given by:

$$f(x, y) = \begin{cases} x \, exp\{-x\,(1+y)\}; & x, y > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Note that the value of the project can be well approximated before execution. The organization wants to develop a regression equation of Y on X to get an idea about the productivity on the basis of the value of a project. The organization wanted to develop the equation empirically and accordingly paired observations on (x, y) were collected for a number of completed projects. Do you think that the values of X and / or Y needs to be transformed before fitting the equation? If yes, what is the transformation? Explain.

4. A town has N taxis numbered 1 through N. A person standing on roadside notices the taxi numbers on n taxis that pass by. Let M_n be the largest number observed. Assuming independence of the taxi numbers and sampling with replacement, show that

$$\hat{N} = (n+1)\, M_n / n$$

is an approximately unbiased estimator of N for large N.

5.a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the rectangular population with density

$$f(x) = \begin{cases} \dfrac{1}{\theta}, & 0 < x < \theta \\ 0, & otherwise \end{cases}$$

Consider the critical region $x_{(n)} > 0.8$ for testing the hypothesis $H_0 : \theta = 1$, where $x_{(n)}$ is the largest of $x_1, x_2, \ldots, x_n$. What is the associated probability of type I error and what is the power function?

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population having p.d.f.

$$f(x, \theta) = \begin{cases} \dfrac{\theta^3}{\Gamma(3)} e^{-\theta x} x^2, & 0 < x < \infty \\ 0, & otherwise \end{cases}$$

Obtain the maximum likelihood estimate of θ and also obtain the Cramer Rao lower bound to the variance of an unbiased estimator of θ.

6. Suppose in a randomised block design for v treatments in b blocks, an observation under treatment i in block j is missing. Use the missing plot technique to give an "estimate" for the missing cell. Obtain $V(\hat{\tau}_1 - \hat{\tau}_2)$ where $(\hat{\tau}_1 - \hat{\tau}_2)$ is the BLUE of the elementary treatment contrast $(\tau_1 - \tau_2)$ of the effects of the treatments *1* and *2*. Also, give the analysis of variance for testing the equality of the treatment effects under the missing data situation.

7. If X_1, X_2, X_3 constitute a random sample from a Bernoulli population with mean p, show why $[X_1 + 2X_2 + 3X_3] / 6$ is *not* a sufficient statistic for p.

8. If X and Y follow a trinomial distribution with parameters n, θ_1 and θ_2, show that

   (a) $E(Y / X = x) = \dfrac{(n-x)\theta_2}{1-\theta_1}$,

   (b) $V(Y / X = x) = \dfrac{(n-x)\theta_2(1-\theta_1-\theta_2)}{(1-\theta_1)^2}$

9. Life distributions of two independent components of a machine are known to be exponential with means μ and λ respectively. The machine fails if at least one of the components fails. Compute the chance that the machine will fail due to the second component. Out of n independent prototypes of the machine m of them fail due to the second component. Show that $m / (n - m)$ approximately estimates the odds ratio $\theta = \lambda/\mu$.

GROUP S2: Probability

1. A boy goes to his school either by bus or on foot. If one day he goes to the school by bus, then the probability that he goes by bus the next day is 7/10. If one day he walks to the school, then the probability that he goes by bus the next day is 2/5.

   (a) Given that he walks to the school on a particular Tuesday, find the probability that he will go to the school by bus on Thursday of that week.

   (b) Given that the boy walks to the school on both Tuesday and Thursday of that week, find the probability that he will also walk to the school on Wednesday.

   [You may assume that the boy will not be absent from the school on Wednesday or Thursday of that week.]

2. Two absent-minded roommates forget their umbrellas in some way or the other. One roommate Mr. Singh always takes his umbrella whenever he goes out but has a tendency to forget about it and leave his umbrella wherever he visits about 30 % of the time.

   On the other hand, his roommate Mr. Parihar takes his umbrella only 50 % of the time but forgets about it 70 % of the time each place he visits.

   On a certain day, they go out together and after stopping at a bank, shopping at shoe store and dining in a restaurant, return home exhausted.

   (a) What is the probability that the very next day they discover only one umbrella at home?

   (b) What is the probability that Mr. Parihar has lost his umbrella in such a situation?

3. (a) A coin is tossed an odd number of times. If the probability of getting more heads than tails in these tosses is equal to the probability of getting more tails than heads then show that the coin is unbiased.

   (b) For successful operation of a machine, we need at least three components (out of five) to be in working phase. Their respective chances of failure are 7%, 4%, 2%, 8% and 12%. To start with, all the components are in working phase and the operation is initiated. Later it is observed that the machine has stopped but the first component is found to be in working phase. What is the likelihood that the second component is also in working phase?

(c) A lot contains 20 items in which there are 2 or 3 defective items with probabilities 0.4 and 0.6 respectively. Items are tested one by one from the lot unless all the defective items are tested. What is the probability that the testing procedure will continue up to the twelfth attempt?

4.(a) Let S and T be distributed independently as exponential with means $1/\lambda$ and $1/\mu$ respectively. Let $U = \min\{S, T\}$ and $V = \max\{S, T\}$. Find $E(U)$ and $E(U+V)$.

(b) Let X be a random variable with $U(0,1)$ distribution. Find the p.d.f. of the random variable $Y = X / (1 + X)$.

5. (a) Let U and V be independent and uniformly distributed random variables on [0, 1] and let θ_1 and θ_2 (both greater than 0) be constants.

Define $X = -\frac{1}{\theta_1}\ln U$ and $Y = -\frac{1}{\theta_2}\ln V$. Let $S = \min\{X, Y\}$, $T = max\{X, Y\}$ and $R = T - S$.

(i) Find $P[S=X]$.
(ii) Show that S and R are independent.

