

Test Code: CS (Short answer type) 2007

M.Tech. in Computer Science

The candidates for M.Tech. in Computer Science will have to take two tests – Test MIII (objective type) in the forenoon session and Test CS (short answer type) in the afternoon session. The CS test booklet will have two groups as follows.

GROUP A

A test for all candidates in analytical ability and mathematics at the B.Sc. (pass) level, carrying 30 marks.

GROUP B

A test, divided into several sections, carrying equal marks of 70 in mathematics, statistics, and physics at the B. Sc. (Hons.) level and in computer science, and in engineering and technology at the B.Tech. level. A candidate has to answer questions from ***only one*** of these sections according to his/her choice.

The syllabi and sample questions of the CS test are given below.

Note: All questions in the sample set are not of equal difficulty. They may not carry equal marks in the test.

Syllabus

GROUP A

Elements of set theory. Permutations and combinations. Functions and relations. Theory of equations. Inequalities.

Limit, continuity, sequences and series, differentiation and integration with applications, maxima-minima, complex numbers and De Moivre's theorem.

Elementary Euclidean geometry and trigonometry.

Elementary number theory, divisibility, congruences, primality.

Determinants, matrices, solutions of linear equations, vector spaces, linear independence, dimension, rank and inverse.

GROUP B

Mathematics (B.Sc. Hons. level)

In addition to the syllabus of Mathematics in Group A, the syllabus includes:

Calculus and real analysis – Real numbers, basic properties; convergence of sequences and series; limits, continuity, uniform continuity of functions; differentiability of functions of one or more variables and applications. Indefinite integral, fundamental theorem of Calculus, Riemann integration, improper integrals, double and multiple integrals and applications. Sequences and series of functions, uniform convergence.

Linear algebra - Vector spaces and linear transformations; matrices and systems of linear equations, characteristic roots and characteristic vectors, Cayley-Hamilton theorem, canonical forms, quadratic forms.

Graph Theory - Connectedness, trees, vertex coloring, planar graphs, Eulerian graphs, Hamiltonian graphs, digraphs and tournaments.

Abstract algebra – Groups, subgroups, cosets, Lagrange's theorem; normal subgroups and quotient groups; permutation groups; rings, subrings, ideals, integral domains, fields, characteristics of a field, polynomial rings, unique factorization domains, field extensions, finite fields.

Differential equations – Solutions of ordinary and partial differential equations and applications.

Statistics (B.Sc. Hons. level)

Notions of sample space and probability, combinatorial probability, conditional probability, Bayes theorem and independence, random variable and expectation, moments, standard univariate discrete and continuous distributions, sampling distribution of statistics based on normal samples, central limit theorem, approximation of binomial to normal. Poisson law, Multinomial, bivariate normal and multivariate normal distributions.

Descriptive statistical measures, product-moment correlation, partial and multiple correlation; regression (simple and multiple); elementary theory and methods of estimation (unbiasedness, minimum variance, sufficiency, maximum likelihood method, method of moments, least squares methods). Tests of hypotheses (basic concepts and simple applications of Neyman-Pearson Lemma). Confidence intervals. Tests of regression. Elements of non-parametric inference. Contingency tables and Chi-square, ANOVA, basic designs (CRD/RBD/LSD) and their analyses. Elements of factorial designs. Conventional sampling techniques, ratio and regression methods of estimation.

Physics
(B.Sc. Hons. level)

General properties of matter – Elasticity, surface tension, viscosity.
Classical Dynamics – Lagrangian and Hamiltonian formulation, Symmetries and conservation laws, Motion in central field of force, collision and scattering, mechanics of many system of particles, small oscillation and normal modes, wave motion, Special theory of relativity.
Electrodynamics – Electrostatics, magnetostatics, electromagnetic induction, self and mutual inductance, capacitance, Maxwell's equation in free space and linear isotropic media, boundary conditions of fields at interfaces.

Physical optics – Interference, diffraction, polarization.

