

Syllabus and Sample questions for QEA 2014

Syllabus

1. Permutations and combinations.
2. Elementary set theory; Functions and relations; Matrices.
3. Functions of one and several variables: limits, continuity, differentiation, applications, integration of elementary functions, definite integrals, theory of quadratic equations.
4. Constrained and unconstrained optimization, convexity of sets and concavity and convexity of functions.
5. Elements of probability theory, discrete and continuous random variables, expectation and variance, joint conditional and marginal distributions, distributions of functions of random variable.

Sample questions are given in the next two pages.

- 1) If $[x]$ stands for the greatest integer less than or equal to x and $|x|$ is the absolute value of x , then find the value of the integral $\int_{-2}^2 ([x] + |x|) dx$.
- 2) Let X and Y be disjoint sets containing p and q elements respectively, and let the set Z be defined as $Z = XUY$. Find the number of subsets S of Z that contains m elements and also has the property that $S \cap X$ has n elements.
- 3) Let A be the set of all functions $f: \mathbf{R} \rightarrow \mathbf{R}$ such that $f(xy) = xf(y)$ for all $x, y \in \mathbf{R}$. If $f \in A$, then show that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbf{R}$.
- 4) Find

$$\lim_{x \rightarrow 2} \frac{e^{x^2} - e^{2x}}{(x-2)e^{2x}}.$$

- 5) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

- 6) If $|a| < 1$ and $|b| < 1$, then show that the series $a(a+b) + a^2(a^2+b^2) + a^3(a^3+b^3) + \dots$ converges to $\frac{a^2}{1-a^2} + \frac{ab}{1-ab}$.

- 7) Consider the function $f(x) = x^2 - 2x + 2 + \frac{4}{x^2 - 2x + 2}$, $x \in \mathbf{R}$. Then show that $f(x) - 5 = 0$ has exactly three solutions.

- 8) Find the number of maps f from the set $\{1,2,3\}$ into the set $\{1,2,3,4,5\}$ such that (i) $f(1) \neq f(3)$ and (ii) $f(i) \leq f(j)$ whenever $i < j$.
- 9) Find the maximum and minimum values of $(x^2 + y^2)$ subject to the restrictions $|x| + |y| \leq 1$.
- 10) A and B are two independent witnesses (i.e., there is no collusion) in a judicial case. The probability that A will speak the truth is p and the probability that B will speak the truth is q . Now, A and B have agreed in a certain statement. Then find the probability that both have spoken the truth.