(b) A sequence of random variables $\{X_n \mid n = 1, 2, \ldots\}$ is called a *martingale* if

(i) $E(|X_n|) < \infty$
(ii) $E(X_{n+1} \mid X_1, X_2, \ldots, X_n) = X_n$ for all $n = 1, 2, \ldots$

Let $\{Z_n \mid n = 1, 2, \ldots\}$ be a sequence of iid random variables with $P[Z_n = 1] = p$ and $P[Z_n = -1] = q = 1-p$, $0 < p < 1$. Let $X_n = Z_1 + Z_2 + \ldots + Z_n$ for $n = 1, 2, \ldots$. Show that $\{X_n \mid n = 1, 2, \ldots\}$, is a martingale if and only if $p = q = ½$.

6. A manufacturer sells a bottle of mineral water at a fixed price of Rs.10. If the volume of water in the bottle is less than 800 ml then he is unable to sell it and it represents a total loss. The filled bottles have a normally distributed volume with mean μ ml and standard deviation 100 ml. The cost of filling per bottle is Rs. c, where $c = 0.002\mu + 1$. Determine the mean volume μ which will maximize the expected profit of the manufacturer. [Use $\sqrt{-\ln(0.0008\pi)} = 2.447$]

7. (a) In a recent study, a set of n randomly selected items is tested for presence of colour defect. Let A denote the event "colour defect is present" and B denote the event "test reveals the presence of colour defect". Suppose $P(A) = \alpha$, $P(B|A) = 1-\beta$ and $P(\text{Not } B| \text{ Not } A) = 1-\delta$, where $0 < \alpha, \beta, \delta < 1$. Let X be the number of items in the set with colour defects and Y be the number of items in the set detected having colour defects.

(i) Find $E(X | Y)$.

(ii) If the colour defect is very rare and the test is a very sophisticated one such that $\alpha = \beta = \delta = 10^{-9}$, then find the probability that an item detected as having colour defect is actually free from it.

(b) Consider the following bivariate density function

$$f(x,y) = \begin{cases} c.xy, & x > 0, y > 0, x + y < 1 \\ 0, & otherwise \end{cases}$$

(i) Find c.

(ii) Find the conditional expectation, $E(Y|X = x)$, for $0 < x < 1$.

8. Suppose in a big hotel there are N rooms with single occupancy and also suppose that there are N boarders. In a dinner party to celebrate the marriage anniversary of one of the boarders they start drinking alcohol to their hearts' content and as a consequence they become unable to identify their own rooms. What is the probability that at the end of the dinner party none of the boarders occupies the room originally assigned to them? What is the limiting value of this probability as $N \to \infty$?

9. (a) A company desires to operate s identical machines. These machines are subject to failure according to a given probability law. To replace these failed machines, the company orders new machines at the beginning of each week to make up the total s. It takes one week for each new order to be delivered.

Let X_n denote the number of machines in working order at the beginning of the nth week and Y_n denote the number of machines that fail during the nth week ($n = 1,2, \ldots$).

(i) Show that $X_{n+1} = s - Y_n$.

(ii) Show that $\{X_n | n = 1,2, \ldots\}$ constitutes a Markov Chain.

(iii) Assume that the probability distribution of Y_n given $X_n = i$ is given by:

$$P(Y_n = j \mid X_n = i) = \{1 / (i+1)\} \; ; \; j = 0,1,\ldots,i;$$

Obtain the transition probability matrix of $\{X_n | n = 1,2,\ldots\}$.

(b) Consider a Poisson process with parameter λ. Show that for $0 < s < t$, the conditional distribution of $N(s)$ given that $N(t) = n$ is Binomial with parameters n and (s/t), where $N(t)$ denotes the total number of occurrences of the Poisson event in $(0,t)$.

PART II: ENGINEERING STREAM

GROUP E1: Mathematics

1. (a) Let $f(x)$ be a polynomial in x and let a, b be two real numbers where $a \neq b$. Show that if $f(x)$ is divided by $(x - a)(x - b)$ then the remainder is

$$\frac{(x-a)f(b)-(x-b)f(a)}{b-a}.$$

(b) Find $\frac{dy}{dx}$ when $y=\left(x^{\log x}\right)\left(\log x\right)^{x}$, $x>1$

2. (a) A variable line through the point (a, b) cuts the axes of reference at A and B respectively. The lines through A and B parallel to the y-axis and x-axis respectively meet at P. Find the locus of P.

(b) Inside a square ABCD with sides of length 12 cm, segment AE is drawn where E is the point on DC such that DE = 5 cm. The perpendicular bisector of AE is drawn and it intersects AE, AD and BC at the points M, P and Q respectively. Find the ratio PM : MQ.

3.(a) Evaluate the value of $3.9^{1/2}.27^{1/4}.81^{1/8}\ldots\ldots\infty$.

(b) Let f be a twice differentiable function such that

$$f''(x) = -f(x);\ f'(x) = g(x) \text{ and } h(x) = f^2(x) + g^2(x).$$

Given that $h(5) = 11$, find $h(10)$.

4. (a) Show that

$$\lim_{n\to\infty} n^2\left\{\sqrt{\left(1-\cos\frac{1}{n}\right)\sqrt{\left(1-\cos\frac{1}{n}\right)\sqrt{\left(1-\cos\frac{1}{n}\right)\ldots\ldots\ldots to\ \infty}}}\right\}=\frac{1}{2}$$

(b) Test the convergence of the series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\cdots\infty$.

Assume $x > 0$ and examine all possibilities.

5.(a) Find the limit of the following function as $x \to 0$.

$$\frac{|x|}{\sqrt{\left(x^4+4x^2+7\right)}}\sin\left(\frac{1}{3\sqrt{x}}\right).$$

(b) If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$ then show that $a \cdot b < 0$.

6.(a) If ω is a complex cube root of unity then show that

$$a^3+b^3+c^3-3abc=(a+b+c)\left(a+b\omega+c\omega^2\right)\left(a+b\omega^2+c\omega\right).$$

(b) Show that $\left[\frac{\sum_{r>s} x^r}{r!}\right] \div \left[\frac{\sum_{r>s} y^r}{r!}\right] > \frac{x^s}{y^s}$, whenever $x > y > 0$.

