Test Code: PCB (short answer type) 2013

M.Tech. in Computer Science

Syllabus and Sample Questions

The selection test for M.Tech. in Computer Science will consist of two parts.

- Test **MMA** (objective type) in the forenoon session is the 1^{st} part, and
- Test **PCB** (short answer type) in the afternoon session is the 2^{nd} part.

  The **PCB** test will consist of two groups.

  ◇ **Group A** (24 Marks) : All candidates have to answer questions on analytical ability and mathematics at the undergraduate level.

  ◇ **Group B** (76 Marks) : A candidate has *to choose exactly one of the following five sections*, from which questions have to be answered:
  (i) Mathematics, (ii) Statistics, (iii) Physics, (iv) Computer Science, and (v) Engineering and Technology.
  While questions in the first three sections will be at postgraduate level, those for the last two sections will be at B.Tech. level.

The syllabus and sample questions for the **MMA** test are available separately. The syllabus and sample questions for the **PCB** test are given below.

Note:

1. Not all questions in this sample set are of equal difficulty. They may not carry equal marks in the test. More sample questions are available on the website for M.Tech(CS) at *http://www.isical.ac.in/~deanweb/MTECHCSSQ.html*

2. Each of the two tests **MMA** and **PCB**, will have individual qualifying marks.

SYLLABUS for **Test PCB**

Group A

Elements of set theory. Permutations and combinations. Functions and relations. Theory of equations. Inequalities.
Limits, continuity, sequences and series, differentiation and integration with applications, maxima-minima.

Elementary Euclidean geometry and trigonometry.
Elementary number theory, divisibility, congruences, primality.
Determinants, matrices, solutions of linear equations, vector spaces, linear independence, dimension, rank and inverse.

Group B

Mathematics

In addition to the syllabus for Mathematics in Group A, the syllabus includes:

Calculus and real analysis – real numbers, basic properties, convergence of sequences and series, limits, continuity, uniform continuity of functions, differentiability of functions of one or more variables and applications, indefinite integral, fundamental theorem of Calculus, Riemann integration, improper integrals, double and multiple integrals and applications, sequences and series of functions, uniform convergence.
Linear algebra – vector spaces and linear transformations, matrices and systems of linear equations, characteristic roots and characteristic vectors, Cayley-Hamilton theorem, canonical forms, quadratic forms.
Graph Theory – connectedness, trees, vertex coloring, planar graphs, Eulerian graphs, Hamiltonian graphs, digraphs and tournaments.
Abstract algebra – groups, subgroups, cosets, Lagrange's theorem, normal subgroups and quotient groups, permutation groups, rings, subrings, ideals, integral domains, fields, characteristics of a field, polynomial rings, unique factorization domains, field extensions, finite fields.
Differential equations – solutions of ordinary and partial differential equations and applications.

Statistics

Notions of sample space and probability, combinatorial probability, conditional probability, Bayes' theorem and independence.
Random variable and expectation, moments, standard univariate discrete and continuous distributions, sampling distribution of statistics based on normal samples, central limit theorem, approximation of binomial to normal, Poisson law.
Multinomial, bivariate normal and multivariate normal distributions.
Descriptive statistical measures, product-moment correlation, partial and multiple correlation.
Regression – simple and multiple.
Elementary theory and methods of estimation – unbiasedness, minimum variance, sufficiency, maximum likelihood method, method of moments, least squares methods.
Tests of hypotheses – basic concepts and simple applications of Neyman-Pearson lemma, confidence intervals.

Tests of regression, elements of non-parametric inference, contingency tables and Chi-square, ANOVA, basic designs (CRD/RBD/LSD) and their analyses, elements of factorial designs.
Conventional sampling techniques, ratio and regression methods of estimation.

Physics

General properties of matter – elasticity, surface tension, viscosity.
Classical dynamics – Lagrangian and Hamiltonian formulation, symmetries and conservation laws, motion in central field of force, planetary motion, collision and scattering, mechanics of system of particles, small oscillation and normal modes, wave motion, special theory of relativity.
Electrodynamics – electrostatics, magnetostatics, electromagnetic induction, self and mutual inductance, capacitance, Maxwell's equation in free space and linear isotropic media, boundary conditions of fields at interfaces. Nonrelativistic quantum mechanics – Planck's law, photoelectric effect, Compton effect, wave-particle duality, Heisenberg's uncertainty principle, quantum mechanics, Schrodinger's equation, and some applications.
Thermodynamics and statistical Physics – laws of thermodynamics and their consequences, thermodynamic potentials and Maxwell's relations, chemical potential, phase equilibrium, phase space, microstates and macrostates, partition function free energy, classical and quantum statistics.
Atomic and molecular physics – quantum states of an electron in an atom, Hydrogen atom spectrum, electron spin, spin-orbit coupling, fine structure, Zeeman effect, lasers.
Condensed matter physics – crystal classes, 2D and 3D lattice, reciprocal lattice, bonding, diffraction and structure factor, point defects and dislocations, lattice vibration, free electron theory, electron motion in periodic potential, energy bands in metals, insulators and semiconductors, Hall effect, thermoelectric power, electron transport in semiconductors, dielectrics, Claussius Mossotti equation, Piezo, pyro and ferro electricity.
Nuclear and particle physics – Basics of nuclear properties, nuclear forces, nuclear structures, nuclear reactions, interaction of charged particles and e-m rays with matter, theoretical understanding of radioactive decay, particle physics at the elementary level.
Electronics – semiconductor physics; diodes - clipping, clamping, rectification; Zener regulated power supply, transistor - CC, CB, and CE configuration; transistor as a switch, OR and NOT gates; feedback in amplifiers.
Operational Amplifier and its applications – inverting, noninverting amplifiers, adder, integrator, differentiator, waveform generator comparator, Schmidt trigger.
Digital integrated circuits – NAND, NOR gates as building blocks, XOR gates, combinational circuits, half and full adder.

