!!! IMPORTANAT NOTE ON NEW SYLLABUS 2009!!!

<u>Prior to 2009 tests, questions from applied mathematics and advanced demography were not asked.</u>

<u>Demography Test 2009 will be on the following new pattern as mentioned below.</u>

There will be two tests RD-1 (Forenoon) and RD-2 (Afternoon). Topics to be covered in these tests along with an outline of the syllabus are given below:

Broad subjects for RD-I and RD-II: Mathematics & Applied Mathematics, Demography and Statistics. (There will be equal number of questions from applied mathematics, demography and statistics).

Outline of Syllabus for Mathematics and Applied Mathematics

Real analysis: Basic number systems, Completeness, Sequence and series, Continuity and differentiability of real values functions of one and several variables, applications, Uniform convergence and The Reimann integral.

Linear algebra: Vector spaces, Linear transformations, matices, Determinants, Characteristic roots, System of linear equations.

Complex analysis: Analytic functions, Cauchy’s theorem and Cauchy Integral Formula, Laurent Series, Singularities, Theory of residues, Contour integration

Ordinary differential equations: First order ODE and their solutions, Initial value and boundary value problems, General theory of homogeneous and nonhomogenous linear differential equations, Second order ODE and their solutions, Stability of systems.

Partial Differential equations: First order and second order PDE, Equations of parabolic, hyperbolic, and elliptic type.

Outline of Syllabus for Demography

Life Tables: Complete and abridged life tables in both discrete and continuous versions, Relation between mortality rates and probability of deaths, Applications of life tables.

Stable population models: Lotka's intrinsic growth models, stable and stationery populations, population dynamics, Equilibrium points.

Population models: Exponential, Logistic growth models in population, Birth process, birth-death process and birth-death-migration process.

Population Projection: Life table methods and Difference equations methods in projections, component method of projections, Construction of survival matrices.

Outline of Syllabus for Statistics

See Statistics part of the JRF in Statistics (RS-I & RS-II) available online.

Year 2009

SAMPLE

BOOKLET No. **TEST CODE: RD I**

Forenoon

Sample Questions
Time: 2 hours

Note: Answer all questions. Full credit will be given for complete and rigorous arguments.

RD I

Answer as many questions as you can.

1. Let $h(x) = \sum_{n=1}^{\infty} \frac{1}{x^2 + n^2}$.. Show that h. is continuous function defined on all of the real line. If h is differentiable then is the derivative function h' continuous ?

2. Given a function f on $[a,b]$, define the *total variation* of f to be

$$Vf = \sup\left\{ \sum_{k=1}^{n} |f(x_k) - f(x_{k-1})| \right\},$$ where the supr emum is taken over all partitions P of $[a,b]$. If f is continuously differentiable then show that $Vf \leq \int_a^b |f'|$.

3. Let X_1, X_2 and X_3 be independently and identically distributed random variables taking values 1 and -1 with probability p and 1−p, respectively. Write $Y_1 = X_1X_2$, $Y_2 = X_2X_3$ and $Y_3 = X_1X_3$. Derive the marginal distributions of Y_1, Y_2 and Y_3. Prove that, for $i \neq j$, Y_i and Y_j are not independent, when $p \neq 0.5$.

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4. Consider a population made up of three different categories with corresponding proportions θ^2, $2\theta(1-\theta)$ and $(1-\theta)^2$. Derive the maximum likelihood estimate of the ratio $\theta/(1-\theta)$ based on a random sample of n individuals of whom n_1, n_2 and n_3 fall into the three categories, respectively.

5. Observations $x_1, \ldots, x_n$ are drawn from normal populations with the same mean μ but with different standard deviations $\sigma_1, \ldots, \sigma_n$ which are known. What is the maximum likelihood estimate of μ? Is it unbiased?

6. A continuous random variable (r.v.) X has the distribution function F(x) proportional to $\alpha x^{\beta} - \beta x^{\alpha}, \alpha > \beta \geq 1, 0 \leq X < 1$. Obtain the r^{th} raw moment of X.

7. The r.v. X has a probability distribution defined by the function $f(x) = \frac{1}{\pi(1+x^2)}$. ,

$-\infty < x < \infty$

Let Y=1 if $X \geq 1$

= 0 if $X < 1$.

Find the distribution of Y.