7.(a) Cable of a suspension bridge hangs in the form of a parabola and is attached to the supporting pillars 200 m apart. The lowest point of the cable is 40 m below the point of suspension. Find the angle between the cable and the supporting pillars. State all the assumptions involved.

(b) Let A, B and C be the angles of a triangle with angle C as the smallest of them. Show that

(i) $\sin\left(\frac{C}{2}\right) \le \frac{1}{2}$

(ii) Hence, or otherwise, show that $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) < \frac{1}{4}$.

8.(a) Evaluate the following two integrals directly and compare them.

$$\iint_{ax^2+by^2\le 1} dxdy \quad \text{and} \quad \iint_{\sqrt{a}|x|\le 1,\sqrt{b}|y|\le 1} dxdy.$$

(b) Determine x, y and z so that the 3×3 matrix with the following row vectors is orthogonal : $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, $(1/\sqrt{2}, -1/\sqrt{2}, 0)$, (x, y, z).

GROUP E2: Engineering Mechanics

1.(a) A screw jack has a thread of 12 mm pitch. What effort needs to be applied the end of a handle of 450 mm to lift a load of 2.5 kN, if the corresponding efficiency is 50%?

(b) Derive the expression for moment of inertia I_{YY} of the shaded hollow rectangular section (Fig. 1).

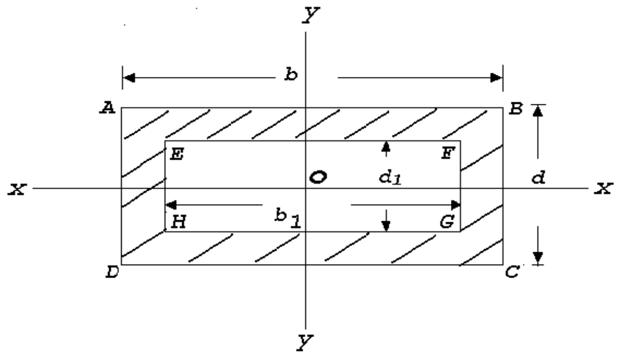


Fig. 1

(c) As illustrated in Fig.-2, a particle of weight *W* attached to a fixed point O by a string of length *L* whirls in a horizontal circular path of radius *r* with uniform speed *V* so that the string generates a height *h*. Show that the relation between *V*, *r*, *h* and the tensile force *T* in the string is

$$T = W\sqrt{\left(1+\frac{r}{h}\right)^2}$$

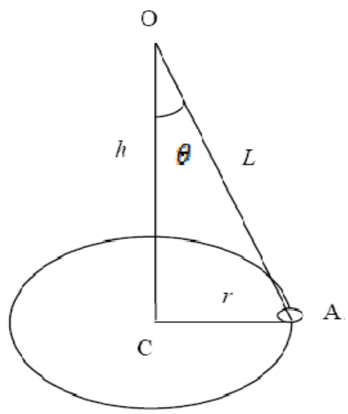


Fig. 2

2.(a) A turbine rotor weighs 20 tonnes and has a radius of gyration of 1.75 meter when running at 200 rpm. It is suddenly relieved of part of its load and its speed rises to 205 rpm in 1 sec. Find the unbalanced uniform turning moment.

(b) An Aluminium thin-walled tube (radius/thickness = 20) is closed at each end and pressurized by 6 MPa to cause plastic deformation. Neglect the elastic strain and find the plastic strain in the circumferential (hoop) direction of the tube. The plastic stress-strain curve is given by $\bar{\sigma} = 170(\textit{strain rate})^{0.25}$.

3.(a) A uniform ladder 5 m long and 14 kg mass is placed against a vertical wall at an angle 50^0 to the horizontal ground. The co-efficient of friction between ladder and wall is 0.2 and between ladder and ground in 0.5. Calculate how far up the ladder a man of 63 kg. can climb before the ladder shifts.

(b) Determine the diameter of a steel shaft rotating at an angular velocity of 300 rpm transmitting 500 HP. The allowable stress = 800 kg/cm^2. The allowable angle of twist = 0.5^0 per m, G = 8×10^5 kg/cm^2. What would be the savings if a hollow shaft is used to transmit the same power under the same condition, the ratio of diameters being 0.9?

4.(a) For the beam and loading shown in Fig.3, determine the equation defining the shear and bending moment at any point and at point D.

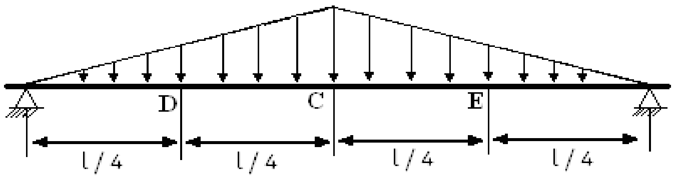


Fig. 3

(b) As illustrated in the given Fig.4 a metal punch (similar in principle to a paper punch) is used to punch holes in thin steel sheet that will be used to make a metal cabinet. To punch a 60 mm diameter disk or "slug" out of the sheet metal that is 10 mm thick requires a punch force of P=500 kg. Determine the average shear stress in the sheet metal resulting from the punching operation.

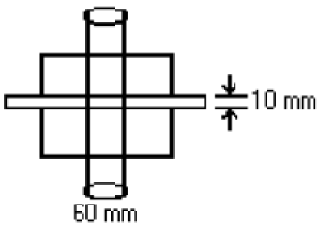


Fig. 4

5. (a) A tie rod in the suspension of a car is to be constructed from a grade of steel, which has 0.1% proof stress equal to 250 MN/m^2. The tie rod is to be constructed as a solid round bar of length 350 mm long. If the tie rod is subjected to a maximum axial force of 10 KN,

   (i) Determine the minimum diameter of the tie rod
   (ii) The extension of the tie rod under load (E= 2094 GN/m^2)
   (iii) The minimum diameter of the tie rod if a factor of safety of 2.5 is applied to the proof stress

   (b) Find the width of the belt necessary to transmit 11.25 KW power to a pulley of diameter 300mm when the pulley makes 1600 rpm. Assume the co-efficient of friction between the belt and the pulley is 0.22 and angle of contact is 210^0. Maximum tension in the belt will not exceed 10N/mm width.

