INDIAN STATISTICAL INSTITUITE

STUDENTS' BROCHURE

MASTER OF MATHEMATICS

203, BARRACKPORE TRUNK ROAD
KOLKATA 700108

INDIAN STATISTICAL INSTITUTE

STUDENTS' BROCHURE

M.MATH. PROGRAMME

1. **DETAILED COURSE STRUCTURE**

1.1 M. Math. Curriculum

First Year

Semester I

1. Analysis of Several Variables
2. Topology I
3. Linear Algebra
4. Algebra I
5. Measure Theoretic Probability

Semester II

1. Complex Analysis
2. Functional Analysis
3. Algebra II
4. Topology II
5. Differential Geometry I

Second Year

Semester I

1. Basic Probability Theory/Elective
2. Fourier Analysis
3. Differential Topology
4. Elective Course
5. Elective Course

Semester II

1. Elective Course
2. Elective Course
3. Elective Course
4. Elective Course
5. Elective Course

1.2 List of Compulsory Courses

1. Analysis of Several Variables
2. Topology I
3. Linear Algebra
4. Algebra I
5. Measure Theoretic Probability
6. Complex Analysis
7. Functional Analysis
8. Algebra II
9. Topology II
10. Differential Geometry I
11. Basic Probability Theory (for non-BMath non-BStat students)
12. Fourier Analysis
13. Differential Topology

1.3 List of Elective Courses

Group A (*At least one elective course is to be chosen from this group*.)

1.3.1 Number Theory
1.3.2 Advanced Number Theory
1.3.3 Algebraic Number Theory

Group B

1. Differential Equations
2. Graph Theory and Combinatorics
3. Advanced Functional Analysis
4. Operator Theory
5. Partial Differential Equations
6. Advanced Linear Algebra
7. Advanced Probability
8. Markov Chains
9. Ergodic Theory
10. Stochastic Processes
11. Topology III
12. Topology IV
13. Differential Geometry II
14. Algebra III
15. Commutative Algebra I
16. Commutative Algebra II
17. Algebraic Geometry (PRQ: Op 1)
18. Elliptic Curves
19. Representations of Locally Compact Groups
20. Lie Groups & Lie Algebra
21. Linear Algebraic Groups
22. Mathematical Logic
23. Set Theory
24. Game Theory
25. Automata, Languages and Computation
26. Advanced Fluid Dynamic
27. Quantum Mechanics I
28. Quantum Mechanics II
29. Analytical Mechanics
30. Special Topics (to be suggested by the faculty)
31. Projects I and II

2. BRIEF SYLLABI

2.1 Compulsory Courses

C1. Analysis of Several Variables

Differentiability of maps from $\mathbf{R}^m$ to $\mathbf{R}^n$ and the derivative as a linear map. Higher derivatives, Chain Rule, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier. Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e., product of intervals. Multiple integrals expressed as iterated simple integrals. Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustrated with plenty of examples. Inverse and implicit functions theorems. Picard's Theorem.

Curves in $\mathbf{R}^2$ and $\mathbf{R}^3$ Line integrals, Surfaces in $\mathbf{R}^3$, Surface integrals, Integration of forms, Divergence, Gradient and Curl operations, Green's, Strokes' and Gauss' (Divergence) theorems.

References :

- M. Spivak: *Calculus on manifolds*, Benjamin (1965).
- W. Rudin: *Principles of mathematical analysis*, Mc Graw-Hill.
- T. Apostol: *Mathematical Analysis*.
- R. Courant and F. John: *Introduction to Calculus and Analysis.*
- T. Apostol: *Calculus* (Vol 2), John Wiley.

C2. Topology I

Topological spaces, open and closed sets, basis, closure, interior and boundary. Subspace topology, Hausdorff spaces. Continuous maps: properties and constructions; Pasting Lemma. Homeomorphisms. Countability and separation axioms. Product topology, Urysohn embedding lemma and metrization theorem for second countable spaces. Quotient topology and examples of Topological Manifolds. Connected, path-connected and locally connected spaces. Lindelof and Compact spaces, Locally compact spaces, one-point compactification and Tychonoff's theorem. Paracompactness and Partitions of unity. Urysohn's lemma, Tietze extension theorem and applications. Completion of metric spaces. Baire Category Theorem and applications.

Covering spaces, Path Lifting and Homotopy Lifting Theorems, Fundamental Group.

References :

- J. R. Munkres, *Topology*: a first course, Prentice-Hall (1975).
- G.F. Simmons, *Introduction to Topology and Modern Analysis*, TataMcGraw-Hill (1963).
- M.A. Armstrong, *Basic Topology*, Springer.
- J.L. Kelley, *General Topology*, Springer-Verlag (1975).
- J. Dugundji, Topology, UBS (1999).
- I. M. Singer and J. A. Thorpe, *Lecture notes on elementary topology and geometry*, UTM, Springer.

C3. Linear Algebra

1. Review of linear transformations and matrices. Eigenvectors, characteristic polynomial, orthogonal matrices and rotations. Inner product spaces, Hermitian, unitary and normal transformations, spectral theorems, bilinear and quadratic forms. Multilinear forms, wedge and alternating forms.
2. Review of basic concepts from rings and ideals required for module theory (if necessary). Modules over commutative rings: examples. Basic concepts: submodules, quotients modules, homomorphisms, isomorphism theorems, generators, annihiliators, torsion, direct product and sum, direct summand, free modules, finitely generated modules, exact and split exact sequences.
3. Properties of $K[X]$ over a field K. Structure theorem for finitely generated modules over a PID; applications to Abelian groups, rational and Jordan canonical forms.

Time permitting, snake's lemma, complexes and homology sequences may be introduced.

<u>References</u> :

- D.S. Dummit and R.M. Foote, *Abstract Algebra*, John Wiley (Asian reprint 2003).
- S. Lang, *Algebra*, GTM (211), Springer (Indian reprint 2002).
- K. Hoffman and R. Kunze, *Linear Algebra*, Prentice-Hall of India (1998).
- N.S. Gopalakrishnan, *University Algebra*, Wiley Eastern (1986).

C4. Algebra I

1. Commutative rings with unity: examples, ring homomorphisms, ideals, quotients, isomorphism theorems with applications to non-trivial examples. Prime and maximal ideals, Zorn's Lemma and existence of maximal ideals. Product of rings, ideals in a finite product, Chinese Remainder Theorem. Prime and maximal ideals in a quotient ring and a finite product. Field of fractions of an integral domain. Irreducible and prime elements; PID and UFD.
2. Polynomial Ring: universal property; division algorithm; roots of polynomials. Gauss' Theorem (R UFD implies $R[X]$ UFD); irreducibility criteria. Symmetric polynomials: Newton's Theorem. Power Series.
3. Noetherian rings and modules, algebras, finitely generated algebras, Hilbert Basis Theorem. Tensor product of modules: definition, basic properties and elementary computations. Time permitting, introduction to projective modules.
4. Groups: Review of normal subgroups, quotient groups and homomorhism theorems. Group actions with examples, class equations and their applications, Sylow's Theorems; groups and symmetry. Direct sum and free Abelian groups. Time permitting: composition series, exact sequences, direct product and semidirect product with examples.