Nonrelativistic quantum mechanics – Wave particle duality, Heisenberg's uncertainty principle, Schrodinger's equation, particle in a box, harmonic oscillator, tunneling through a barrier, motion in a central potential, orbital angular momentum, angular momentum algebra.

Thermodynamics and Statistical Physics – Laws of thermodynamics and their consequences, thermodynamic potentials and Maxwell's relations, chemical potential, phase equilibrium, phase space, microstates and macrostates, partition function free energy, classical and quantum statistics.

Electronics – Semiconductor physics, diode as a circuit element, clipping, clamping, rectification, Zener regulated power supply, transistor as a circuit element, CC CB CE configuration, transistor as a switch, OR and NOT gates feedback in amplifiers.

Operational Amplifier and its applications – Inverting, noninverting amplifiers, adder, integrator, differentiator, waveform generator comparator and Schmitt trigger.

Digital integrated circuits – NAND, NOR gates as building blocks, XOR gates, combinational circuits, half and full adder.

Atomic and molecular physics – Quantum states of an electron in an atom, Hydrogen atom spectrum, electron spin, spin-orbit coupling, fine structure, Zeeman effect, Lasers.

Condensed matter physics – Crystal classes, 2D and 3D lattice, reciprocal lattice, bonding, diffraction and structure factor, point defects and dislocations, lattice vibration, free electron theory, electron motion in periodic potential, energy bands in metals, insulators and semiconductors, Hall effect, thermoelectric power, electron transport in semiconductors, dielectrics, Clausius Mossotti equation, Piezo, pyro and ferro electricity.

Nuclear and particle physics – Basics of nuclear properties, nuclear forces, nuclear structures, nuclear reactions, interaction of charged particles and e-m rays with matter, theoretical understanding of radioactive decay, particle physics at the elementary level.

Computer Science (B.Tech. level)

Data structure – Arrays, stack, queue, linked list, binary tree, heap, AVL tree, B-tree.

Programming languages – Fundamental concepts, abstract data types, procedure call and parameter passing, languages like C and C++.

Design and analysis of algorithms – Sorting, selection, searching.

Computer organization and architecture – Number representation, computer arithmetic, memory organization, I/O organization, microprogramming, pipelining, instruction level parallelism.

Operating systems – Memory management, processor management, critical section, deadlocks, device management.

Formal languages and automata theory – Finite automata and regular expression, pushdown automata, context-free grammars, Turing machines, elements of undecidability.

Principles of Compiler Construction – Lexical analyzer, parser, code optimization, symbol table.

Database management systems – Relational model, relational algebra, relational calculus, functional dependency, normalization (up to 3rd normal form).

Computer networks – OSI, TCP/IP protocol; internetworking; LAN technology – Bus/tree, Ring, Star; MAC protocols; WAN technology - Circuit switching, packet switching; data communications, data encoding, routing, flow control, error detection/correction.

Switching Theory and Logic Design – Boolean algebra, minimization of Boolean functions, combinational and sequential circuits – synthesis and design.

Engineering and Technology
(B.Tech. level)

Moments of inertia, motion of a particle in two dimensions, elasticity, friction, strength of materials, surface tension, viscosity and gravitation.

Laws of thermodynamics, and heat engines.

Electrostatics, magnetostatics and electromagnetic induction.

Magnetic properties of matter – dia, para and ferromagnetism.

Laws of electrical circuits – RC, RL and RLC circuits, measurement of current, voltage and resistance.

D. C. generators, D. C. motors, induction motors, alternators, transformers.

p-n junction, bipolar & FET devices, transistor amplifier, oscillator, multi-vibrator, operational amplifier.

Digital circuits – Logic gates, multiplexer, de-multiplexer, counter, A/D and D/A converters.

Boolean algebra, minimization of switching function, combinational and sequential circuits.

Microprocessor/assembly language programming, C.

Sample Questions

GROUP A

Mathematics

A1. If $1, a_1, a_2, \dots, a_{n-1}$ are the n roots of unity, find the value of $(1 - a_1)(1 - a_2) \dots (1 - a_{n-1})$.