Computer Science

Data structures – array, stack, queue, linked list, binary tree, heap, AVL tree, B-tree.
Programming languages – Fundamental concepts – abstract data types, procedure call and parameter passing, languages like C and C++.
Design and analysis of algorithms – Asymptotic notation, sorting, selection, searching.
Computer organization and architecture – Number representation, computer arithmetic, memory organization, I/O organization, microprogramming, pipelining, instruction level parallelism.
Operating systems – Memory management, processor management, critical section problem, deadlocks, device management, file systems.
Formal languages and automata theory – Finite automata and regular expressions, pushdown automata, context-free grammars, Turing machines, elements of undecidability.
Principles of Compiler Construction – Lexical analyzer, parser, syntax-directed translation, intermediate code generation.
Database management systems – Relational model, relational algebra, relational calculus, functional dependency, normalization (up to 3^rd normal form).
Computer networks – OSI, LAN technology – Bus/tree, Ring, Star; MAC protocols; WAN technology – circuit switching, packet switching; data communications – data encoding, routing, flow control, error detection/correction, Internet working, TCP/IP networking including IPv4.
Switching Theory and Logic Design – Boolean algebra, minimization of Boolean functions, combinational and sequential circuits - synthesis and design.

Engineering and Technology

C Programming language.
Moments of inertia, motion of a particle in two dimensions, elasticity, friction, strength of materials, surface tension, viscosity and gravitation.
Laws of thermodynamics and heat engines.
Electrostatics, magnetostatics and electromagnetic induction.
Magnetic properties of matter – dia, para and ferromagnetism.
Laws of electrical circuits – RC, RL and RLC circuits, measurement of current, voltage and resistance.
D.C. generators, D.C. motors, induction motors, alternators, transformers.
p-n junction, bipolar & FET devices, transistor amplifier, oscillator, multi-vibrator, operational amplifier.
Digital circuits – logic gates, multiplexer, de-multiplexer, counter, A/D and D/A converters.
Boolean algebra, minimization of switching functions, combinational and sequential circuits.

SAMPLE QUESTIONS

Group A

A1. How many times will the digit '7' be written when listing the integers from 1 to 1000? Justify your answer.

A2. For sets A and B, define $A\Delta B = (\bar{A} \cap B) \cup (A \cap \bar{B})$. Show the following for any three sets A, B and C.

(a) $A\Delta A = \emptyset$.

(b) $A\Delta(B\Delta C) = (A\Delta B)\Delta C$.

(c) If $A\Delta B = A\Delta C$ then $B = C$.

A3. In a group of n persons, each person is asked to write down the sum of the ages of all the other $(n-1)$ persons. Suppose the sums so obtained are $s_1, \ldots, s_n$. It is now desired to find the actual ages of the persons from these values.

(a) Formulate the problem in the form of a system of linear equations.

(b) Can the ages be always uniquely determined? Justify your answer.

A4. Evaluate

$$\lim_{x \to 0} \left(x^2 \left(1 + 2 + 3 + \cdots + \left[\frac{1}{|x|} \right] \right) \right).$$

For any real number a, $[a]$ is the largest integer not greater than a.

Group B

Mathematics

M1. (a) Evaluate

$$\lim_{k\to\infty} \int_0^\infty \frac{dx}{1+kx^{10}} \; .$$

(b) Is it possible to define $f : S \to T$ such that f is continuous and onto for each of the following pairs of S and T? For each pair, provide an example of one such f, if possible; otherwise, show that it is impossible to define one such f.

(i) $S = (0,1) \times (0,1)$ and T is the set of rational numbers.

(ii) $S = (0,1) \times (0,1)$ and $T = [0,1] \times [0,1]$.

M2. (a) Let B be a non-singular matrix. Then prove that λ is an eigenvalue of B if and only if $1/\lambda$ is an eigenvalue of B^{-1}.

(b) If rank$(A) =$ rank(A^2) then show that

$$\{x \; : \; Ax = 0\} \; = \; \{x \; : \; A^2x = 0\}.$$

(c) Let

$$A = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}.$$

Which of the following statements are true? In each case, justify your answer.

(i) The rank of A is equal to the trace of A.

(ii) The determinant of A is equal to the determinant of A^n for all $n > 1$.

M3. (a) (i) For $0 \le \theta \le \pi/2$, show that $\sin\theta \ge 2\theta/\pi$.