GROUP E3: Electrical and Electronics Engineering

1.(a) On full-load unity power factor test, a meter having specification of 235 V and 5A makes 60 revolutions in 6 minutes, but its normal speed is 520 revolution/KWh. Does the meter have any inaccuracy? If so, find the percentage error.

(b) Write down the transfer function of the given system (as shown in the following figure-5) and find the values of K for which the system will be stable but under damped.

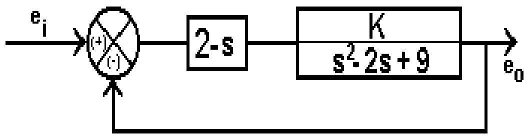


Fig - 5

2. (a) By intelligent selection of loop currents write down the mesh equations of the given circuit (as shown in Fig. 6) and determine the current flowing through that branch of the circuit containing capacitor. (All resistances/ reactance's are in ohms).

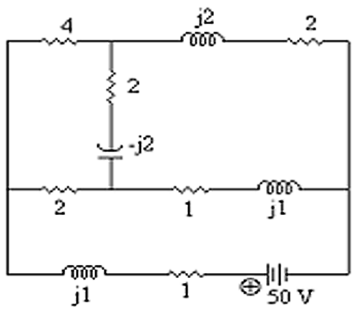


Fig. 6

(b)

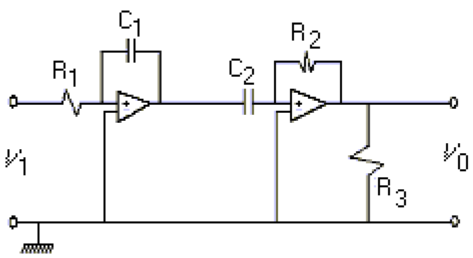


Fig. 7

Refer Fig. 7. Find the expression for V_0. What would be the nature of V_0 when $R_1 = R_2$ and $C_1 = C_2$? (Consider the Op-amps to be identical).

3. (a) A series ac circuit that resonates at 48 Hz consists of a coil (having R = 30 Ω and L = 500 mH) and a capacitor. If the supply voltage is 100 volt determine the value of the capacitor.

(b) Calculate the value of a capacitor which when connected across the circuit (as of Q. 4(a) above), enhances the resonant frequency to 60 Hz. Compare the value of the source current in both the cases.

(c) The rms value of a sinusoidal alternating voltage at a frequency of 50 Hz is 155volt. If at $t = 0$ it crosses the zero axis in a positive direction, determine the time taken to attain the first instantaneous value of 155 volt. How much time it takes to fall from the maximum peak value to its half? Explain with suitable waveform.

5. (a) A 200/400 - V, 10KVA, 50Hz single phase transformer has, at full load, a Cu loss of 120W. If it has an efficiency of 98% at full load unity power factor, determine the iron losses. What would be the efficiency of the transformer at half load 0.8 power factor lagging?

(b) In the 2-port network given below, the parameters at two parts are related by the equations,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

(i) Find expressions for A, B, C and D
(ii) Show that AD - BC = 1
(iii) What are the physical interpretations of the above coefficients?

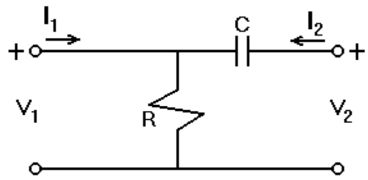


Fig. 8

GROUP E4: Thermodynamics

1.(a) In a thermodynamic system of a perfect gas, let $U = f(V,T)$ where U, V and T refer to internal energy, volume of a gram-molecule of the substance and temperature (in absolute scale) respectively. An amount of heat δQ is added so that the volume expands by δV against a pressure P. Prove that:

$$C_P - C_V = \left[P + \left(\frac{\delta U}{\delta V}\right)_T\right]\left(\frac{\delta V}{\delta T}\right)_P$$

where C_p and C_V stand for specific heat at constant pressure and specific heat at constant volume respectively.

(b) 0.15 cu.m. of air at a pressure of 1.06 kg/cm^2 is compressed to a volume of 0.008 cu.m. at 361 kg/cm^2. Calculate (i) the quantity of heat rejected, (ii) change in internal energy if the process of compression is a) Adiabatic b) Polytropic with $n = 1.3$.

2.(a) A compression ignition engine has a stroke of 28 cm and a cylinder diameter of 18 cm. The clearance volume is 475 cm^3. The fuel injection takes place at constant pressure for 4.5% of the stroke. Find the air standard efficiency of the engine assuming that it works on diesel cycle. If the fuel injection takes place at 10% of the stroke, find the loss in air standard efficiency.

(b) A diesel engine has a compression ratio 14 to 1 and the fuel supply is cut off at 0.08 of the stroke. If the relative efficiency is 0.52, estimate the weight of fuel of a calorific value 10400 k.cal per kg that would be required per horsepower.

3.(a) Calculate the change in entropy of saturated steam at a given pressure such that the boiling point = 152.6^{o}C and the latent heat at this temperature = 503.6 cal/gm. [Use $\text{Log}_e 1.56 = 0.445$.]

(b) Draw the pv and $T - \Phi$ diagrams for a diesel cycle in which 1 kg of air at 1 kg / cm^2 and 90 ^{0}C is compressed through a ratio of 14 to 1. Heat is then added until the volume is 1.7 times the volume at the end of compression, after which the air expands adiabatically to its original volume. Take $C_v = 0.169$ and $\gamma = 1.41$.

4.(a) The approximated equation for adiabatic flow of super heated steam through a nozzle is given by $pv^n = \text{constant.}$ Show that

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{n/n+1}$$

where $p_1 =$ pressure of steam at entry; $p_2 =$ pressure of steam at throat and p_2 / p_1 is the critical pressure ratio.

(b) The dry saturated steam is expanded in a nozzle from pressure of 10 bar to pressure of 4 bar. If the expansion is super saturated, find the degree of under cooling.