A2. Let

$$S = \{(a_1, a_2, a_3, a_4) : a_i \in \mathbb{R}, i=1, 2, 3, 4 \text{ and } a_1 + a_2 + a_3 + a_4 = 0\}$$

and

$$\Gamma = \{(a_1, a_2, a_3, a_4) : a_i \in \mathbb{R}, i=1, 2, 3, 4 \text{ and } a_1 - a_2 + a_3 - a_4 = 0\}.$$

Find a basis for $S \cap \Gamma$.

A3. Provide the inverse of the following matrix:

$$\begin{pmatrix} c_0 & c_1 & c_2 & c_3 \\ c_2 & c_3 & c_0 & c_1 \\ c_3 & -c_2 & c_1 & -c_0 \\ c_1 & -c_0 & c_3 & -c_2 \end{pmatrix}$$

$$\text{where } c_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}, c_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}, c_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}, \text{ and } c_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}.$$

(Hint: What is $c_0^2 + c_1^2 + c_2^2 + c_3^2$?)

A4. For any real number x and for any positive integer n show that

$$[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + \dots + \left[x + \frac{n-1}{n}\right] = [nx]$$

where $[a]$ denotes the largest integer less than or equal to a .

A5. Let $b_q b_{q-1} \dots b_1 b_0$ be the binary representation of an integer b , i.e.,

$$b = \sum_{j=0}^q 2^j b_j, b_j = 0 \text{ or } 1, \text{ for } j = 0, 1, \dots, q.$$

Show that b is divisible by 3 if $b_0 - b_1 + b_2 - \dots + (-1)^q b_q = 0$.

A6. A sequence $\{x_n\}$ is defined by $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2+x_n}$, $n=1, 2, \dots$

Show that the sequence converges and find its limit.

- A7. Is $\sin(x|x|)$ differentiable for all real x ? Justify your answer.
- A8. Find the total number of English words (all of which may not have proper English meaning) of length 10, where all ten letters in a word are not distinct.
- A9. Let $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \dots + \frac{a_n}{n+1} = 0$, where a_i 's are some real constants. Prove that the equation $a_0 + a_1x + a_2x^2 + \dots + a_nx^n = 0$ has at least one solution in the interval $(0, 1)$.
- A10. Let $\phi(n)$ be the number of positive integers less than n and having no common factor with n . For example, for $n = 8$, the numbers 1, 3, 5, 7 have no common factors with 8, and hence $\phi(8) = 4$. Show that
- $\phi(p) = p - 1$,
 - $\phi(pq) = \phi(p)\phi(q)$, where p and q are prime numbers.
- A11. A set S contains integers 1 and 2. S also contains all integers of the form $3x + y$ where x and y are distinct elements of S , and every element of S other than 1 and 2 can be obtained as above. What is S ? Justify your answer.
- A12. Let f be a real-valued function such that $f(x+y) = f(x) + f(y) \forall x, y \in \mathbf{R}$. Define a function ϕ by $\phi(x) = c + f(x)$, $x \in \mathbf{R}$, where c is a real constant. Show that for every positive integer n ,
- $$\phi^n(x) = (c + f(c) + f^2(c) + \dots + f^{n-1}(c)) + f^n(x);$$
- where, for a real-valued function g , $g^n(x)$ is defined by
- $$g^0(x) = 0, g^1(x) = g(x), g^{k+1}(x) = g(g^k(x)).$$
- A13. Consider a square grazing field with each side of length 8 metres. There is a pillar at the centre of the field (i.e. at the intersection of the two diagonals). A cow is tied with the pillar using a rope of length $\frac{8}{\sqrt{3}}$ metres. Find the area of the part of the field that the cow is allowed to graze.

- A14. Let $f: [0,1] \rightarrow [-1,1]$ be such that $f(0) = 0$ and $f(x) = \sin \frac{1}{x}$ for $x > 0$. Is it possible to get three sequences $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ satisfying all the three properties P1, P2 and P3 stated below? If so, provide an example sequence for each of the three sequences. Otherwise, prove that it is impossible to get three such sequences.