Note: It is desirable that Item No. 1 of Algebra-I is covered before Item No. 2 of Linear Algebra begins.

<u>References</u> :

- D.S. Dummit and R.M. Foote, *Abstract Algebra*, John Wiley (Asian reprint 2003).

- N. Jacobson, *Basic Algebra* Vol. I, W.H. Freeman and Co (1985).
- S. Lang, *Algebra*, GTM (211), Springer (Indian reprint 2004).
- N.S. Gopalakrishnan, *University Algebra*, Wiley Eastern (1986).
- N.S. Gopalakrishnan, *Commutative Algebra* (Chapter 1), Oxonian Press (1984).
- J.J. Rotman, *An Introduction to the theory of groups*, GTM (148), Springer-Verlag (2002).

C5. Measure Theoretic Probability

Measure and Integration: σ-algebras of sets, Monotone Class Theorem, Probability and -finite Measures, Construction of Lebesgue measure, Integration, Fatou Lemma, Monotone and Dominated Convergence Theorems, Radon-Nikodym theorem, product measures, Fubini's theorem.

Probability: (If needed a quick review of concepts and results (without proof) from basic Discrete and Continuous Probabilty.) Distribution Functions of Probabilty Measures on R, Borel-Cantelli Lemma, Weak and Strong Laws of Large Numbers in i.i.d. case, various Modes of Convergende, Characteristic Functions, Uniqueness/Inversion/Levy Continuity Theorems, Proof of the Central Limit Theorem for i.i.d. case with Finite Variance.

References :

- W. Rudin, *Real and complex analysis*, McGraw-Hill Book Co. (1987).
- P. Billingsley, *Probability and measure*, John Wiley (1995).
- K. R. Parthasarathy, *Introduction to probability and measure*, TRIM (33), Hindustan Book Agency (2005).
- J. Nevue, *Mathematical foundations of the calculus of probability*, Holden-Day (1965).
- I. K. Rana, *An introduction to measure and integration*, Narosa Publishing House (1997).

C6. Functional Analysis

Normed linear spaces, Banach spaces. Bounded linear operators. Dual of a normed linear space. Hahn-Banach theorem, uniform boundedness principle, open mapping theorem, closed graph theorem. Computing the dual of wellknown Banach spaces. Weak and weak* topologies, Banach-Alaoglu Theorem. The double dual.

L^p spaces, Riesz representation theorem for the space $C[0, 1]$.

Hilbert spaces, adjoint operators, self-adjoint and normal operators, spectrum, spectral radius, analysis of the spectrum of a compact operator on a Banach space, spectral theorem for bounded self-adjoint operators.

Time permitting: Goldsteins Theorem, reflexivity; spectral theorem for normal and unitary operators.

References :

- W. Rudin, *Real and complex analysis*, McGraw-Hill (1987).
- W. Rudin, *Functional analysis*, McGraw-Hill (1991).
- J. B. Conway, *A course in functional analysis*, GTM (96), Springer-Verlag (1990).
- K. Yosida, *Functional analysis*, Grundlehren derMathematischen Wissenschaften (123), Springer-Verlag (1980).

C7. Complex Analysis

A review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates. power series expansions, maximum modulus principle, Classification of singularities and calculus of residues. Normal families, Arzela's theorem. Product developments, functions with prescribed zeroes and poles, Hadamard's theorem. Conformal mappings, the Riemann mapping theorem, the linear fractional transformations.

Depending on time available, some of the following topics may be done:
(i) Subharmonic functions, the Dirichlet problem and Green's functions.
(ii) An introduction to elliptic functions.
(iii) Introduction to functions of several complex variables.

References :

- L. V. Ahlfors, *Complex analysis. An introduction to the theory of analytic functions of one complex variable*, McGraw-Hill (1978).
- J. B. Conway, *Functions of one complex variable* II. GTM (159), Springer-Verlag, 1995.
- W. Rudin, *Real and complex analysis*, McGraw-Hill (1987).
- R. Narasimhan and Y. Nievergelt, *Complex Analysis in One Variable*, Birkhauser, 2001.

C8. Algebra II

Results on finite groups: permutation groups, simple groups, solvable groups, simplicity of An. Topics like semi-direct product (if not covered in Algebra-I).

Algebraic and transcendental extensions; algebraic closure; splitting fields and normal extensions; separable, inseparable and purely inseparable extensions; finite fields.

Galois extensions and Galois groups, Fundamental theorem of Galois theory, cyclic extensions, solvability by radicals, constructibility of regular n-gons, cyclotomic extensions.

Time permitting, additional topics may be selected from:
(i) Traces and norms, Hilbert theorem 90, Artin-Schrier theorem, Galois cohomology, Kummer extension.
(ii) Transcendental extensions.
(iii) Real fields: ordered fields, real closed fields, Sturm's theorem, real zeros and homomorphisms.
(iv) Integral extensions and the Nullstellensatz.

References :

- D.S. Dummit and R.M. Foote, *Abstract Algebra*, John Wiley (Asian reprint 2003).
- S. Lang, *Algebra*, GTM (211), Springer (Indian reprint 2004).
- M. Nagata, *Field theory*, Marcel-Dekker (1977).
- N.S. Gopalakrishnan, *University Algebra*, Wiley Eastern (1986).
- N. Jacobson, *Basic Algebra*, W.H. Freeman and Co (1985).
- G. Rotman, *Galois theory*, Springer (Indian reprint 2005).
- TIFR pamphlet on *Galois theory*.

C9. Topology II

Review of fundamental groups, necessary introduction to free product of groups, Van Kampen's theorem. Covering spaces, lifting properties, Universal cover, classification of covering spaces, Deck transformations, properly discontinuous action, covering manifolds, examples.

Categories and functors. Singular homology groups, axiomatic properties, Mayer-Vietoris sequence, homology with coefficients, statement of universal coefficient theorem for homology, simple computation of homolgy groups.

CW-complexes and Cellular homology, Simplicial complex and simplicial homology as a special case of Cellular homology, Relationship between fundamental group and first homology group.

References :

- A. Hatcher, *Algebraic Topology*, Cambridge University Press (2002).
- W. S. Massey, *A basic course in algebraic topology*, GTM (127), Springer (1991).
- J. R. Munkres, *Elements of algebraic topology*, Addison-Wesley (1984).
- M. J. Greenberg, *Lectures on algebraic topology*, Benjamin (1967).
- I. M. Singer and J. A. Thorpe, *Lecture notes on elementary topology and geometry*, UTM, Springer.
- E. Spanier, *Algebraic Topology,* Springer-Verlag (1982).

C10. Differential Geometry I

Parametrized curves in $\mathbf{R}^3$, length of curves, integral formula for smooth curves, regular curves, parametrization by arc length. Osculating plane of a space curve, Frenet frame, Frnet formula, curvatures, invariance under isometry and reparametrization. Discussion of the cases for plane curves, rotation number of a closed curve, osculating circle, 'Umlaufsatz'.

Smooth vector fields on an open subset of $\mathbf{R}^3$, gradient vector field of a smooth function, vector field along a smooth curve, integral curve of a vector field. Existence theorem of an integral curve of a vector field through a point, maximal integral curve through a point.