5. A mass of m_1 kg of a certain perfect gas at a temperature T_1 ^{0}K is mixed at constant pressure with m_2 kg of mass of the same gas at a temperature T_2 ^{0}K ($T_1 > T_2$). The system is thermally insulated. Find the change in entropy of the mixture and deduce the same for equal masses of the gas. Show that the change in entropy for equal masses of the gas is necessarily positive.

GROUP E5: Engineering Drawing

1.(a) A hollow cube of 5cm side is lying on HP and one of its vertical face is touching VP. A slim rod, to be taken as its solid diagonal, is placed within it. Draw top and front / side views of solid diagonal and, from the drawn figure determine its true length.

(b) Two balls are vertically erected to 18 cm and 30 cm respectively above the flat ground. These balls are away from a 3 cm thick wall (on the ground) by 12 cm and 21 cm respectively but on either side of the wall. The distance between the balls, measured along the ground and parallel to the wall is 27 cm. Determine their approximate distance.

2. (a) Sketch the profile of a square thread, knuckle thread and a white-worth thread showing all relevant dimensions in terms of the pitch.

(b) Sketch:

(i) single riveted lap joint,
(ii) double riveted lap joint chain-riveting,
(iii) double riveted lap joint zigzag-riveting, and
(iv) single cover single riveted butt joint.

3.(a) Draw the isometric view of an octahedron erected vertically up on one of its vertices. (Distinct free hand sketch only.)

(b) You are given two square prisms of same height of 10 cm. Prism A has side 7 cm and prism B has side of 5 cm respectively. Longer face of B is lying on H.P. with its base perpendicular to V.P. Base of A is lying on H.P. but equally inclined to V.P. You are instructed to remove by cutting a portion of bottom base of A so that within the cavity maximum of B may be placed accordingly. Note that vertical face of B may be parallel to V.P. but just touch the central axis of A. Draw the sectional view of the combination and determine the volume of material to be removed from A.

4. A parallelepiped of dimension 100×60×80 is truncated by a plane which passes through 85, 45 and 65 unit distance on the associated edges from the nearest top point of the object. Draw the isometric view of the truncated solid object. In third angle projection method, draw its plan. (All dimensions are in mm).

Note: A copy of one of the previous year's TEST CODE: PQB Question paper is appended to the following pages to give an idea to the candidate. Please note that for the year 2014, **Group E5: Engineering Properties of Metals** will not be under the syllabus.

2013

BOOKLET No. **TEST CODE: PQB**

Afternoon

Time: 2 hours

Group	Number of Questions: Total	Number of Questions: To be Answered	Maximum Marks
Part I (for Statistics/Mathematics Stream)			
S1 (Statistics)	5	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM EACH GROUP	120
S2 (Probability)	5		
Part II (for Engineering Stream)			
E1 (Mathematics)	3	A TOTAL OF SIX [6] TAKING AT LEAST TWO [2] FROM E1	120
E2 (Engineering Mechanics)	2		
E3 (Electrical and Electronics Engineering)	2		
E4 (Thermodynamics)	2		
E5 (Engineering Properties of Metals)	2		
E6 (Engineering Drawing)	2		

On the answer-booklet write your Name, Registration Number, Test Code, Number of this Booklet, etc. in the appropriate places.

There are two parts in this booklet as detailed above. Candidates having Statistics/Mathematics background are required to answer questions from Part I as per instructions given. Those having Engineering background are required to answer questions from Part II as per instructions given.

USE OF CALCULATORS IS NOT ALLOWED.

STOP! WAIT FOR THE SIGNAL TO START

PART I (FOR STATISTICS / MATHEMATICS STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS, TAKING AT LEAST TWO [2] FROM EACH GROUP. *(Note: Partial credit may be given for partially correct answer)*

GROUP S1
Statistics

1. Let $y_1, y_2, \ldots, y_6$ be uncorrelated random variables with common variance $\sigma^2 > 0$. Consider the following linear model, where β_0, β_1 and β_2 are unknown parameters.

$$E(y_1) = \beta_0$$
$$E(y_2) = E(y_3) = \beta_0 + \beta_1$$
$$E(y_4) = E(y_5) = E(y_6) = \beta_0 + \beta_1 + \beta_2$$

Find the least square estimators of the parameters β_0, β_1 and β_2. Obtain an expression for the residual (error) sum of squares and an estimator of σ^2. Also obtain the variances and covariances of the estimators of the parameters β_0, β_1 and β_2.

[10+6+4=20]

2. (a) A large IT service company recruits many students from campus. The students are trained and are subsequently allocated to three different business units A, B and C. About 2500 students are recruited each year and about 500 are allocated to unit A while about 1000 each are allocated to units B and C respectively as these units are about twice as big as unit A. Before allocation, the students have to go through a series of internal examinations and are subsequently placed in certain percentiles like ≤ 20, between 20 and 40, between 40 and 60, between 60 and 80, and > 80. The future performance of the students on the job is supposed to be highly correlated with the percentile where they are placed. The Human Resource department that makes the allotment tries to ensure that students are allocated fairly in the sense that each unit gets about the same percentage of people from each percentile block. However, due to many practical problems, the exact percentage varies and all units claim that they are getting the worst

people. In order to verify the claim of the units, the following allocation data were collected:

Percentile (X)	Unit	# Students Allocated
X ≤ 20	A	152
X ≤ 20	B	117
X ≤ 20	C	231
20 < X ≤ 40	A	108
20 < X ≤ 40	B	232
20 < X ≤ 40	C	160
40 < X ≤ 60	A	96
40 < X ≤ 60	B	247
40 < X ≤ 60	C	157
60 < X ≤ 80	A	106
60 < X ≤ 80	B	222
60 < X ≤ 80	C	172
X > 80	A	38
X > 80	B	182
X > 80	C	280

Examine the data given above and give your comments. Do you think that the claims made by the units are true? Is it true for a particular unit? Give reasons.