P1: $a_n > 0, b_n > 0, c_n > 0$, for all n .

P2: $\lim_{n \rightarrow \infty} a_n = 0, \lim_{n \rightarrow \infty} b_n = 0, \lim_{n \rightarrow \infty} c_n = 0$.

P3: $\lim_{n \rightarrow \infty} f(a_n) = 0, \lim_{n \rightarrow \infty} f(b_n) = 0.5, \lim_{n \rightarrow \infty} f(c_n) = 1$.

- A15. Let $a_1 a_2 a_3 \dots a_k$ be the decimal representation of an integer a ($a_i \in \{0, \dots, 9\}$ for $i = 1, 2, \dots, k$). For example, if $a = 1031$, then $a_1=1, a_2=0, a_3=3, a_4=1$. Show that a is divisible by 11 if and only if

$$\sum_{i \text{ odd}} a_i - \sum_{i \text{ even}} a_i$$

is divisible by 11.

GROUP B

Mathematics

- M1. Let $0 < x_1 < 1$. If $x_{n+1} = \frac{x_n + 3}{3x_n + 1}$, $n = 1, 2, 3, \dots$

(i) Show that $x_{n+2} = \frac{5x_n + 3}{3x_n + 5}$, $n = 1, 2, 3, \dots$

(ii) Hence or otherwise, show that $\lim_{n \rightarrow \infty} x_n$ exists.

(iii) Find $\lim_{n \rightarrow \infty} x_n$.

- M2. (a) A function f is defined over the real line as follows:

$$f(x) = \begin{cases} x \sin \frac{\pi}{x}, & x > 0 \\ 0, & x = 0. \end{cases}$$

Show that $f'(x)$ vanishes at infinitely many points in $(0,1)$.

(b) Let $f : [0,1] \rightarrow \mathfrak{R}$ be a continuous function with $f(0) = 0$. Assume that f' is finite and increasing on $(0,1)$.

Let $g(x) = \frac{f(x)}{x}$ $x \in (0,1)$. Show that g is increasing.

M3. Let $a_1=1$, and

$$a_n = n(a_{n-1}+1) \text{ for } n = 2, 3, \dots$$

$$\text{Let } P_n = (1 + \frac{1}{a_1})(1 + \frac{1}{a_2}) \cdots (1 + \frac{1}{a_n})$$

Find $\lim_{n \rightarrow \infty} P_n$.

M4. Consider the function of two variables

$$F(x,y) = 21x - 12x^2 - 2y^2 + x^3 + xy^2.$$

(a) Find the points of local minima of F .

(b) Show that F does not have a global minimum.

M5. Find the volume of the solid given by $0 \leq y \leq 2x$, $x^2 + y^2 \leq 4$ and $0 \leq z \leq x$.

M6. (a) Let A , B and C be $1 \times n$, $n \times n$ and $n \times 1$ matrices respectively. Prove or disprove: $\text{Rank}(ABC) \leq \text{Rank}(AC)$.

(b) Let S be the subspace of $\mathbf{R}^4$ defined by

$$S = \{(a_1, a_2, a_3, a_4) : 5a_1 - 2a_3 - 3a_4 = 0\}.$$

Find a basis for S .

M7. Let A be a 3×3 matrix with characteristic equation $\lambda^3 - 5\lambda^2 = 0$.

(i) Show that the rank of A is either 1 or 2.

(ii) Provide examples of two matrices A_1 and A_2 such that the rank of A_1 is 1, rank of A_2 is 2 and A_i has characteristic equation $\lambda^3 - 5\lambda^2 = 0$ for $i = 1, 2$.

M8. Define B to be a multi-subset of a set A if every element of B is an element of A and elements of B need not be distinct. The ordering of elements in B is not important.