Level sets, examples of surfaces in $\mathbf{R}^3$. Tangent spaces at a point. Vector fields on surfaces. Existence theorem of integral curve of a smooth vector field on a surface through a point. Existence of a normal vector of a connected surface. Orientation, Gauss map. The notion of geodesic on a surface. The existence and uniqueness of geodesic on a surface through a given point and with a given velocity vector thereof. Covariant derivative of a smooth vector field. Parallel vector field along a curve. Existence and uniqueness theorem of a parallel vector field along a curve with a given initial vector. The Weingarten map of a surface at a point, its self-adjointness property. Normal curvature of a surafce at a point in a given direction. Principal curvatures, first and second fundamental forms, Gauss curvature and mean curvature. Surface area and volume. Surfaces with boundary, local and global stokes theorem. Gauss-Bonnet theorem.

References :

- B O'Neill, *Elementary Differential Geometry*, Academic Press (1997).
- A. Pressley, *Elementary Differential Geometry*, Springer (Indian Reprint 2004).
- J. A. Thorpe, *Elementary topics in Differential Geometry*, Springer (Indian reprint, 2004).

C11. Basic Probability Theory
(*Compulsory for non BStat/BMath Students. B.Stat./B.Math. students can't opt for this.*)

Orientation. Combinatorial probability. Fluctuations in Coin Tossing and Random Walks. Combination of Events, Occupancy and Matching Problems. Conditional probabilities. Urn Models. Independence.

Random Variables, discrete distributions, Expectation, variance and moments, probability generating functions and moment generating functions, Tchebychev's inequality. Standard discrete distributions: uniform, binomial, Poisson, geometric, hypergeometric, negative binomial. Continuous random variables: univariate densities and distributions, Expectations, variance and moments, standard univariate densities: normal, exponential, gamma, beta, chi-square, Cauchy.

Joint and conditional distributions, Independence of random variables, Transformation of variables.

Laws of Large Numbers (proofs optional).

References :

- S. Ross, *First course in probability theory*, Mac Millan (1989).
- P.G. Hoel, S.C. Port and C.J. Stone, *Introduction to probability theory* Vols 1 and 3, Houghton Mifflin Co (1971).
- W. Feller, *An introduction to probability theory and its applications* Vol 1, Wiley (1950).
- K.L. Chung, *Elementary probability theory with stochastic processes*, Springer (1974).

C12. Differential Topology

Manifolds in RN, submanifolds, smooth maps of manifolds, derivatives and tangents, Inverse function theorem and immersions, submersions, Transversality, Homotopy and stability, Sard's theorem and Morse functions, embedding manifolds in Euclidean space.

Differential Forms and Integration of forms, Stokes' Theorem, Definition of DeRham Cohomology.

References :

- V. Guillemin and A. Pollack, *Differential Topology*, Prentice-Hall.
- J.W. Milnor, *Topology from the Differentiable Viewpoint*, Univ Press of Virginia (1965).

C13. Fourier Analysis

Fourier and Fourier-Stieltjes' series, summability kernels, convergence tests. Fourier transforms, the Schwartz space, Fourier Inversion and Plancherel theorem. Maximal functions and boundedness of Hilbert transform. Paley-Wiener Theorem. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem. Introduction to wavelets and multi-resolution analysis.

References :

- Y. Katznelson, *An introduction to harmonic analysis*, Dover Publications (1976).
- E. M. Stein and Rami Shakrachi, *Fourier Analysis: An Introduction*, Princeton Lectures in Analysis.
- E. Hernez and G. Weiss, *A first course on wavelets*, Studies in Advanced Mathematics. CRC Press (1996).

2.2 Elective Courses

Group A

(A student must choose *at least one* elective from this group.)

A1. Number Theory

Review of unique factorization; properties of the rings $\mathbf{Z}[i]$ and $\mathbf{Z}[\omega]$ (chapter 1 of IR).

Review of congruences, Euler's --function, results of Fermat, Euler and Wilson; linear congruences, Chinese remainder theorem. Primitive roots and the group structure of $U(\mathbf{Z}/n\mathbf{Z})$; applications to congruences of higher degree; Hensel's Lemma (chapter 4 of IR and sections 2.1 to 2.7 of NZM).

Quadratic Reciprocity: Quadratic Residues, Gaussian reciprocity law, the Jacobi symbol (chapter 5 of IR).

Arithmetic Functions, Moebius inversion formula and combinatorial methods like principle of inclusion-exclusion and pigeonhole etc (sections 4.2,4.3,4.5 of NZM).

Diophantine equations. Linear equations, the equation $x^2+y^2 = z^2$. Method of Descent; the equation $x^4+y^4= z^2$ (section 5.1 to 5.4 of NZM).

Binary Quadratic forms. Sum of two squares. Legendre's Theorem (section 3.4 to 3.7 of NZM).

Simple continued fractions. Infinite continued fractions and irrational numbers. Periodic continued fractions, algorithms for solving Brahmagupta-Pell equation, numerical computations. Dirichlet's box principle and solution of Pell's equation (chapter 7 of NZM).

Elementary results on the function $\pi(x)$, Bertrand's postulate (sections 8.1, 8.2 of NZM).

Time permitting, additional topics may be chosen from :

(i) Partitions. Euler's identity and Euler's formula (sections 10.1 to 10.4 of NZM).
(ii) Gauss and Jacobi Sums, Cubic and Biquadratic Reciprocity (from chapters 8,9 of IR).
(iii) Irrational numbers: Hurwitz's theorem on rational approximations; irrationality of certain values of trigonometric functions; irrationality of π (chapter 6 of NZM).
(iv) Diophantine equations over finite fields (chapter 10 of IR).
(v) Introduction to Zeta function, Dirichlet's L-functions and Elliptic Curves (from IR).

References :

- (IR) K. Ireland and M. Rosen, *A Classical Introduction to Modern Number Theory* Second Edition, Springer (Indian reprint 2004).
- (NZM) I. Niven, H.S. Zuckerman and H. Montgomery, *An Introduction to the Theory of Numbers*, John Wiley.
- J.H. Silverman, *A Friendly Introduction to Number Theory,* Prentice-Hall.
- J. Stillwell, *Mathematics and Its History* Second Edition, Springer (Indian reprint 2004).

A2. Advanced Number Theory

Review of finite fields; Polynomial equations over finite fields: theorems of Chevalley and Warning;

Quadratic Forms over prime fields. Review of the law of quadratic reciprocity.

The ring of *p*-adic integers; the field of *p*-adic numbers; completion; padic equations and Hensel's lemma; Quadratic Forms with *p*-adic coefficients. Hilbert's symbol.

Dirichlet series: abscissa of convergence and absolute convergence. Riemann Zeta function and Dirichlet *L*-functions. Dirichlet's theorem on primes in arithmetic progression. Functional equation and Euler product for *L*-functions.

Modular forms and the modular group $SL(2,\mathbf{R})$. Eisenstein series. Zeros and poles of modular functions. Dimensions of the spaces of modular forms. The *j*-invariant and Picard's Theorem. *L*-function and Ramanujan's τ-function.

<u>References</u> :

- J.P. Serre: *A Course in Arithmetic*, Springer.
- Z. Borevich and I. Shafarevich: *Number Theory* (Chapter 1), Academic Press.
- K. Chandrasekharan: *Introduction to Analytic Number Theory*, Springer.

