Test Code PSB (Short answer type) 2013

Syllabus for Mathematics

Combinatorics; Elements of set theory. Permutations and combinations. Binomial and multinomial theorem. Theory of equations. Inequalities.

Linear Algebra: Vectors and vector spaces. Matrices. Determinants. Solution of linear equations. Trigonometry. Co-ordinate geometry.

Complex Numbers: Geometry of complex numbers and De Moivres theorem.

Calculus: Convergence of sequences and series. Functions. Limits and continuity of functions of one or more variables. Power series. Differentiation. Leibnitz formula. Applications of differential calculus, maxima and minima. Taylor's theorem. Differentiation of functions of several variables. Indefinite integral. Fundamental theorem of calculus. Riemann integration and properties. Improper integrals. Double and multiple integrals and applications.

Syllabus for Statistics and Probability

Probability and Sampling Distributions: Notions of sample space and probability. Combinatorial probability. Conditional probability and independence. Random variables and expectations. Moments and moment generating functions. Standard univariate discrete and continuous distributions. Joint probability distributions. Multinomial distribution. Bivariate normal and multivariate normal distributions. Sampling distributions of statistics. Weak law of large numbers. Central limit theorem.

Descriptive Statistics: Descriptive statistical measures. Contingency tables and measures of association. Product moment and other types of correlation. Partial and multiple correlation. Simple and multiple linear regression.

Statistical Inference: Elementary theory of estimation (unbiasedness, minimum variance, sufficiency). Methods of estimation (maximum likelihood method, method of moments). Tests of hypotheses (basic concepts and simple applications of Neyman-Pearson Lemma). Confidence intervals. Inference related to regression. ANOVA. Elements of nonparametric inference.

Design of Experiments and Sample Surveys: Basic designs such as CRD, RBD, LSD and their analyses. Elements of factorial designs. Conventional sampling techniques (SRSWR/SRSWOR) including stratification. Ratio and regression methods of estimation.

Sample Questions

1. Suppose V is the space of all $n \times n$ matrices with real elements. Define $T : V \to V$ by $T(A) = AB - BA$, $A \in V$, where $B \in V$ is a fixed matrix. Show that for any $B \in V$

   (a) T is linear;

   (b) T is not one-one;

   (c) T is not onto.

2. Let f be a real valued function satisfying

$$|f(x) - f(a)| \leq C\,|x - a|^{\gamma},$$

   for some $\gamma > 0$ and $C > 0$.

   (a) If $\gamma = 1$, show that f is continuous at a;

   (b) If $\gamma > 1$, show that f is differentiable at a.

3. Suppose integers are formed by taking one or more digits from the following

$$2, 2, 3, 3, 4, 5, 5, 5, 6, 7.$$

   For example, 355 is a possible choice while 44 is not. Find the number of distinct integers that can be formed in which

   (a) the digits are non-decreasing;

   (b) the digits are strictly increasing.

4. Consider n independent observations $\{(x_i, y_i) : 1 \leq i \leq n\}$ from the model

$$Y = \alpha + \beta x + \epsilon,$$

where ϵ is normal with mean 0 and variance σ^2. Let $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\sigma}^2$ be the maximum likelihood estimators of α, β and σ^2, respectively. Let v_{11}, v_{22} and v_{12} be the estimated values of $\text{Var}(\hat{\alpha})$, $\text{Var}(\hat{\beta})$ and $\text{Cov}(\hat{\alpha}, \hat{\beta})$, respectively.

(a) What is the estimated mean of Y when $x = x_0$? Estimate the mean squared error of this estimator.

(b) What is the predicted value of Y when $x = x_0$? Estimate the mean squared error of this predictor.

5. A box has an unknown number of tickets serially numbered $1, 2, \ldots, N$. Two tickets are drawn using simple random sampling without replacement (SRSWOR) from the box. If X and Y are the numbers on these two tickets and $Z = \max(X, Y)$, show that

(a) Z is not unbiased for N;

(b) $aX + bY + c$ is unbiased for N if and only is $a + b = 2$ and $c = -1$.

6. Suppose X_1, X_2 and X_3 are three independent and identically distributed Bernoulli random variables with parameter p, $0 < p < 1$. Verify if the following statistics are sufficient for p:

(a) $X_1 + 2X_2 + X_3$;

(b) $2X_1 + 3X_2 + 4X_3$.

7. Suppose X_1 and X_2 are two independent and identically distributed random variables with Normal $(\theta, 1)$ distribution. Further, consider a Bernoulli random variable V with $P[V = 1] = 1/4$, which is independent of X_1 and X_2. Define X_3 as

$$X_3 = \begin{cases} X_1 & \text{when } V = 0, \\ X_2 & \text{when } V = 1. \end{cases}$$

For testing $H_0 : \theta = 0$ against $H_1 : \theta = 1$ consider the test:

$$\text{Reject } H_0 \text{ if } \left(X_1 + X_2 + X_3\right)/3 > c.$$

Find c such that the test has size 0.05.

8. Suppose X_1 is a standard normal random variable. Define

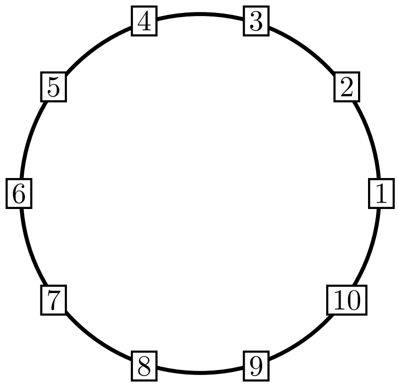
$$X_2 = \begin{cases} -X_1 & \text{if } |X_1| < 1, \\ X_1 & \text{otherwise.} \end{cases}$$

   (a) Show that X_2 is also a standard normal random variable.

   (b) Obtain the cumulative distribution function of X_1+X_2 in terms of the cumulative distribution function of a standard normal random variable.

9. Envelopes are on sale for Rs. 30 each. Each envelope contains exactly one coupon, which can be one of four types with equal probability. Suppose you keep on buying envelopes and stop when you collect all the four types of coupons. What will be your expected expenditure?

10. There are 10 empty boxes numbered $1, 2, \ldots, 10$ placed sequentially on a circle as shown in the figure.



We perform 100 independent trials. At each trial, one box is selected with probability $1/10$ and one ball is placed in each of the two neighbouring boxes of the selected one.

Define X_k to be the number of balls in the k^{th} box at the end of 100 trials.

(a) Find $\mathrm{E}[X_k]$ for $1 \leq k \leq 10$.

(b) Find $\mathrm{Cov}(X_k, X_5)$ for $1 \leq k \leq 10$.

Note: For more sample questions you can visit
`http://www.isical.ac.in/~deanweb/MSTATSQ.html`.