(b) Let R and s be the range and standard deviation of n observations $x_1, x_2, ..., x_n$. Show that, $\frac{R^2}{2n} \leq s^2 \leq \frac{R^2}{4}$.

[12+8=20]

3. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with p.d.f. $f(x, \theta) = e^{-(x-\theta)}, \theta < x < \infty; -\infty < \theta < \infty$. Obtain a sufficient statistic for θ.

(b) Given the probability density function

$$f(x, \theta) = \left[\frac{1}{\pi} \frac{1}{1+(x-\theta)^2}\right]; -\infty < x < \infty, \ -\infty < \theta < \infty$$

show that the Cramer-Rao lower bound on the variance of an unbiased estimator of θ is $\frac{2}{n}$, where n is the size of the random sample from this distribution.

[10+10=20]

4. From a population of size 3 an SRSWOR sample of size 2 is drawn. Consider the following estimator

$$\hat{Y}_{12} = 3\left(\frac{1}{2}y_1 + \frac{1}{2}y_2\right), \hat{Y}_{13}=3\left(\frac{1}{2}y_1 + \frac{2}{3}y_3\right), \hat{Y}_{23}=3\left(\frac{1}{2}y_2 + \frac{1}{3}y_3\right)$$

where $\hat{Y}_{ij}$ is an estimator of the population total Y based on the sample that has units i, j.

(a) Prove that $\hat{Y}_{ij}$ is an unbiased estimator of the population total Y.

(b) Obtain the sampling variance of $\hat{Y}_{ij}$.

(c) Hence or otherwise show that $Var(\hat{Y}_{ij}) < Var(3\bar{y})$ if $y_3(3y_2 - 3y_1 - y_3) > 0$, where $\bar{y}$ is the sample mean.

[5+9+6=20]

5. (a) Let $x_1, x_2, \ldots, x_n$ denote a random sample from a normal population $N(\theta, \sigma^2)$, where σ^2 is known. Using Neyman-Pearson Lemma find the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 > \theta_0$. Hence derive the expression for power of the test.

(b) In a cable manufacturing process, twinning is an important stage through which two insulated copper wires are twinned. Resistance is a very important quality characteristic of a cable wire necessitating its measurement before and after each stage of the manufacturing process. The concerned engineers perceive that the variance of the output resistance after twinning is twice the variance of the input resistance before twinning. Accordingly, they have set the specification of resistance before twinning and after twinning. To verify their perception, they have collected n number of observations (x_i, y_i), i = 1, 2, ..., n, where x_i represents the input resistance before twinning and y_i represents the output resistance after twinning corresponding to ith observation. Formulate a statistical test of hypothesis to verify the engineers' perception and suggest a suitable test procedure for the same. State the assumptions clearly.

[10+10=20]

GROUP S2
Probability

6. (a) A closet contains n pairs of shoes. If $2r$ shoes are taken at random ($2r < n$), what is the probability that the chosen shoes will contain no matching pairing?

   (b) In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices, where only one answer is correct. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copies, is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he answered it correctly.

[10+10=20]

7. (a) If X_1 and X_2 are independent rectangular variables on $[0, a]$ then find the distribution of $X_1 - X_2$.

   (b) Two points are selected randomly on a line segment of length a units. Find the probability that the distance between the points is at most d, where $0 < d < a$.

[15+5=20]

8. (a) How many times a fair coin must be tossed in order that the ratio of the observed number of heads to the number of tosses, lying between 0.4 and 0.6, will have a probability not less than 0.9?

   (b) $\{X_n\}$ is a sequence of independent random variables such that

$$P\left(X_n = \frac{1}{\sqrt{n}}\right) = p_n \text{ and } P\left(X_n = 1 + \frac{1}{\sqrt{n}}\right) = 1 - p_n$$

   Examine whether the weak law of large numbers is applicable to the sequence $\{X_n\}$.

[10+10=20]

9. (a) Given $f\ (x, y) = xe^{-x(y+1)}$; $x \geq 0,\ y \geq 0,$ find the regression curve of Y on X.

(b) Let X and Y be independent Poisson variables with parameters λ and μ respectively. Show that the conditional distribution of X given $X+Y$ is Binomial. If X and Y are independent Binomial variables with parameters (n, p) and (m, p) respectively, what will be the conditional distribution of X given $X+Y$?

[12+8=20]

10. (a) A factory has two machines and one repairman. Only one machine is used at any given time. Assume that a machine breaks down only at the end of a day with probability p. The repairman can work on only one machine at a time. When a machine breaks down at the end of the previous day, the repair can be completed in 1 day or 2 days with probability r and $(1 - r)$ respectively. All repairs are completed at the end of the day and no repair takes more than two days. Assume that all breakdowns and repairs are independent events and the factory works on all days.

(i) Model this process as a Markov Chain, i.e., identify the states and compute the transition probability matrix.
(ii) In what percentage of days none of the machines will be available for production?

(b) Let $X_{(1)}, X_{(2)}$ and $X_{(3)}$ be the order statistics of iid random variables X_1, X_2 and X_3 with common pdf

$$f(x) = \begin{cases} \beta e^{-x\beta}, & x > 0 \\ 0 & otherwise \end{cases}$$

Let $Y_1 = X_{(3)} - X_{(2)}$ and $Y_2 = X_{(2)}$. Show that Y_1 and Y_2 are independent.

[12+8=20]

PART II (FOR ENGINEERING STREAM)

ATTENTION: ANSWER A TOTAL OF SIX [6] QUESTIONS TAKING AT LEAST TWO [2] FROM E1. *(Note: Partial credit may be given for partially correct answer)*

GROUP E1
Mathematics

1. (a) If A be a skew-symmetric matrix, i.e., $A = -A^T$ and $(I + A)$ be a non-singular matrix, then show that $B = (I - A)(I + A)^{-1}$ is orthogonal.

   (b) Consider the system of linear equations $Ax = b$ of which

$$A = \begin{bmatrix} 1 & 5 & -3 \\ -1 & -4 & 1 \\ -2 & -7 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} -4 \\ 3 \\ k \end{bmatrix}$$

   Find the value of k with justification so that the system has many solutions, where k is a real number.