A3. Algebraic Number Theory

Review of norm and trace, Number fields and their rings of integers, Prime decomposition in number rings, Kummer-Dedekind discriminant criterion for ramification, The Ideal Class Group, ray class group, their finiteness and Dirichlet's Unit theorem, Valuations and completions of number fields, Decomposition and inertia groups, Frobenius automorphism, Artin symbol, Dedekind zeta function and the Distribution of ideals in a number ring, Kronecker limit formula, Frobenius density theorem. Time permitting, introduction to class field theory.

<u>References</u> :

- G.J. Janusz: *Algebraic Number Fields* (chapter 1-4), Academic Press.
- D.A. Marcus: *Number Fields,* Springer.
- J. Neukirch: *Algebraic Number Theory*, Springer.
- I. Stewart and D. Tall: *Algebraic Number Theory and Fermat's Last Theorem*, A.K. Peters.
- J. Esmonde and M. Ram Murty: *Problems in Algebraic Number Theory*, Springer.
- TIFR pamphlet on *Algebraic Number Theory*.

Note: For students opting for "Algebraic Number Theory", a prior knowledge of "Commutative Algebra" is desirable.

Group B

B1. Differential Equations

(*Only for non BStat/BMath students*)

Ordinary differential equations – first order equations, Picard's theorem (existence and uniqueness of solution to first order ordinary differential equation), Second order linear equations – second order linear differential equations with constant co-efficients, Systems of first order differential equations, Equations with regular singular points, Introduction to power series and power series solutions, Special ordinary differential equations arising in physics and some special functions (eg. Bessel's functions, Legendre polynomials, Gamma functions).

Partial differential equations – elements of partial differential equations and the three equations of physics i.e. Laplace, Wave and the Heat equations, at least in 2-dimensions. Lagrange's method of solving first order quasi linear equations.

Note: The course may be combined with the B.Math./B.Stat. course on "Differential Equations".

References :

- G.F. Simmons, *Differential Equations*, McGraw-Hill.

B2. Graph Theory and Combinatorics

Graphs and digraphs, connectedness, trees, degree sequences, connectivity, Eulerian and hamiltonian graphs, matchings and SDR's, chromatic numbers and chromatic index, planarity, covering numbers, flows in networks, enumeration, Inclusion-exclusion, Ramsey's theorem, recurrence relations and generating functions.

References :

- F. Harary, *Graph Theory*, Addision-Wesley (1969), Narosa (1988).
- D. B. West, *Introduction to Graph Theory*, Prentice-Hall (Indian Edition 1999).
- J. A. Bondy and U. S. R. Murthy, *Graph Theory and Applications*, MacMillan (1976).
- H. J. Ryser, *Combinatorial Mathematics*, Carus Math. Monograph, MAA (1963).
- M. J. Erickson, *Introduction to Combinatorics*, John Wiley (1996).
- L. Lovasz, *Combinatorial Problems and Exercises*, AMS Chelsea (1979).

Note: The course may be combined with the "Graph Theory and Combinatorics" course of M.Stat. 2nd year.

B3. Advanced Functional Analysis

Brief introduction to topological vector spaces (TVS) and locally convex TVS. Linear Operators. Uniform Boundedness Principle. Geometric Hahn-Banach theorem and applications (Markov-Kakutani fixed point theorem, Haar Measure on locally compact abelian groups, Liapounovs theorem). Extreme points and Krein-Milman theorem. In addition, one of the following topics:
(a) Geometry of Banach spaces: vector measures, Radon-Nikodym Property and geometric equivalents. Choquet theory. Weak compactness and Eberlein-Smulian Theorem. Schauder Basis.
(b) Banach algebras, spectral radius, maximal ideal space, Gelfand transform.
(c) Unbounded operators, Domains, Graphs, Adjoints, spectral theorem.

References :

- N. Dunford and J. T. Schwartz, *Linear operators*. Part II: *Spectral theory. Self adjoint operators in Hilbert space*, Interscience Publishers, John Wiley (1963).
- Walter Rudin, *Functional analysis* Second edition, International Series in Pure and Applied Mathematics. McGraw-Hill (1991).
- K. Yosida, *Functional analysis*, Grundlehren derMathematischen Wissenschaften (123), Springer-Verlag (1980).

- J. Diestel and J. J. Uhl, Jr., *Vector measures*, Mathematical Surveys (15), AMS (1977).

B4. Operator theory

1. Compact operators on Hilbert Spaces: Fredholm Theory, Index.
2. C^*-algebras—noncommutative states and representations, Gelfand-Neumark representation theorem.
3. Von-Neumann Algebras; Projections, Double Commutant theorem, L^∞ functional Calculus.
4. Toeplitz operators.

References :

- W. Arveson, *An invitation to C*-algebras*, GTM (39), Springer-Verlag (1976).
- N. Dunford and J. T. Schwartz, *Linear operators*. Part II: *Spectral theory. Self adjoint operators in Hilbert space*, Interscience Publishers John Wiley (1963).
- R. V. Kadison and J. R. Ringrose, *Fundamentals of the theory of operator algebras*. Vol. I. *Elementary theory*, Pure and Applied Mathematics (100), Academic Press (1983).
- V. S. Sunder, *An invitation to von Neumann algebras*, Universitext, Springer- Verlag (1987).

B5. Partial Differential Equations

Theory of Schwartz distributions and Sobolev spaces; local solvability and Lewys example; existence of fundamental solutions for constant coefficient differential operators; Laplace, heat and wave equations, hypoelliptic and analytic hypoelliptic operators, elliptic boundary value problems interior regularity, local existence.

References :

- G. B. Folland, *Introduction to partial differential equation*s, Princeton University Press (1995).
- F. Trs, *Basic linear partial differential equations*, Pure and Applied Mathematics (62), Academic Press (1975).
- J. Rauch, *Partial differential equations*, GTM (128), Springer-Verlag (1991).
- E. DiBenedetto, *Partial differential equations*, Birkhauser (1995).
- L. C. Evans, *Partial differential equations*, Graduate Studies in Mathematics (19), AMS (1998).
- L. Hormander, *The analysis of linear partial differential operators*. I. *Distribution theory and Fourier analysis*. Grundlehren der Mathematischen Wissenschaften (256), Springer-Verlag (1990).

B6. Advanced Linear Algebra

Topics from:
Majorization and doubly stochastic matrices. Matrix Decomposition Theorems (Polar, QR, LR, SVD etc.) and their applications. Perturbation Theory. Nonnegative matrices and their applications. Wavelets and the Fast Fourier Transform. Basic ideas of matrix computations.

References :

- R. Bhatia, *Matrix Analysis*, GTM (169), Springer-Verlag (1997).

B7. Advanced Probability

Independence, Kolmogorov Zero-one Law, Kolmogorov Three-series theorem, Strong law of large Numbers. LevyCramer Continuity theorem, CLT for i.i.d. components, Infinite Products of probability measures, Kolmogorovs Consistency theorem, Radon-Nikodym Theorem, Conditional expectations.

Discrete parameter martingales with applications.