(ii) Hence or otherwise show that for $\lambda < 1$,

$$\lim_{x\to\infty} x^\lambda \int_0^{\pi/2} e^{-x\sin\theta} d\theta = 0.$$

(b) Let $a_n \ge 0, n = 1, 2, \ldots$ be such that $\sum a_n$ converges. Show that $\sum \sqrt{a}_n n^{-p}$ converges for every $p > 1/2$.

M4. (a) Let $a_1, a_2, \ldots$ be integers and suppose there exists an integer N such that $a_n = (n-1)$ for all $n \ge N$. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n!}$ is rational.

(b) Let $0 < s_1, s_2, s_3 < 1$. Show that there exists exactly one $x \in (0, \infty)$ such that

$$s_1^x + s_2^x + s_3^x = 1.$$

M5. (a) Let A be an $n \times n$ symmetric matrix and let $l_1, l_2, \ldots, l_{r+s}$ be $(r+s)$ linearly independent $n \times 1$ vectors such that for all $n \times 1$ vectors x,

$$x'Ax = (l_1'x)^2 + \cdots + (l_r'x)^2 - (l_{r+1}'x)^2 - \cdots - (l_{r+s}'x)^2.$$

Prove that rank$(A) = r + s$.

(b) Let A be an $m \times n$ matrix with $m < n$ and rank$(A) = m$. If $B = AA'$, $C = A'A$, and the eigenvalues and eigenvectors of B are known, find the non-zero eigenvalues and corresponding eigenvectors of C.

M6. (a) If T is an injective homomorphism of a finite dimensional vector space V onto a vector space W, prove that T maps a basis of V onto a basis of W.

(b) Find a polynomial of degree 4 which is irreducible over $GF(5)$. Justify your answer.

M7. (a) Let S and T be two subsets of a finite group $(G, +)$ such that $|S| + |T| > |G|$. Here $|X|$ is the number of elements in a set X. Then prove that

$$S + T = G, \quad \text{where} \quad S + T = \{s + t \ : \ s \in S, t \in T\}.$$

(b) A number x is a square modulo p if there is a y such that $y^2 \equiv x \bmod p$. Show that for an odd prime p, the number of squares modulo p is exactly $\frac{p+1}{2}$.

(c) Using (a), (b) or otherwise prove that for any integer n and any odd prime p, there exist x, y such that $n \equiv (x^2 + y^2) \bmod p$.

M8. (a) Give an example of a 3-regular graph on 16 vertices whose chromatic number is 4. Justify your answer.

(b) Give an example of a graph G such that both G and $\overline{G}$ are not planar. Justify your answer.

(c) A graph is said to be 2-connected if deleting any one vertex does not make the graph disconnected. Let G be a 2-connected graph.

(i) Suppose $e = (u, v)$ is an edge of G and x is a vertex of G where x is distinct from u and v. Show that there is a path from x to u which does not go through v.

(ii) Hence or otherwise, show that if e_1 and e_2 are two distinct edges of G, then they lie on a common cycle.

Statistics

S1. Let $p_1 > p_2 > 0$ and $p_1 + p_2 + p_3 = 1$. Let $Y_1, Y_2, \ldots$ be independent and identically distributed random variables where, for all i, $Pr[Y_i = j] = p_j$, $j = 1, 2, 3$. Let $S_{j,n}$ denote the number of Y_i's among $Y_1, \ldots, Y_n$ for which $Y_i = j$. Show that

$$\lim_{n\to\infty} Pr[S_{1,n} - S_{2,n} \ \geq \ 2] = 1.$$

S2. The random variables $X_1, X_2, \ldots, X_k$ are defined iteratively as follows:

X_1 is uniformly distributed on $\{1, \ldots, n\}$ and for $i \geq 2$, the distribution of X_i given $(X_1, \ldots, X_{i-1})$ is uniform on $\{1, 2, \ldots, X_{i-1}\}$.

Find $E(X_k)$ and compute $\lim\limits_{k\to\infty} E(X_k)$.

S3. Let X_1, X_2, X_3 and X_4 be independent random variables having a normal distribution with zero mean and unit variance. Show that

$$\frac{\sqrt{2}(X_1X_3 + X_2X_4)}{{X_3}^2 + {X_4}^2}$$

has a t distribution.

S4. (a) Let $X_1, \ldots, X_n$ be independent Poisson random variables with common expectation λ. Let $\widehat{\lambda} = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} X_i$. Is $\exp(-\widehat{\lambda})$ an unbiased estimator of $\exp(-\lambda)$? Justify your answer.

(b) Let X_1, X_2 and X_3 be independent random variables such that X_i is uniformly distributed in $(0, i\theta)$ for $i = 1, 2, 3$. Find the maximum likelihood estimator of θ and examine whether it is unbiased for θ.

S5. Consider the following Gauss-Markov linear model:

$$\begin{aligned}
E(y_1) &= \theta_0 + \theta_1 + \theta_2, \\
E(y_2) &= \theta_0 + \theta_1 + \theta_3, \\
E(y_3) &= \theta_0 + \theta_2 + \theta_3, \\
E(y_4) &= \theta_0 + \theta_1 + \theta_2.
\end{aligned}$$

(a) Determine the condition under which the parametric function $\displaystyle\sum_{i=0}^{3} c_i\theta_i$ is estimable for known constants $c_i,\ i = 0, 1, 2, 3$.