For example, if $A = \{1,2,3,4,5\}$ and $B = \{1,1,3\}$, B is a 3-element multi-subset of A . Also, multi-subset $\{1,1,3\}$ is the same as the multi-subset $\{1,3,1\}$.

- (a) How many 5-element multi-subsets of a 10-element set are possible?
- (b) Generalize your result to m -element multi-subsets of an n -element set ($m < n$).

M9. Consider the vector space of all $n \times n$ matrices over $\mathfrak{R}$.

- (a) Show that there is a basis consisting of only symmetric and skew-symmetric matrices.
- (b) Find out the number of skew-symmetric matrices this basis must contain.

M10. Let $\mathbf{R}$ be the field of reals. Let $\mathbf{R}[x]$ be the ring of polynomials over $\mathbf{R}$, with the usual operations.

- (a) Let $I \subseteq \mathbf{R}[x]$ be the set of polynomials of the form $a_0 + a_1x + \dots + a_nx^n$ with $a_0 = a_1 = 0$. Show that I is an ideal.
- (b) Let $\mathbf{P}$ be the set of polynomials over $\mathbf{R}$ of degree ≤ 1 . Define $\oplus$ and $\odot$ on $\mathbf{P}$ by $(a_0 + a_1x) \oplus (b_0 + b_1x) = (a_0 + b_0) + (a_1 + b_1)x$ and $(a_0 + a_1x) \odot (b_0 + b_1x) = a_0b_0 + (a_1b_0 + a_0b_1)x$. Show that $(\mathbf{P}, \oplus, \odot)$ is a commutative ring. Is it an integral domain? Justify your answer.

- M11. (a) If G is a group of order 24 and H is a subgroup of G of order 12, prove that H is a normal subgroup of G .
- (b) Show that a field of order 81 cannot have a subfield of order 27.

M12. (a) Consider the differential equation:

$$\frac{d^2y}{dx^2} \cos x + \frac{dy}{dx} \sin x - 2y \cos^3 x = 2 \cos^5 x.$$

By a suitable transformation, reduce this equation to a second order linear differential equation with constant coefficients. Hence or otherwise solve the equation.

- (b) Find the surfaces whose tangent planes all pass through the origin.

M13. (a) Draw a simple graph with the degree sequence (1,1,1,1,4).

- (b) Write down the adjacency matrix of the graph.
- (c) Find the rank of the above matrix.
- (d) Using definitions of characteristic root and characteristic vectors only, find out all the characteristic roots of the matrix in (b).

- M14. (a) Show that a tree on n vertices has at most $n-2$ vertices with degree > 1 .
 (b) Show that in an Eulerian graph on 6 vertices, a subset of 5 vertices cannot form a complete subgraph.
- M15. (a) Show that the edges of K_4 can be partitioned into 2 edge-disjoint spanning trees.
 (b) Use (a) to show that the edges of K_6 can be partitioned into 3 edge-disjoint spanning trees.
 (c) Let K_n denote the complete undirected graph with n vertices and let n be an even number. Prove that the edges of K_n can be partitioned into exactly $n/2$ edge-disjoint spanning trees.

Statistics

- S1. (a) X and Y are two independent and identically distributed random variables with $\text{Prob}[X = i] = p_i$, for $i = 0, 1, 2, \dots$. Find $\text{Prob}[X < Y]$ in terms of the p_i values.
 (b) Based on one random observation X from $N(0, \sigma^2)$, show that $\sqrt{\pi/2} |X|$ is an unbiased estimate of σ .
- S2. (a) Let $X_0, X_1, X_2, \dots$ be independent and identically distributed random variables with common probability density function f . A random variable N is defined as

$$N = n \text{ if } X_1 \leq X_0, X_2 \leq X_0, \dots, X_{n-1} \leq X_0, X_n > X_0, n = 1, 2, 3, \dots$$
 Find the probability of $N = n$.
 (b) Let X and Y be independent random variables distributed uniformly over the interval $[0, 1]$. What is the probability that the integer closest to $\frac{X}{Y}$ is 2?
- S3. If a die is rolled m times and you had to bet on a particular number of sixes occurring, which number would you choose? Is there always one best bet, or could there be more than one?