8. A population grows as $P(t) = (\alpha/(t_0 - t))^2$ where $t \leq t_0$ and $\alpha > 0$. What is the rate of increase at time t? What does this say about the growth rate as $t \rightarrow t_0$?

9. Let l_x and d_x represent number of persons and deaths at age x in a life table. Write d_{x+t}, for $0<t<1$, as the number of deaths between ages (x+t) and (x+t+1). Then show that

$d_{x+t} = (1-t)\, d_x + t\, d_{x+1}$. Assume uniform distribution of deaths.

10. Prove that projection of P_1 at exponential rate r_1 added to projection of P_2 at exponential rate r_2 gives a higher result than projection of $P_1 + P_2$ at exponential rate

$r = (P_1 r_1 + P_2 r_2)/(P_1 + P_2)$.

11. The proportion of individuals between ages x to x+dx in a stable population is $be^{-rx}\, s(x)dx$, where b is a positive constant, r is the rate of increase, s(x) is the probability of survival upto age x. Suppose a stable population is increasing at 1 percent per year, and with all individuals subject to a death rate of 1.5 percent uniformly at all ages. What fraction will be over 50 years of age?

YEAR 2009

SAMPLE

BOOKLET No. **TEST CODE: RD II**

Afternoon

Sample Questions
Time: 2 hours

Note: Answer all questions. Full credit will be given for complete and rigorous arguments.

RD II

Answer as many questions as you can. Questions are of 10 marks each.

1. Show that the differential equation $2xz + q^2 = x(xp + yq)$ has a complete integral $z + a^2 x = axy + bx^2$, and deduce that $x(y + hx)^2 = 4(z - kx^2)$ is also a complete solution.

2. Show that $v(x, y; \alpha, \beta) = \dfrac{(x + y)[2xy + (\alpha - \beta)(x - y) + 2\alpha\beta]}{(\alpha + \beta)^3}$ is the Riemann function for the second order PDE $u_{xy} + \dfrac{2}{x + y}\left(u_x + u_y\right) = 0.$

3. By solving the differential equation $y' = 1 + y^2 = 0$ show that no solution through the origin exists for $-\infty < t < \infty.$

4. Write down the joint probability mass function of (X, Y) having a trinomial distribution giving details of the observations and parameter ranges.
   (a) Derive the marginal distribution of Y.
   (b) Derive the conditional distribution of X,
   given Y = y.

5. Consider the linear model

$Y_i = \rho\left(\frac{i}{n}\right) + \varepsilon_i \quad i = 1, \cdots, n,$

where ρ is the unknown regression parameter of interest and ε_i's are independent error variables satisfying $E(\varepsilon_i) = 0$ and $V(\varepsilon_i) = \sigma^2$ (unknown). Find out the least square estimator of ρ and prove that it is the Best Linear Unbiased Estimator (BLUE), that is, it has the smallest variance among all the linear (in Y_i's) unbiased estimators.

6. Of a total population of animals of size K, "*a*" are captured at random marked and released. On a second occasion n are captured of which "*r*" are found to be marked. If n is small compared with K, then find the maximum likelihood estimator of K.

7. If the force of mortality $\mu(x) = A + B c^x$, prove that de_x^0/dA does not involve A, B or c (symbols have their usual significance).

8. Consider a closed population P(t) where $\frac{dP(t)}{dt} = 0.01\, P(t) - 5 \times 10^{-9}[P(t)]^2$. Given that the population starts at 1,000,000, obtain the population size at time t. What is the ultimate upper limit on its size? Does it depend on the starting population? [Hint: You may either use a transformation of the type Q(t)= (a/P(t))-1, where a is a constant , or use the variable separation method]

9. a) The proportion of individuals between ages x to x+dx in a stable population is $be^{-rx}\, s(x)\, dx$, where b is a positive constant, r is the rate of increase, s(x) is the probability of survival upto age x. All the parameters are gender independent. How fast does the community have to be increasing so that each man of age 40 has two women of age 20 to marry.
   b) In a stable population, the male and the female death rates are constant but different, the rate of increase for the populations of two sexes is the same, and the values of the constant b for male and female are in the ratio k. Find the sex ratio for persons aged x in the population.

10. a) Set down an equation showing the number of births in one generation in terms of the number of births in the preceding generation, age specific rates of birth and death being given and fixed. Confine the model to females.
    b) Solve the equation of the above problem for the implied rate of increase on the supposition that the trajectory is an exponential starting with B_0 at time zero.