(b) Obtain the least squares estimates of the parameters $\theta_0, \theta_1, \theta_2$ and θ_3.

(c) Obtain the best linear unbiased estimator of $(2\theta_1 - \theta_2 - \theta_3)$ and also determine its variance.

S6. Suppose Y is regressed on X_1, X_2 and X_3 with an intercept term and the following are computed:

$$Y'Y = 5000;\ Y'X = (20, 30, 50, -40);\ X'X = \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 19 & 3 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}.$$

(a) Compute the regression coefficients.

(b) Compute the ANOVA table.

(c) Compute the estimate of the error variance, and the estimates of the variances of all the regression coefficients.

S7. Suppose that a coin is tossed 10 times.

(a) Find the most powerful test at level $\alpha = 0.05$ to test whether the coin is fair against the alternative that the coin is more likely to show up heads.

(b) What will be the conclusion of the test if there are exactly 7 heads in 10 tosses?

(c) Find the power function of this test.

S8. (a) Consider a randomized block design with v treatments, each replicated r times. Let t_i be the effect of the i-th treatment. Find $Cov(\sum a_i\hat{t}_i, \sum b_i\hat{t}_i)$ where $\sum a_i\hat{t}_i$ and $\sum b_i\hat{t}_i$ are the best linear unbiased estimators of $\sum a_i t_i$ and $\sum b_i t_i$ respectively and $\sum a_i = \sum b_i = \sum a_i b_i = 0$.

(b) A sample S_1 of n units is selected from a population of N units using SRSWOR. Observations on a variable Y are obtained for the n_1 units of S_1 who responded. Later, a further sub-sample S_2 of m units is selected using SRSWOR out of the $(n - n_1)$ units of S_1 who did not respond. Assuming that Y could be observed for all the m units of S_2, find the following:

(i) an unbiased estimator of the population mean $\bar{Y}$ on the basis of the available observations on Y,

(ii) an expression for the variance of the proposed estimator.

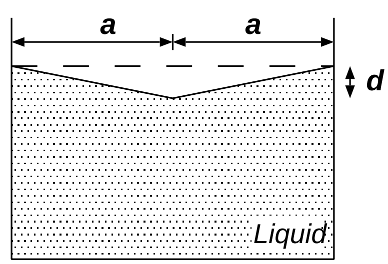
Physics

P1. (a) Consider the central force $\overrightarrow{F}(\overrightarrow{r}) = F(r)\frac{\overrightarrow{r}}{r}$. Show that $\overrightarrow{\nabla} \times \overrightarrow{F} = 0$.

(b) An electron is describing an orbit of radius a around the z-axis and in a plane perpendicular to it. A uniform magnetic field $\overrightarrow{B}$ is acting along the z-axis. Determine the magnitude of the angular momentum of the electron.

(c) In an X-ray machine, an accelerating potential of 50 KeV is applied on the electrons emitted by the cathode. What is the minimum wavelength present in the X-rays emitted?

(d) A wire of negligible thickness having mass per unit length λ is floating in a liquid kept in a square vessel of side $2a$. The wire is floating parallel to one of the sides of the vessel. The top surface of the liquid (in contact with the wire) dips by a distance d as shown in the following figure. What is the surface tension of the liquid? (Assume $d \ll a$.)



P2. (a) A particle is falling freely from a height h at 30° latitude in the northern hemisphere. Show that the particle will undergo a deflection of $\omega\sqrt{\frac{2h^3}{3g}}$ in the eastward direction, where ω is the rotational velocity of the earth about its own axis and g is the acceleration due to gravity.

(b) A particle of mass m is moving in a plane in the field of force $\overrightarrow{F} = -\widehat{r}kr\cos\theta$, where k is a constant, $\widehat{r}$ is the radial unit vector and θ is the polar angle.

(i) Write the Lagrangian of the system.

(ii) Show that the Lagrange's equations of motion are:

A. $m\ddot{r} - mr\dot{\theta}^2 + kr\cos\theta = 0$;

B. $mr^2\dot{\theta} \neq$ constant.

(iii) Interpret (ii)B in the context of Kepler's second law.

P3. (a) In an inertial frame, two events have the space-time coordinates $\{x_1, y, z, t_1\}$ and $\{x_2, y, z, t_2\}$ respectively. Let

$(x_2 - x_1) = 3c(t_2 - t_1)$, where c represents the velocity of light in vacuum. Consider another inertial frame which moves with velocity u along x-axis with respect to the first frame. Find the value of u for which the events are simultaneous in the latter frame.

(b) Consider a particle of mass m inside a one-dimensional box of length L. Let the state $\psi(x,t)$ of the particle at $t = 0$ be given by the following:

$$\psi(x,0) = \begin{cases} A\sin(\frac{4\pi x}{L})\cos(\frac{3\pi x}{L}), & 0 \le x \le L \\ 0, & \text{otherwise} \end{cases}$$

(i) Calculate the value of A.