   (c) For a fixed positive integer n, if $D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$

   then show that $\dfrac{D}{(n!)^3} - 4$ is divisible by n.

[5+7+8=20]

2. (a) If $x^2 - x + 1 = 0$ then find the value of $\sum_{n=1}^{5} \left(x^n + \dfrac{1}{x^n} \right)^2$.

   (b) Let $f(x) = e^x \sin x$ be the equation of a curve. If at $x = a$, $0 \le a \le 2\pi$, the slope of the tangent is the maximum, find the value of a.

(c) The rate of cooling of a substance in moving air is proportional to the difference of temperature of the substance and the air. A substance cools from 36^0C to 34^0C in 15 minutes. When will the substance reach the temperature of 32^0C, given that the constant temperature of air is 30^0C?

[8+5+7=20]

3. (a) Show that the series $\frac{x}{1}+\frac{1}{2}\frac{x^3}{3}+\frac{1.3}{2.4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+\cdots\cdots \quad (x>0)$

is convergent when $x^2 \leq 1$ and divergent when $x^2 > 1$.

(b) Show that $\int_0^9 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}dx = 9$

(c) Evaluate $\int \frac{1}{e^x+1}dx$

[10+6+4=20]

GROUP E2
Engineering Mechanics

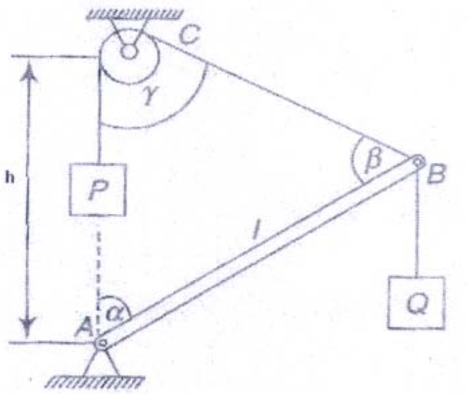
4. (a) Consider four points P, Q, R and S on a horizontal rod PS of 12m length, where PQ = QR = RS = 4m. Forces of 1000, 1500, 1000 and 500 N act at P, Q, R and S respectively with downward direction. The lines of action of these forces make angles of 90^0, 60^0, 45^0, and 30^0 respectively with PS. Find the magnitude, direction and position of the resultant force.

(b) Two parallel shafts, about 600 mm apart, are to be connected by spur gears. One shaft is to run at 360 rpm and the other at 120 rpm. Design the gears, if the pitch of the teeth is to be 25 mm.

[10+10=20]

5. (a) A prismatic bar AB of negligible weight and length *l* is hinged at A and supported by a string that passes over a pulley C and carries a load P at its free end. Assuming that the distance *h* between the hinge A and

pulley C is equal to the length *l* of the base, find the angle α at which the system will be in equilibrium.



(b) A 'V' belt pulley is used to transmit motion. The angle of the 'V' groove is α and angle of wrap is β. If the tension in the belt on the tight side is T_1 and that on the slack side is T_2, what is the relation between T_1 and T_2 for impending slippage?

[10+10=20]

GROUP E3
Electrical & Electronics Engineering

6. (a) A balanced star-connected load of (8+j6)Ω per phase is connected to a balanced three-phase 400 Volt supply. Find the line current, power factor, power and total Volt-Ampere.

(b) The armature supply voltage of a dc motor is 230 V. The armature current is 12 A, the armature resistance is 0.8Ω and the speed is 100 radian/sec. Calculate the (i) induced emf (ii) electromagnetic torque (iii) electrical power input to the armature (iv) mechanical power developed by the armature and (v) the armature copper loss.

(c) A three-phase, 50 Hz, 4-pole induction motor runs at a slip of 4 per cent at full-load. Calculate the rotor speed.

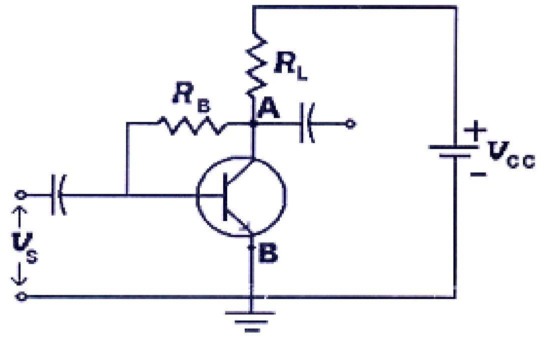
[6+10+4=20]

7. (a) An *npn* transistor shown in the following figure is used in common-emitter amplifier mode with $\beta = 49$, $V_{cc} = 10$V, and $R_L = 2$kΩ. If a 100 kΩ resistor R_B is connected between the collector and the base of the transistor, then calculate

(i) the quiescent collector current, and

(ii) the collector to emitter voltage drop between points A and B.

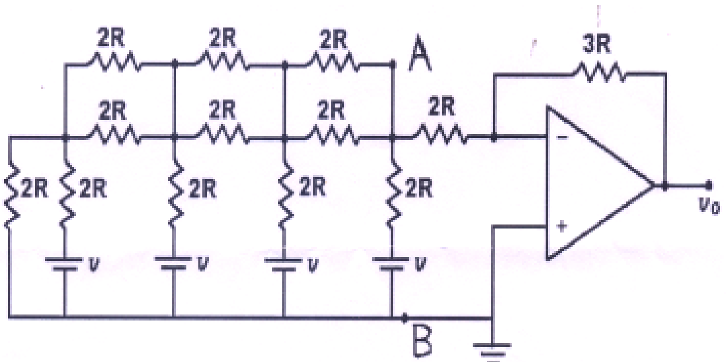
Assume base to emitter voltage drop is 0.7 volts.



(b) Consider the circuit shown in the figure below.

(i) Find the Thevenin equivalent for the network to the left of terminals AB and

(ii) Find the output voltage v_0 using the Thevenin's voltage across terminals AB.