- S4. Let X_1, X_2 and X_3 be independent random variables with X_i following a uniform distribution over $(0, i\theta)$, for $i = 1, 2, 3$. Find the maximum likelihood estimate of θ based on observations x_1, x_2, x_3 on X_1, X_2, X_3 respectively. Is it unbiased? Find the variance of the estimate.
- S5. New laser altimeters can measure elevation to within a few inches, without bias. As a part of an experiment, 25 readings were made on the elevation of a mountain peak. These averaged out to be 73,631 inches with a standard deviation (SD) of 10 inches. Examine each of the following statements and ascertain whether the statement is true or false, giving reasons for your answer.
- (a) 73631 ± 4 inches is a 95% confidence interval for the elevation of the mountain peak.
 - (b) About 95% of the readings are in the range 73631 ± 4 inches.
 - (c) There is about 95% chance that the next reading will be in the range of 73631 ± 4 inches.
- S6. Consider a randomized block design with two blocks and two treatments A and B . The following table gives the yields:

	Treatment A	Treatment B
Block 1	a	b
Block 2	c	d

- (a) How many orthogonal contrasts are possible with a, b, c and d ? Write down all of them.
 - (b) Identify the contrasts representing block effects, treatment effects and error.
 - (c) Show that their sum of squares equals the total sum of squares.
- S7. Let X be a discrete random variable having the probability mass function

$$p(x) = \Lambda^x (1 - \Lambda)^{1-x}, x = 0, 1,$$

where Λ takes values ≥ 0.5 only. Find the most powerful test, based on 2 observations, for testing $H_0 : \Lambda = \frac{1}{2}$ against $H_1 : \Lambda = \frac{2}{3}$, with level of significance 0.05.

S8. (a) Let X_i , $i = 1, 2, 3, 4$ be independently and identically distributed $N(\mu, \sigma^2)$ random variables. Obtain three non-trivial linear combinations of X_1, X_2, X_3, X_4 such that they are also independently and identically distributed.

(b) Let X be a continuous random variable such that X and $-X$ are identically distributed. Show that the density function of X is symmetric.

S9. Let $t_1, t_2, \dots, t_k$ be k independent and unbiased estimators of the same parameter θ with variances $\sigma_1^2, \sigma_2^2, \dots, \sigma_k^2$. Define $\bar{t}$ as $\sum_{i=1}^k \frac{t_i}{k}$. Find

$E(\bar{t})$ and the variance of $\bar{t}$. Show that $\sum_{i=1}^k (t_i - \bar{t})^2 / \{k(k-1)\}$ is an unbiased estimator of $\text{var}(\bar{t})$.

S10. Consider a simple random sample of n units, drawn without replacement from a population of N units. Suppose the value of Y_1 is unusually low whereas that of Y_n is very high. Consider the following estimator of $\bar{Y}$, the population mean.

$$\hat{\bar{Y}} = \begin{cases} \bar{y} + c, & \text{if the sample contains unit 1 but not unit } N; \\ \bar{y} - c, & \text{if the sample contains unit } N \text{ but not unit 1;} \\ \bar{y}, & \text{for all other samples;} \end{cases}$$

where $\bar{y}$ is sample mean and c is a constant. Show that $\hat{\bar{Y}}$ is unbiased. Given that

$$V(\hat{\bar{Y}}) = (1-f) \left[\frac{S^2}{n} - \frac{2c}{N-1} (Y_N - Y_1 - nc) \right]$$

where $f = \frac{n}{N}$ and $S^2 = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})^2$, comment on the choice of c .

S11. In order to compare the effects of four treatments A, B, C, D , a block design with 2 blocks each having 3 plots was used. Treatments A, B, C were given randomly to the plots of one block and treatments A, B, D were given randomly to the plots of the other block. Write down a set of 3 orthogonal contrasts with the 4 treatment effects and show that all of them are estimable from the above design.