(ii) What is the average momentum of the system at $t = 0$?

(iii) If a measurement is performed to find the energy of the system at $t = 0$, what is the probability of obtaining the value of energy as $\frac{\pi^2\hbar^2}{2mL^2}$?

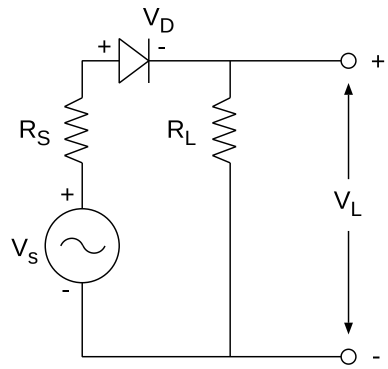
P4. (a) The conductivity parameter of a conducting medium is σ and it contains no free charges. Obtain in explicit form the plane wave solutions of Maxwell's electromagnetic field equations for this medium. How do these solutions physically differ from the corresponding ones for a non-conducting (di-electric) medium?

(b) Show that the electromagnetic field energy is almost exclusively magnetic in a good conductor.

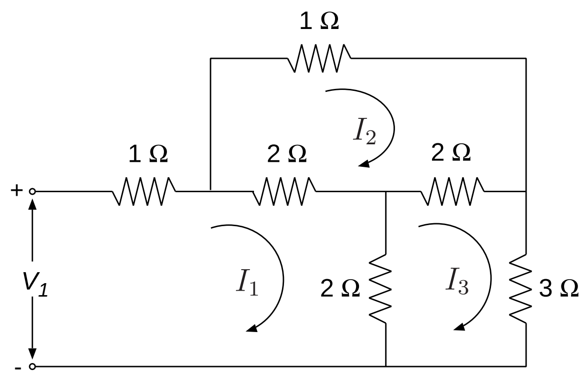
P5. (a) How does one understand molecular mean free path in the context of molecular kinetic theory of gases? Obtain the analytic form of the law governing the distribution of free paths in an ideal gas.

(b) Calculate the mean free path, the collision rate and the molecular diameter for Hydrogen gas molecules having the following particulars: molecular weight of Hydrogen = 2.016 gm; viscosity, $\eta = 85 \times 10^{-6}$ dynes/cm^2/velocity gradient; mean speed, $\overline{c} = 16 \times 10^4$ cm/sec; density, $\rho = 0.000089$ gm/cc.

P6. (a) Consider the following circuit. Assume the diode to be ideal and $R_L = R_S = 100\ \Omega$. Sketch the waveforms of the diode voltage V_D and load voltage V_L if the source voltage varies with time t as $V_S = \sin\omega t$, where ω is the angular frequency.



(b) Determine the port current I_1 in terms of port voltage $V_1 = 9.1$ V for the following one-port network. Determine the equivalent resistance across the one-port network.



(c) Suppose a one-port network is made up of a non-linear resistor such that the current is described by $i = k_1 \exp(v/k_2)$, where k_1 and k_2 are constants and v is the port voltage. Determine v. Show that the equivalent resistance across the one-port network is zero for large values of v.

Computer Science

C1. (a) How many asterisks (`*`) in terms of k will be printed by the following C function, when called as $count(m)$ where $m = 3^k$? Justify your answer. Assume that 4 bytes are used to store an integer in C and k is such that 3^k can be stored in 4 bytes.

```
void count(int n)
{
     printf("*");
     if(n>1)
     {
         count(n/3);
         count(n/3);
         count(n/3);
     }
}
```

(b) A 64000-byte message is to be transmitted over a 2-hop path in a store-and-forward packet-switching network. The network limits packets to a maximum size of 2032 bytes including a 32-byte header. The transmission lines in the network are error free and have a speed of 50 Mbps. Each hop is 1000 km long and the signal propagates at the speed of light (3×10^8 meters per second). Assume that queuing and processing delays at the intermediate node are negligible. How long does it take to deliver the entire message from the source to the destination?

C2. Give an efficient implementation for a data structure `STACK_MAX` to support an operation `max` that reports the current maximum among all elements in the stack. Usual stack operations (`createEmpty`, `push`, `pop`) are also to be supported.

How many bytes are needed to store your data structure after the following operations: `createEmpty`, `push`(5), `push`(6), `push`(7), `pop`, `max`, `push`(6), `push`(8), `pop`, `pop`, `max`, `push`(5). Assume that an integer can be stored in 4 bytes.

C3. You are given an array $X[\,]$. The size of the array is very large but unknown. The first few elements of the array are distinct positive integers in sorted order. The rest of the elements are 0. The number of positive integers in the array is also not known.

Design an algorithm that takes a positive integer y as input and finds the position of y in X. Your algorithm should return "Not found" if y is not in the array. You will get no credit if the complexity of your algorithm is linear (or higher) in the number of positive integers in X.

C4. (a) Prove or disprove the following statement: *The union of a regular language with a disjoint non-regular language over the same alphabet can never be regular*.

[Hint: You may use the closure properties of regular languages.]