[(4+4)+(8+4)=20]

GROUP E4
Thermodynamics

8. (a) Which is the more effective way to increase the efficiency of a Carnot engine out of the following two situations?

    (i) to increase the source temperature T_1, keeping T_2, the sink temperature constant,

    (ii) to decrease the sink temperature T_2, keeping T_1, the source temperature constant.

    (b) Show that for an adiabatic or isentropic process, the work done W_{1-2} by m kg of a gas during expansion from State 1 to State 2 is

$$W_{1-2} = \frac{1}{1-\gamma} m R (T_2 - T_1)$$

where γ = heat capacity ratio
R = the gas constant
T_1 = initial temperature of the gas
T_2 = final temperature of the gas

    (c) Two identical bodies of constant heat capacity are at the same initial temperature T_i. A refrigerator operates between these two bodies until one body is cooled to temperature T_2. If the bodies remain at constant pressure and undergo no change of phase, show that the minimum amount of work done is

$$W(\min) = C_p \left(\frac{T_i^2}{T_2} + T_2 - 2T_i \right)$$

[6+4+10=20]

9. (a) Compare among the air standard efficiencies of Otto cycle and Diesel cycle under the following conditions:

    (i) the compression ratio and heat rejections are same in two cycles

    (ii) the peak pressure, peak temperature and heat rejections are same in two cycles.

    (b) The specific heat of an ideal gas at constant pressure is given by $C_p = a + bT + cT^2$ kJ/kg K where a = 0.85 kJ/kg K, b = 0.00004 kJ/kg

K^2, c = 0.00005 kJ/kg K^3 and T is in Kelvin. Calculate the change in internal energy and enthalpy of 1 kg of gas in raising its temperature from 300 K to 2300 K. Take the ratio of specific heats as 1.5.

(c) A four-stroke petrol engine has to produce 75 kW output at a brake thermal efficiency of 20%. It uses a fuel-air ratio of 0.075:1. If the calorific value of fuel is 45000 kJ/kg, density of air is 1.2 kg/m^3, and density of fuel vapour is four times to that of air, determine

(i) the required mass of air that will be consumed
(ii) the volume of air and fuel that will be consumed
(iii) the volume of the required mixture.

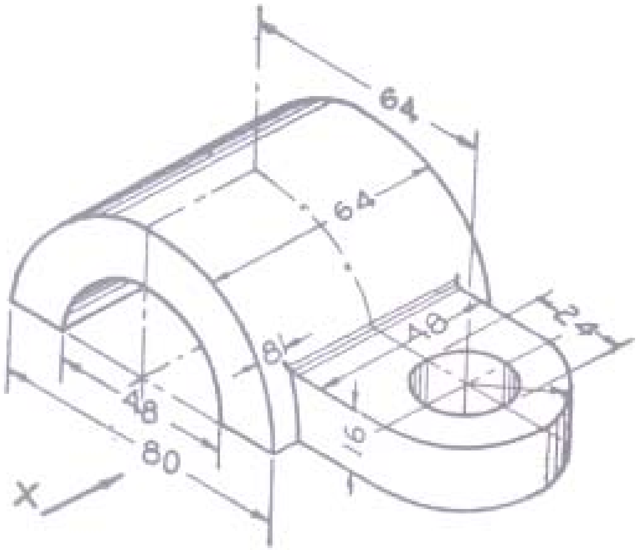
[9+5+6=20]

GROUP E5
Engineering Properties of Metals

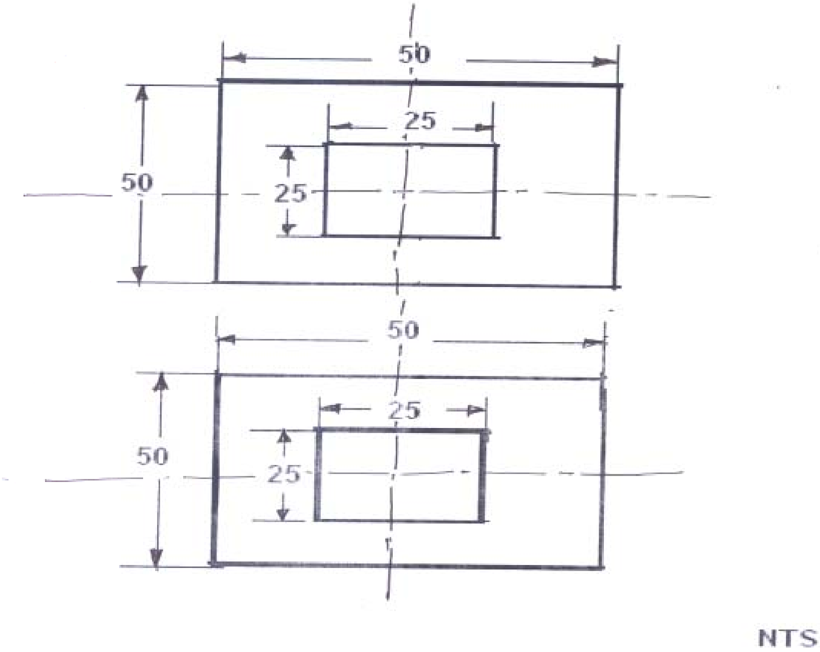
Note: *Engineering Properties of Metals* has been dropped from the 2014 syllabus and consequently the questions of this section have been excluded.

GROUP E6
Engineering Drawing

12. (a) Sketch the (i) front view (ii) side view from the right and (iii) top view of the object given below.



(b) Two views of an object are given below. Develop the third view.



[10+10=20]

13. Sketch two views of a CI flat pulley. The pulley has four arms with elliptical cross section. Show the revolved section of an arm. Given that

External diameter of the pulley = 250 mm
Face width of the pulley = 100 mm
Bore of the pulley = 30 mm
Boss diameter of the pulley = 43 mm
Length of the boss = 65 mm
Rim thickness of the pulley = 4 mm
Thickness of arm near boss = 28 mm
Thickness of arm near rim = 18 mm
Crown height of the pulley = 15 mm per meter of face width.

Assume all other relevant data.

[20]