References :

- J. Nevue, *Mathematical foundations of the calculus of probability*, Holden-Day (1965).
- P. Billingsley, *Probability and measure*, John Wiley (1995).
- Y. S. Chow and H. Teicher, *Probability theory. Independence, interchangeability, martingales*, Springer Texts in Statistics. Springer-Verlag (1997).

Note: The syllabus for the "Advanced Probability" course of M.Stat. 2nd year may also be used.

B8. Markov Chains

Finite State Markov Chains. Examples, Classification of States, Stationary Distribution.

Rates of convergence to stationarity, Dirichlet Form and Spectral gap methods, Some Coupling methods with applications, Random walk on Finite Groups, Poisson Processes, Continuous time Markov Chains, Birth-and-death processes.

Suggested Texts:

- S. M. Ross, *Stochastic processes*, John Wiley (1996).
- R. N. Bhattacharya and E. C. Waymire, *Stochastic processes with applications*.
- E. GinG, R. Grimmett and L. Saloff-Coste, *Lectures on probability theory and statistics*.

B9. Ergodic Theory

Measure preserving systems; examples: Hamiltonian dynamics and Liouvilles theorem, Bernoulli shifts, Markov shifts, Rotations of the circle, Rotations of the torus, Automorphisms of the Torus, Gauss transformations, Skew-product.

Poincare Recurrence lemma: Induced transformation: Kakutani towers: Rokhlins lemma. Recurrence in Topological Dynamics, Birkhoffs Recurrence theorem.

Ergodicity, Weak-mixing and strong-mixing and their characterisations. Ergodic Theorems of Birkhoff and Von Neumann. Consequences of the Ergodic theorem. Invariant measures on compact systems. Unique ergodicity and equidistribution. Weyls theorem.

The Isomorphism problem; conjugacy, spectral equivalence.

Transformations with discrete spectrum, Halmosvon Neumann theorem.

Entropy. The Kolmogorov-Sinai theorem. Calculation of Entropy. The Shannon Mc-Millan Breiman Theorem.

Flows. Birkhoffs ergodic Theorem and Wieners ergodic theorem for flows. Flows built under a function.

<u>References</u> :

- Peter Walters, *An introduction to ergodic theory,* GTM (79), Springer-Verlag (1982).
- Patrick Billingsley, *Ergodic theory and information*, Robert E. Krieger Publishing Co. (1978).
- M. G. Nadkarni, *Basic ergodic theory*, TRIM 6, Hindustan Book Agency (1995).
- H. Furstenberg, *Recurrence in ergodic theory and combinatorial number theory,* Princeton University Press (1981).
- K. Petersen, *Ergodic theory*, Cambridge Studies in Advanced Mathematics (2), Cambridge University Press (1989).

B10. Stochastic Processes

Weak Convergence of probability measures on Polish spaces including *C*[0, 1], Brownian motion; construction, simple properties of paths, Connections between Brownian Motion / Diffusion and PDEs. Time permitting: Stationary processes, Markov processes and generators.

<u>References</u> :

- P. Billingsley, *Convergence of probability measures*, John Wiley (1999).
- K. Ito, *Stochastic processes*, Lecture Notes Series, No. 16 Matematisk Institut, Aarhus Universitet, Aarhus (1969).
- D. Revuz andM. Yor, *Continuous martingales and Brownian motion*, Grundlehren der Mathematischen Wissenschaften (293), Springer-Verlag (1999).

Note: (1) *For students opting for this elective, a prior knowledge of the contents of the courses Probability* I,II, III *of the B.Stat. programme* (ISI) *is essential*. (2) *The course may be combined with the course "Stochastic Processes-I" of M.Stat. 2nd year.*

B11. Topology III

CW-complexes, cellular homology, comparison with singular theory, computation of homology of projective spaces.

Definition of singular cohomology, axiomatic properties, statement of universal coefficient theorem for cohomology. Betti numbers and Euler characterisitic. Cup and cap product, Poincare duality. Cross product and statement of Kunneth theorem. Degree of maps with applications to spheres.

Definition of higher homotopy groups, homotopy exact sequence of a pair. Definition of fibration, examples of fibrations, homotopy exact sequence of a fibration, its application to computation of homotopy groups. Hurewicz homomorphism, The Hurewicz theorem. The Whitehead Theorem.

<u>References</u> :

- A. Hatcher, *Algebraic Topology*, Cambridge University Press (2002).
- M. J. Greenberg and J.R. Harper, *Algebraic topology*: *A First Course*, Benjamin/Cummings (1981).
- E. Spanier, *Algebraic Topology*, Springer-Verlag (1982).
- J. Vick, *Homology Theory,* Springer.
- J. R. Munkres, *Elements of algebraic topology*, Addison-Wesley (1984).

B12. Topology-IV

Smooth manifolds, Differential forms on manifolds, Integration on manifolds, Stoke's theorem, computation of cohomology rings of projective spaces, Borsuk-Ulam theorem.

Degree, linking number and index of vector fields, The Poincare-Hopf theorem.

Definition and examples of principal bundles and fibre bundles, clutching construction, description of classification theorem (without proof).

References :

- R. Bott and L. W. Tu, *Differential forms in algebraic topology*, GTM (82), Springer-Verlag (1982).
- M. J. Greenberg, *Lectures on algebraic topology*, Benjamin (1967).
- F. W. Warner, *Foundations of differentiable manifolds and Lie groups*, GTM (94), Springer-Verlag (1983).

B13. Differential Geometry II

A quick review of tensors, alternating forms, manifolds, immersion, submersion and submanifolds.

Tangent bundle, vector bundles, vector fields, flows and the fundamental theorem of ODE's. Embedding in Euclidean space, tubular neighbourhood. Differential forms and integration, Stoke's theorem. Transversality, Riemann metrics, Riemannian connection on Riemannian manifolds, Gauss-Bonnet theorem. Parallel transport, geodesics and geodesic completeness, the theorem of Hopf-Rinow.

References :

- F. W. Warner, *Foundations of differentiable manifolds and Lie groups*, GTM (94), Springer-Verlag (1983).
- S. Helgason, *Differential geometry, Lie groups, and symmetric space*, Graduate Studies in Mathematics (34), AMS (2001).

B14. Algebra III

Modules over noncommutative rings: Noetherian and Artinian rings and modules. Modules of finite length. Krull-Schmidt theorem.

Balanced maps, Tensor product of modules and algebras: definitions, basic properties, right exactness, change of base.

Semisimple rings and modules; Wedderburn's structure theorem.

Nilradical and Jacobson radical; NAK lemma; Jacobson radical of an Artinian ring is nilpotent; Ring semi-simple if and only if Artinian with trivial radical; Artinian ring is Noetherian.

Central Simple Algebras and the Brauer Group; Examples.

Representation of finite groups: group algebra, Maschke's Theorem, Simple Modules over

Group Algebras; Characters and Orthogonality relations; Burnside's two-prime theorem; Induced representation; Frobenius reciprocity; Brauer's theorem on induced characters.

References :

- T.Y. Lam, *A First Course in Noncommutative Rings*, Springer.
- C.W. Curtis and I. Reiner, *Representation Theory of finite Groups and Associative Algebras*, Springer.
- P.M. Cohn, *Further Algebra and Applications*, Springer.
- S. Lang, Algebra, Springer.
- D.S. Dummit and R.M. Foote, *Abstract Algebra* (Part VI), John Wiley.
- TIFR notes on *Semisimple rings and modules*.