- S12. Let $X_1, X_2, \dots, X_n$ ($X_i = (x_{i1}, x_{i2}, \dots, x_{ip})$, $i=1, 2, \dots, n$) be n random samples from a p -variate normal population with mean vector $\underline{\mu}$ and covariance matrix I .

Further, let $S = ((s_{jk}))$ denote the sample sums of squares and products matrix, namely

$$s_{jk} = \sum_{i=1}^n (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k), 1 \leq j, k \leq p, \text{ where}$$

$$\bar{x}_j = \frac{1}{n} \sum_{i=1}^n x_{ij}, 1 \leq j \leq p.$$

Obtain the distribution of $\ell' S \ell$ where $\ell \in \mathfrak{R}^k, \ell \neq 0$.

- S13. Let $Y_i = \sum_{j=1}^4 \beta_j X_{ij} + \epsilon_i, i=1, 2, \dots, k,$

where Y_i 's and X_{ij} 's are known, and ϵ_i 's are independent and each ϵ_i 's follows $N(0, \sigma^2)$.

Derive the likelihood ratio tests for the following hypotheses indicating their distributions under the respective null hypothesis.

(a) $H_0: \beta_2 = 3\beta_1$ against $H_1: \beta_2 \neq 3\beta_1$, and

(b) $H_0: \beta_1 = \beta_2, \beta_3 = \beta_4, \beta_3 = 2\beta_2$ against

H_1 : at least one of the equalities in H_0 is not true

- S14. For the following sampling scheme, compute the first and second order inclusion probabilities:

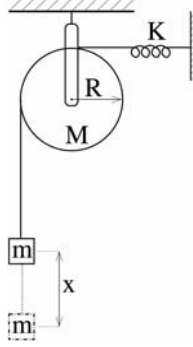
From a group of 15 male and 10 female students, one male and one female students are selected using SRS. After these selections, from the remaining 23 students, two are chosen using SRSWR, thus selecting a sample of size 4.

Physics

- P1. In a photoelectric emission experiment, a metal surface was successively exposed to monochromatic lights of wavelength λ_1, λ_2 and λ_3 . In each case, the maximum velocity of the emitted photo

electrons was measured and found to be α , β and γ , respectively. λ_3 was 10% higher in value than λ_1 , whereas λ_2 was 10% lower in value than λ_1 . If $\beta : \gamma = 4 : 3$, then show that $\alpha : \beta = 9\sqrt{3} : 8\sqrt{5}$.

- P2. (a) Two bodies A and B have constant heat capacities $2C$ and $\frac{2}{3}C$ respectively. The initial temperatures of A and B are $3T$ and $2T$, respectively, in Kelvin scale. A refrigerator working between these two bodies cools down B to a temperature of $\frac{T^0}{4} K$. What is the minimum amount of work required to do this?
- (b) A 60Ω resistor carrying a current of 4 ampere for 11 seconds is kept at a constant temperature of $47^\circ C$ by a stream of cooling water. Calculate the change in entropy of (i) the resistor, and (ii) the universe.
- P3. The nucleus B_Z^A decays by *alpha* (He_2^4) emission with a *half-life* T to the nucleus C_{Z-2}^{A-4} which in turn, decays by *beta* (electron) emission with a *half-life* $\frac{T}{4}$ to the nucleus D_{Z-1}^{A-4} . If at time $t = 0$, the decay chain $B \rightarrow C \rightarrow D$ had started with B_0 number of B nuclei only, then find out the time t at which the number of C nuclei will be maximum.
- P4. p_1 and p_2 are two relativistic protons traveling along a straight line in the same direction with kinetic energies $\frac{n}{n+1}$, and $\frac{14n}{14n+1}$ fractions of their respective total energies. Upon entering a region where a uniform magnetic field B acts perpendicularly on both, p_1 and p_2 describe circular paths of radii r_1 and r_2 respectively. Determine the ratio $\rho = \frac{r_1}{r_2}$. What is the value of ρ when $n = 5$?
- P5. (a) A mass m is attached to a massless spring of spring constant K via a frictionless pulley of radius R and mass M as shown in following Figure. The mass m is pulled down through a small distance x and released, so that it is set into a simple harmonic motion. Find the frequency of the vertical oscillation of the mass m .