(b) It is known that the language $L_1 = \{0^n 1^n 2^i \mid i \neq n\}$ is not a context free language (CFL). Now consider the language $L_2 = \{0^i 1^n 2^n \mid i \neq n\}$. We can prove L_2 is not a CFL by converting L_2 into L_1 by applying two operations, both known to be closed on CFLs. What are the two operations you will use for this conversion? Justify your answer.

C5. Consider three relations $R1(\underline{X}, Y, Z)$, $R2(\underline{M}, N, P)$, and $R3(\underline{N, X})$. The primary keys of the relations are underlined. The relations have 100, 30, and 400 tuples, respectively. The space requirements for different attributes are: $X = 30$ bytes, $Y = 10$ bytes, $Z = 10$ bytes, $M = 20$ bytes, $N = 20$ bytes, and $P = 10$ bytes. Let $V(A, R)$ signify the variety of values that attribute A may have in the relation R. Let $V(N, R2) = 15$ and $V(N, R3) = 300$. Assume that the distribution of values is uniform.

(a) If $R1$, $R2$, and $R3$ are to be joined, find the order of join for the minimum cost. The cost of a join is defined as the total space required by the intermediate relations. Justify your answer.

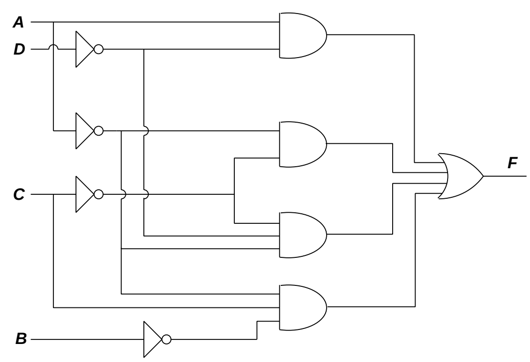
(b) Calculate the minimum number of disk accesses (including both reading the relations and writing the results) required to join $R1$ and $R3$ using block-oriented loop algorithm. Assume that (i) 10 tuples occupy a block and (ii) the smaller of the two relations can be totally accommodated in main memory during execution of the join.

C6. (a) Consider three processes, P_1, P_2, and P_3. Their start times and execution times are given below.

Process	Start time	Execution time
P_1	$t = 0$ ms	100 ms
P_2	$t = 25$ ms	50 ms
P_3	$t = 50$ ms	20 ms

Let Δ be the amount of time taken by the kernel to complete a context switch from any process P_i to P_j. For what values of Δ will the average turnaround time for P_1, P_2, P_3 be reduced by choosing a Shortest Remaining Time First scheduling policy over a Shortest Job First policy?

(b) The circuit shown in the following figure computes a Boolean function F. Assuming that all gates cost Rs. 5 per input (i.e., an inverter costs Rs. 5, a 2-input gate costs Rs. 10, etc.), find the minimum cost realization of F using only inverters, AND / OR gates.



C7. (a) Identifiers in a certain language have the following properties:

- they start with a lower case letter,
- they may contain upper case letters, but each uppercase letter must be followed by one or more lower case letters,
- they may contain digits but only at the end.

Thus, `num` and `varName1` are valid identifiers, but `aBC` and `a2i` are not. Write a regular expression for such identifiers. You may use extended notation if necessary.

(b) Consider the following grammar G.

$$\begin{aligned} S &\rightarrow L = E \\ E &\rightarrow L \\ L &\rightarrow \mathbf{id} \\ L &\rightarrow \mathit{Elist}\ ] \\ \mathit{Elist} &\rightarrow \mathbf{id}\ [\ E \\ \mathit{Elist} &\rightarrow \mathit{Elist}, E \end{aligned}$$

S, L, E, and *Elist* are the non-terminals; all other symbols appearing in the above grammar are terminals. Construct an LL(1) grammar that is equivalent to G.

C8. (a) Let $a_{n-1}a_{n-2}\ldots a_0$ and $b_{n-1}b_{n-2}\ldots b_0$ denote the 2's complement representation of two integers A and B respectively. Addition of A and B yields a sum $S = s_{n-1}s_{n-2}\ldots s_0$. The outgoing carry generated at the most significant bit position, if any, is ignored. Show that an overflow (incorrect addition result) will occur only if the following Boolean condition holds:

$$\overline{s}_{n-1} \oplus (a_{n-1}s_{n-1}) = b_{n-1}(s_{n-1} \oplus a_{n-1})$$

where $\oplus$ denotes the Boolean XOR operation. You may use the Boolean identity: $X + Y = X \oplus Y \oplus (XY)$ to prove your result.

(b) Consider a machine with 5 stages F, D, X, M, W, where F denotes instruction fetch, D - instruction decode and register fetch, X - execute/address calculation, M - memory access, and W - write back to a register. The stage F needs 9 nanoseconds (ns), D needs 3 ns, X requires 7 ns, M needs 9 ns, and W takes 2 ns. Let M_1 denote a non-pipelined implementation of the machine, where each instruction has to be executed in a single clock cycle. Let M_2 denote a 5-stage pipelined version of the machine. Assume that pipeline overhead is 1 ns for each stage. Calculate the maximum clock frequency that can be used in M_1 and in M_2.