B15. Commutative Algebra I

Rings and ideals: review of ideals in quotient rings; prime and maximal ideals, prime ideals under quotient, existence of maximal ideals; operations on ideals (sum, product, quotient and radical); Chinese Remainder theorem; nilradical and Jacobson radical; extension and contraction of ideals under ring homomorphisms; prime avoidance.

Free modules; Projective Modules; Tensor Product of Modules and Algebras; Flat, Faithfully Flat and Finitely Presented Modules; Shanuel's Lemma.

Localisation and local rings, universal property of localisation, extended and contracted ideals and prime ideals under localisation, localisation and quotients, exacteness property. Results on prime ideals like theorems of Cohen and Isaac. Nagata's criterion for UFD and applications; equivalence of PID and one-dimensional UFD.

Modules over local rings. Cayley-Hamilton, NAK lemma and applications. Examples of local-global principles. Projective and locally free modules. Patching up of Localisation.

Polynomial and Power Series Rings. Noetherian Rings andModules. Hilbert's Basis Theorem. Associated Primes and Primary Decomposition. Artininan Modules. Modules of Finite Length.

Integral Extensions: integral closure, normalisation and normal rings. Cohen-Seidenberg Going-Up Theorem. Hilbert's Nullstellensatz and applications.

Valuations, Discrete Valuation Rings, Dedekind domains.

References :

- N.S. Gopalakrishnan, *Commutative Algebra*, Oxonian Press (1984).
- M.F. Atiyah and I.G. Macdonald, *Introduction to commutative algebra*, Addision-Wesley(1969).
- M. Reid: *Undergraduate commutative algebra*, LMS Student Texts (29), Cambridge Univ. Press (1995).
- R.Y. Sharp: *Steps in commutative algebra*, LMS Student Texts, Cambridge Univ. Press.
- E. Kunz: *Introduction to commutative algebra and algebraic geometry*, Birkhauser (1985).
- D.S. Dummit and R.M. Foote: *Abstract Algebra* (Part V), John Wiley (2003).
- D. Eisenbud: *Commutative algebra with a view toward algebraic geometry* GTM (150), Springer-Verlag (1995).

B16. Commutative Algebra-II

Pre-requisite: Commutative Algebra I

Graded Rings and Modules. Artin-Rees Lemma. *I*-adic filtrations. Completion. Exacteness and Flatness properties. Krull's Intersection Theorem. Hensel's Lemma and applications. Weierstrass Preparation theorem.

Associated Primes. Primary Decomposition. Homomorphisms and *AssM*. *SuppM*. Going-up and Going-Down Theorems. Finiteness of Integral Closure. Krull-Akizuki theorem. Dimension Theory: Hilbert Samuel Polynomial. Krull's Principal Ideal Theorem. Dimension Theorem.

Normalisation Lemma. Hilbert's Nullstellensatz. Integral Closure of Affine Domains. Valuations, Discrete Valuation Rings, Dedekind Domains, Fractional and Invertible Ideals, Ramification Formula.

Results on Normal and Regular Rings: Local property of Normal Domains, Normality and DVR at height one primes, Intersection of DVRs; Jacobian criterion for regular local rings (of affine algebras).

Homological Algebra: Projective Resolution. The functors Ext and Tor. Homological Dimension. Injective Modules and Injective Resolution. Injective Dimension and Global Dimension. Global Dimension of Noetherian Local Rings.

Properties of Regular Local Rings. Homological Characterisation of Regular Local Rings. Regular Local Ring is a UFD.

Derivations and Modules of Differentials.

References :

- N.S. Gopalakrishnan, *Commutative Algebra*, Oxonian Press (1984).
- H. Matsumura, *Commutative Algebra*, W.A. Benjamin (1970).
- TIFR pamphlet on *Homological Methods in Commutative Algebra*.
- D. Eisenbud, *Commutative algebra with a view toward algebraic geometry*, GTM (150), Springer-Verlag (1995).
- J.P. Serre, *Local Algebra*, Springer-Verlag (2000).
- N. Bourbaki, *Commutative Algebra*, Springer-Verlag (1995).
- M. Reid, *Undergraduate commutative algebra*, LMS Student Texts (29), Cambridge Univ. Press (1995).

B17. Algebraic Geometry

Topics from: Polynomial rings, Hilbert Basis theorem, Noether normalisation lemma, Hilbert Nullstellensatz, Affine and Projective spaces, Affine Schemes, Elementary dimension theory, Smoothness, Curves, Divisors on curves, Bezouts theorem, Abelian differential, Riemann Roch for curves.

References :

- W. Fulton, Algebraic curves. *An introduction to algebraic geometry*, Addison-Wesley (1989).
- D.S. Dummit and R.M. Foote, *Abstract Algebra* (Part V), John Wiley.
- C.G. Gibson, *Elementary Geometry of Algebraic Curves*, Cambridge.
- I.R. Shafarevich, *Basic algebraic geometry*, Springer.
- J. Harris, *Algebraic geometry.* A first course, GTM (133), Springer-Verlag (1995).
- K. Kendig, *Elementary algebraic geometry*, GTM (44), Springer-Verlag (1977).
- D. Mumford, *The Red Book of Varieties and Schemes*, Springer.
- C. Musili, *Algebraic geometry for beginners*, TRIM (20), HBA (2001).

Note: For students opting for “Algebraic Geometry”, a prior knowledge of “Commutative Algebra” is desirable.

B18. Elliptic Curves

Pre-requisites: A course in complex analysis, a course in number theory, a course in algebraic number theory (could be simultaneous), a course in algebraic geometry (could be simultaneous).

1. Algebraic curves, divisors, Riemann-Roch theorem.
2. Definition of elliptic curves, Weierstrass form, isogeny, Tate module, Weil pairing, Endomorphism ring.
3. Elliptic functions and integrals, Elliptic curves over complex numbers, Uniformization.
4. Elliptic curves over finite fields, Weil conjectures, Hasse invariant.
5. Elliptic curves over local fields, Minimal Weierstrass equation, Torsion, Good and bad reduction and Neron-Ogg-Shafarevich criterion for good reduction.
6. Elliptic curves over global fields, weak Mordell-Weil, Kummer pairing, Mordell-Weil theorem over **Q**.

If time permits : Heights on projective spaces and elliptic curves and Mordell-Weil theorem; Nagell-Lutz theorem.

Regerences:

- J. Silverman, *Arithmetic of elliptic curves* (chapters 2,3,5,6,7 and sections 8.1 to 8.4), GTM 106, Springer-Verlag 1986.

B19. Representations of Locally Compact Groups

Topological Groups, basic properties like subgroups, quotients and products, fundamental systems of neighbourhoods, open subgroups, connectedness and compactness. Existence of Haar measure on locally compact groups, properties of Haar measures.

Group actions on topological spaces, the space *X/G* in the topological as also in the analytical case assuming regularity conditions of the group action.

Representation of a locally compact group on a Hilbert space, the associated representation of group algebra, invariant subspaces and irreducibility, Schurs lemma.