Engineering and Technology

E1. (a) A 44 KW d.c. machine has 110 Ω shunt resistance and 0.05 Ω armature resistance. Calculate the total armature power developed when the machine is working as (i) a generator and (ii) a motor.

(b) Test data of a 200/400 V, 10 KVA transformer is as follows:

(i) Open-circuit test on primary side: 200 V, 50 A, 2500 W

(ii) Short-circuit test on secondary side: 20 V, 10 A, 80 W

Calculate the total loss at full-load with unity power factor.

E2. Two long straight parallel wires stand 2 meters apart in air and carry currents I_1 and I_2 in the same direction. The field intensity at a point midway between the wires is 7.95 Ampere-turn per meter. The force on each wire per unit length is 2.4 $\times 10^{-4}$ N. Assume that the absolute permeability of air is $4\pi \times 10^{-7}$ H per meter.

(a) Explain the nature of the force experienced between the two wires, *i.e.* attractive or repulsive.

(b) Determine I_1 and I_2.

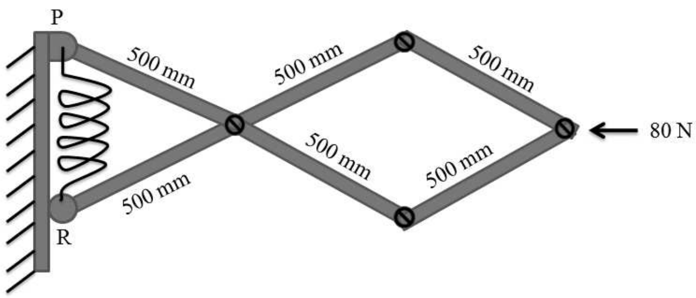
(c) Another parallel wire carrying a current of 50 A in the opposite direction is now placed midway between the two wires and in the same plane. Determine the resultant force on this wire.

E3. A choke coil connected across a 500 V, 50 Hz supply takes 1 A current at a power factor of 0.8.

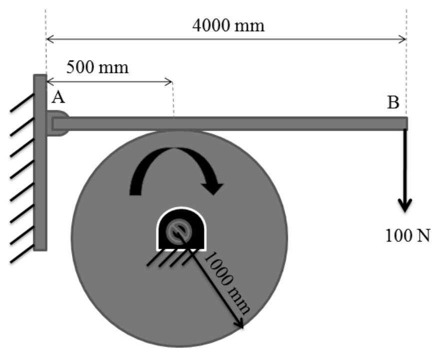
(a) Determine the capacitance that must be placed in series with the choke coil so that it resonates at 50 Hz.

(b) An additional capacitor is now connected in parallel with the above combination in (a) to change the resonant frequency. Obtain an expression for the additional capacitance in terms of the new resonant frequency.

E4. (a) The mechanical system shown in the figure below is loaded by a horizontal 80 N force. The length of the spring is 500 mm. Each arm of the mechanical system is also of length 500 mm as shown in the figure. Under the influence of 80 N load, the spring is stretched to 600 mm but the entire mechanical system including the spring remains in equilibrium. Determine the stiffness of the spring. Note that the spring and the frame are fixed at the pin position P. The other end of the spring is at R which is a frictionless roller free to move along the vertical axis. Assume that the mechanical joints between the arms are frictionless.



(b) A brake system is shown in the figure below. The solid disk of radius 1000 mm is being rotated at 196 rpm. The bar AB, of length 4000 mm, is fixed at the end A and subjected to a downward load of 100 N at the end B to stop the rotation of the disk. The bar AB (assumed to be horizontal) touches the rotating disk at a point 500 mm from the fixed end of the bar. The weight of the disk is 10 Kg and the coefficient of friction between the bar and the disk is 0.5. Calculate the number of revolutions the disk will make before coming to rest.



E5. (a) Air at 90°C and 605 Kg per square meter pressure is heated to 180°C keeping the volume constant at 21 cubic meter. Find

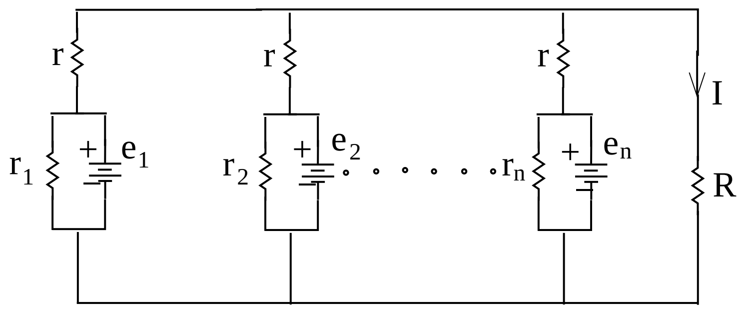
(i) the final pressure, and

(ii) the change in the internal energy.

Note that the specific heat at constant pressure (C_p), the specific heat at constant volume (C_v), and the mechanical equivalent of heat are 0.3, 0.2 and 420 Kg-meter per Kcal, respectively.