Compact groups: Unitarity of finite dimensional representations, Peter-Weyl theory, Representations of *SU*(2,**C**), Representation of a finite group.

Induced representation and Frobenius reciprocity theorem, Representations of Heisenberg groups and of Euclidean motion group, Principal series representations of $SL(2,\mathbf{R})$.

B20. Lie Groups and Lie Algebras

1. Linear Lie groups: the exponential map and the Lie algebra of linear Lie group, some calculus on a linear Lie group, invariant differential operators, finite dimensional representations of a linear Lie group and its Lie algebra. Examples of linear Lie group and their Lie algebras, e.g., Complex groups: $GL(n,\mathbf{C})$, $SL(n,\mathbf{C})$, $SO(n,\mathbf{C})$, Groups of real matrices in those complex groups: $GL(n,\mathbf{R})$, $SL(n,\mathbf{R})$, $SO(n,\mathbf{R})$, Isometry groups of Hermitian forms $SO(m, n)$, $U(m, n)$, $SU(m, n)$. Finite dimensional representations of $\mathfrak{su}(2)$ and $SU(2)$ and their connection. Exhaustion using the lie algebra $\mathfrak{su}(2)$. [2 weeks]
2. Lie algebras in general, Nilpotent, solvable, semisimple Lie algebra, ideals, Killing form, Lies and Engels theorem. Universal enveloping algebra and Poincare-Birkhoff-Witt Theorem (without proof). [6 weeks]
3. Semisimple Lie algebra and structure theory: Definition of Linear reductive and linear semisimple groups. Examples of Linear connected semisimple/reductive Lie groups along with their Lie algebras (look back at 2 above and find out which are reductive/semisimple). Cartan involution and its differential at identity; Cartan decomposition $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$, examples of $\mathfrak{k}$ and $\mathfrak{p}$ for the groups discussed above. Definition of simple and semisimple Lie algebras and their relation, Cartans criterion for semisimplicity. Statements and examples of Global Cartan decomposition, Root space decomposition; Iwasawa decomposition; Bruhat decomposition. [6 weeks]

If time permits, one of the following topics:
(i) A brief introduction to Harmonic Analysis on $SL(2,\mathbf{R})$.
(ii) Representations of Compact Lie Groups and Weyl Character Formula.
(iii) Representations of Nilpotent Lie Groups.

Regerences:

- J.E. Humphreys: *Introduction to Lie algebras and representation theory*, GTM (9), Springer.
- S.C. Bagchi, S. Madan, A. Sitaram and U.B. Tiwari: *A first course on representation theory and linear Lie groups*, University Press.
- Serge Lang: $SL(2,\mathbf{R})$. GTM (105), Springer.
- W. Knapp: *Representation theory of semisimple groups. An overview based on examples*, Princeton Mathematical Series (36), Princeton University Press.

B21. Linear Algebraic Groups

Pre-requisites: A course in commutative algebra, a course in Lie algebras (could be simultaneous), a course in algebraic geometry (could be simultaneous).

Review of background commutative algebra and algebraic geometry (as in chapter 1 of Humphreys's book or chapter 1 of Springer's book). Definition of affine algebraic groups and homomorphisms over algebraically closed fields, examples. Orbit-closures under actions, linearity of affine groups. Lie algebra of an algebraic group and adjoint representation. Homogeneous spaces and quotients, Chevalley's theorem. Correspondence between groups and Lie algebras. Jordan decomposition, Commutative linear algebraic groups, diagonalizable groups and algebraic tori. Definition of weights and roots, Weyl group. Unipotent groups, Lie-Kolchin theorem, Structure theorem for connected solvable groups. Definition of reductive and semisimple groups, Borel subgroups, parabolic subgroups. Basic facts on complete varieties, Borel's fixed point theorem. Conjugacy of maximal tori, Nilpotency of Cartan subgroups. Density theorem and connectedness of centralizers of tori. Normalizer theorem for parabolic subgroups. Regular and singular tori, Structure theorem for groups of semi-simple rank one. Structure theorem for reductive groups, Bruhat decomposition, semisimple groups. Tits system, standard parabolic subgroups, simplicity proof. If time permits, mention (without proof): Representations and classification of semisimple groups and statements for general fields.

<u>References:</u>

- J. E. Humphreys, *Linear algebraic groups*, (chapters 1 to 10), GTM, Springer-Verlag (1975).
- T. A. Springer, *Linear algebraic groups*, (chapters 1 to 8), Progress in Mathematics, Birkhuser (1998).
- R. Steinberg, *Conjugacy classes in algebraic groups*, (Chapters 1 and 2 only as reference, some proofs are not given), Lecture Notes in Mathematics (366), Springer-Verlag (1974).

B22. Mathematical Logic

Syntax of First-Order Logic: First Order Languages, Terms and Formulas of a First Order language, First Order Theories.

Semantics of First-Order Languages: Structures of First-Order Languages, Truth in a Structure, Model of a Theory.

Propositional Logic: Tautologies and Theorems of propositional Logic, Tautology Theorem.

Proof in First Order Logic, Metatheorems of a first order theory, e.g. , theorems on constants, equivalence theorem, deduction and variant theorems etc., Consistency and Completeness, Lindenbaum Theorem.

Henkin Extension, Completeness theorem, Extensions by definition of first order theories, Interpretation theorem.

Model Theory: Embeddings and Isomorphisms, L¨owenheim-Skolem Theorem, Compactness theorem, Categoricity, Complete Theories.

Recursive functions, Arithmatization of first order theories, Decidable Theory, Representability, Godel's first Incompleteness theorem.

<u>References:</u>

- S. M. Srivastava, *A Course on Mathematical Logic*, Universitext, Springer (2008).
- J. R. Shoenfield, *Mathematical logic*, Addison-Wesley (1967).

B23. Set Theory

Either (A) or (B)

(A) Descriptive Set Theory.

1. A quick review of elementary cardinal and ordinal numbers, transfinite induction, induction on trees, Idempotence of Souslin operation.
2. Polish spaces, Baire category theorem, Transfer theorems, Standard Borel spaces, Borel isomorphism theorem, Sets with Baire property, Kuratowski-Ulam Theorem. The projective hierarchy and and its closure properties.
3. Analytic and coanalytic sets and their regularity properties, separation and reduction theorems, Von Neumann and Kuratowski-Ryll Nardzewskis selection theorems, Uniformization of Borel sets with large and small sections. Kondos uniformization theorem.

<u>References:</u>

- S. M. Srivastava, *A course on Borel sets*, GTM (180), Springer-Verlag (1998).
- A. S. Kechris, *Classical descriptive set theory*, GTM (156), Springer-Verlag (1995).

(B) Axiomatic Set Theory

A naive review of cardinal and ordinal numbers including regular and singular cardinals, some large cardinals like inaccessible and measurable cardinals. Martins Axiom and its consequences. Axiomatic development of set theory upto foundation axiom, Class and Class as models, relative consistency, absoluteness, Reflection principle, Mostowski collapse lemma etc. , non-decidability of large cardinal axioms, Godel’s second incompleteness theorem, Godel’s constructible universe, Forcing lemma and independence of CH.