(b) A molten metal is forced through a cylindrical die at a pressure of 168×10^3 Kg per square meter. Given that the density of the molten metal is 2000 Kg per cubic meter and the specific heat of the metal is 0.03, find the rise in temperature during this process. Assume that the mechanical equivalent of heat is 420 Kg-meter per Kcal.

E6. (a) Calculate the current I flowing through the resistor R shown in the following figure ($e_1 < e_2 < \cdots < e_n$).



(b) A parallel plate capacitor is charged to 75 μC at 100 V. After removing the 100 V source, the capacitor is immediately connected to an uncharged capacitor with capacitance twice that of the first one. Determine the energy of the system before and after the connection is made. Assume that all capacitors are ideal.

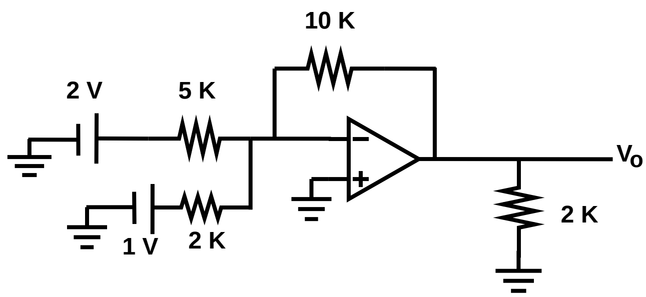
E7. (a) Design a logic circuit to implement the following truth table. What is the function of this logic circuit?

Inputs		Outputs	
A	B	C	D
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

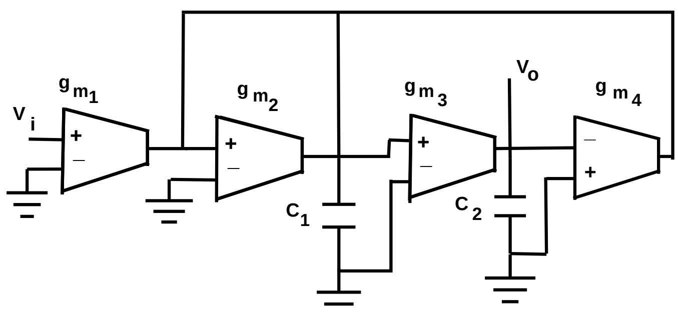
(b) Show that the logic circuit corresponding to the following truth table can be realized by interconnecting two instances of the logic circuit derived in (a) and an additional OR gate.

Inputs			Outputs	
A	B	C	D	E
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

E8. (a) Consider the following circuit with an ideal Op-amp. Calculate V_o.



(b) The following network uses four transconductor amplifiers and two capacitors to produce the output voltage V_o for the input voltage V_i.

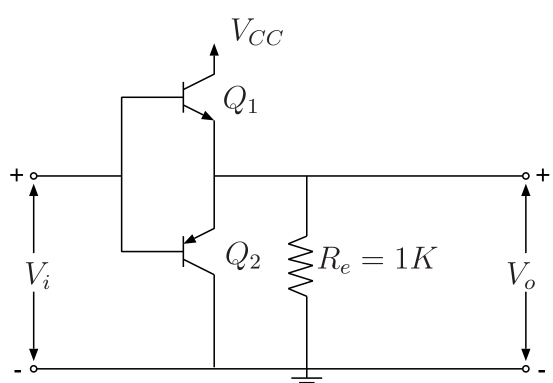


(i) Show that the voltage transfer function $H(s)$ can be expressed as:

$$H(s) = \frac{V_o}{V_i} = \frac{g_{m_1}/g_{m_4}}{1 + (\frac{g_{m_2}C_2}{g_{m_3}g_{m_4}})s + (\frac{C_1C_2}{g_{m_3}g_{m_4}})s^2}.$$

(ii) Does the transfer function suggest a lowpass, bandpass or highpass frequency response? Briefly explain.

E9. Consider the amplifier shown in the following figure.



(i) Draw the equivalent circuit using the small-signal hybrid parameter model.

(ii) For the following values of h parameters for both transistors: $h_{ie} = 1000\ \Omega$, $h_{fe} = 100$, $h_{re} = h_{oe} = 0$, determine the voltage amplification A_v and the input resistance R_{in}.

E10. (i) Find the output of the following C program.

```
#include <stdio.h>
void PRINT1(void)
{
  static int x = 10;
  x += 5;
  printf("%d ", x);
}
void PRINT2(void)
{
  static int x;
  x = 10;
  x += 5;
  printf("%d ", x);
}
int main( )
{
  PRINT1( ); PRINT1( ); PRINT2( ); PRINT2( );
  return 0;
}
```

(ii) Explain the output of the following C program.

```
#include <stdio.h>
int main( )
{
  char string1[15] = "ISI, Kolkata";
  char string2[15] = "ISI, Kolkata";
  if (string1 == string2)
     printf("Two strings are equal");
  else
     printf("Two strings are unequal");
  return 0;
}
```

(iii) Write a C function for finding the second largest element in an array of n integers. Note that the array contains at least two distinct integers. For example, if the array contains $30, 21, 12, 30$, $-5, 21$ and 10, your function should return 21. The function must use as few comparisons as possible.

——x——