<u>References:</u>

- S. M. Srivastava, *A Course on Axiomatic Set Theory*
- K. Kunen, *Set theory*. *An intr*oduction to independence proofs, Studies in Logic and the Foundations of Mathematics (102), North-Holland Publishing Co. (1980).
- T. Jech, *Set theory*, Academic Press (1978).

B24. Game Theory

A. Non-Cooperative Games: Games in normal form. Rationalizability and iterated deletion of never-best responses. Nash equilibrium: existence, properties and applications. Two-person Zero Sum Games. Games in extensive form: perfect recall and behavoiour strategies. Credibility and Subgame. Perfect Nash equilibrium. Bargaining. Repeated Games; Folk Theorems.

B. Introduction to Cooperative Games (TU Games).

Note: The course may be combined with the course “Game Theory-I” of M.Stat. 2nd year or the course “Game Theory-I” of the MSQE programme at ISI.

B25. Automata, Languages and Computation

1. Automata and Languages: Finite automata, regular languages, regular expressions, equivalence of deterministic and non-deterministic finite automata, minimisation of finite automata, closure properties, Kleene’s theorem, pumping lemma and its applications, Myhill-Nerode theorem and

its uses. Context-free grammar, context-free languages, Chomsky normal form, closure properties, pumping lemma for CFL, pushdown automata.

2. Computability: Computable functions, primitive and partial recursive functions, universality and halting problem, recursive and recursively enumerable sets, parameter theorem, diagonalisation and reducibility, Rice's theorem and its applications, Turing machines and its variants, equivalence of different models of computation and Church-Turing thesis.
3. Complexity: Time complexity of deterministic and non-deterministic Turing machines, P and NP, NP-completeness, Cook's theorem: other NP-complete problems.

References:

- N. Cutland, Computability. *An introduction to recursive function theory*, Cambridge University Press (1980).
- M. D. Davis, Ron Sigal and E. J. Weyuker, *Computability, complexity, and languages. Fundamentals of theoretical computer science*, Academic Press (1994).
- J. E. Hopcroft and J. D. Ullman, *Introduction to automata theory, languages, and computation*, Addison-Wesley (1979).
- H. R. Lewis and C. H. Papadimitriou, *Elements of the theory of computation*, Prentice-Hall (1981).
- M. Sipser, *Introduction to the theory of computation*, PWS Pub Co, NY (1999).
- M. R. Garey and D. S. Johnson, *Computers and intractability*. A guide to the theory of NP-completeness, W. H. Freeman and Co. (1979).

Note: The course may be combined with the corresponding course of M. Tech.(CS) 1st Year.

B26. Advanced Fluid Dynamics

Inviscid Incompressible fluid: Two dimensional motion, stream function, complex potential and velocity, sources, sinks. Doublets and their images. Circle theorem, Blasiuss theorem, Kutta-Jokowaski theorem. Axi-symmetric motion, Stokes stream function. Image of a source and a sink with respect to a sphere. Vortex motion, vortex lines and filaments, systems of vortices, rectilinear vortices, vortex pair and doublets. A single infinite row of vortices, Karmans vortex sheet. Linearised gravity waves, progressive waves in deep and shallow water, stationary waves, energy and group velocity, long waves and their energy, capillary waves.

Inviscid compressible fluid: First and second law of thermodynamics, polytropic gas and its entropy, adiabatic and isentropic flow, propagation of small disturbances. Mach number, Mach cone, irrotional motion, Bernoullis Equation, pressure, density and temperature in terms of Mach number. Area velocity relations in one-dimensional flow, concept of subsonic and supersonic flows. Normal shock-wave, Rankine-Hugonoit and Prandtls relations in case of a plane shock wave.

Viscous incompressible fluid: Equations of motion of a viscous fluid, Reynold's number, circulation in a viscous liquid, Flow between parallel plates, flow through pipes of circular, elliptic and annular section under constant pressure gradient. Prandtl's concept of boundary layer.

References:

- L. M. Milne-Thomson, *Theoretical hydrodynamics*, Macmillan (1960).
- L. D. Landau and E. M. Lifshitz, *Fluid mechanics, Course of Theoretical Physics*, Vol. 6

Pergamon Press (1959).

- H. Lamb, *Hydrodynamics*, Cambridge University Press (1993).
- W. H. Besant and A. S. Ramsey, *A treatise of Hydro-mechanics* Part II, ELBS.
- P. K. Kundu, *Fluid mechanics*, Academic Press.

B27. Quantum Mechanics I

1. (i) Physical Basis of Quantum Mechanics. (ii) Old Quantum theory. (iii) Uncertainty, Complimentarity and Duality. (iv) Measurement problems. (v) Heisenberg and Schrodinger representation.
2. (i) Schrodinger wave equation (ii) Perturbation theory.
3. Problem of two or more degrees of freedom without spherical symmetry; Stark effect.
4. Angular momentum, $SU(2)$ algebra.
5. Three-dimensional Schrodinger equation. Problems with spherical symmetry. Harmonic Oscillator.
6. Scattering problem , differential cross section, phase shift, variational principle, SW transformation, Regge poles.
7. WKB approximation.
8. Particles with spin, Pauli matrices, Pauli-Schrodinger equation. Two and three body problems. Hydrogen atom in electric and magnetic field.
9. Quantum Statistics.

References:

- L.I. Schiff, *Quantum Mechanics*.
- J.J. Sakurai, *Modern Quantum Mechanics*.
- L. D. Landau and E. M. Lifshitz, *Quantum mechanics: non-relativistic theory, Course of Theoretical Physics* Vol 3, Pergamon Press Ltd (1958).
- L.M. Falicov, *Group theory and its physical applications*, Univ of Chicago Press (1966).

B28. Quantum Mechanics II

Non stationary problems. Relativistic Dirac equation, Spinors. Scattering by a central force. Radiation theory. Quantization of Schrodinger field. Born approximation. Compton effect (Klein Nishina formula). Bremsstrahlung. Symmetry and conservation laws. Quantum Probability and quantum Statistics. Supersymmetric Quantum Mechanics, SWKB. Path integral method.

References:

- L.I. Schiff, *Quantum Mechanics*.
- P.A.M. Dirac, *The Principles of Quantum Mechanics*, Oxford, Clarendon Press (1947).
- P. A. M. Dirac, *Spinors in Hilbert space*, Plenum Press (1974).
- M. E. Rose, *Elementary theory of angular momentum*, John Wiley.
- R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path integrals*.
- L. D. Landau and E. M. Lifshitz, *Statistical physics, Course of Theoretical Physics*. Vol. 5. Pergamon Press Ltd (1958).
- S. Flugge, *Practical quantum mechanics*, Springer-Verlag (1999).

- H. Weyl, *The theory of groups and Quantum Mechanics*.

B29. Analytical Mechanics

Generalised coordinates, Lagranges Equation. Examples of Lagranges equation. Conservation laws. Motion in a central field. Collision of particles. Small Oscillations. Rotating Coordinate systems. Inertial forces. Dynamics of a rigid body. Hamiltonian Mechanics.

<u>References:</u>

- V. I. Arnold, *Mathematical methods of classical mechanics* GTM (60), Springer- Verlag (1978).
- R. Abraham and J. E. Marsden, *Foundations of mechanics* Second edition, Benjamin/Cummings (1978).