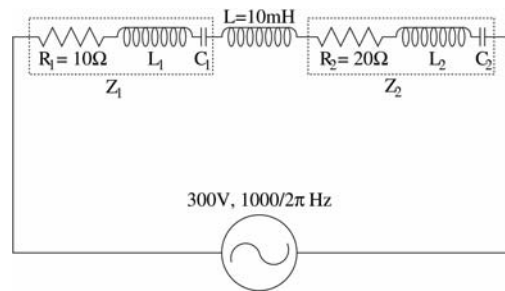
- (b) The Hamiltonian of a mechanical system having two degrees of freedom is:

$$H(x, y; p_x, p_y) = \frac{1}{2m} (p_x^2 + p_y^2) + \frac{1}{2} m \omega^2 (x^2 + y^2),$$

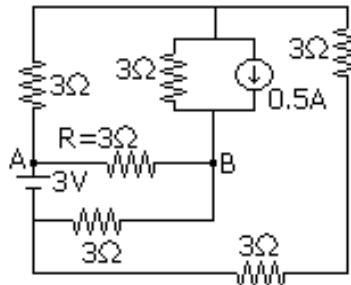
where m and ω are constants; x, y are the generalized co-ordinates for which p_x, p_y are the respective conjugate momenta. Show that the expressions $(x p_y - y p_x)^n$, $n=1,2,3,\dots$ are constants of motion for this system.

- P6. In a Millikan's oil drop experimental setup, two small negatively charged spherical oil droplets having radii $3r$ and $5r$ were allowed to fall freely in the closed chamber filled with air. The downward terminal velocities attained by them were v_1 and v_2 respectively. Subsequently, under the action of a strong electric field, the droplets attained upward terminal velocities $v_1/6$ and $v_2/20$ respectively. Neglecting the bouyant force of air and assuming the charges to be uniformly distributed over the surface of the droplets, compare their surface charge densities.
- P7. An electron is confined to move within a linear interval of length L . Assuming the potential to be zero throughout the interval except for the two end points, where the potential is infinite, find the following:
- probability of finding the electron in the region $0 < x < L/4$, when it is in the lowest (ground) state of energy;
 - taking the mass of the electron m_e to be 9×10^{-31} Kg, Planck's constant h to be 6.6×10^{-34} Joule-sec and $L = 1.1$ cm, determine the electron's quantum number when it is in the state having an energy equal to 5×10^{-32} Joule.

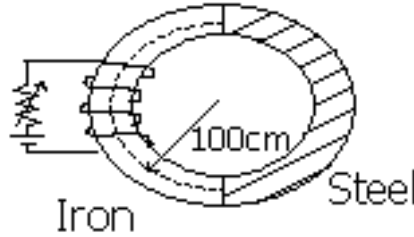
- P8. Two blocks of impedance Z_1 and Z_2 and an inductor L are connected in series across a supply of $300V$, $\frac{1000}{2\pi}$ Hz as shown low. The upper $3dB$ frequency of Z_1 alone is $\frac{1000}{2\pi}$ Hz and the lower $3dB$ frequency of Z_2 alone is also $\frac{1000}{2\pi}$ Hz. Calculate:
- the power dissipated in the circuit, and
 - the power factor.



- P9. (a) Given the circuit shown in the figure, find out the current through the resistance $R = 3\Omega$ between A and B .



- (b) Suppose a metal ring of mean radius 100 cm is made of iron and steel as shown in the figure. The cross-section of the ring is 10 sq.cm. If the ring is uniformly wound with 1000 turns, calculate the current required to produce a flux of 1 milliweber. The absolute permeability of air is $4\pi \times 10^{-7}$ H/m and relative permeability of iron and steel are 250 and 1000, respectively.



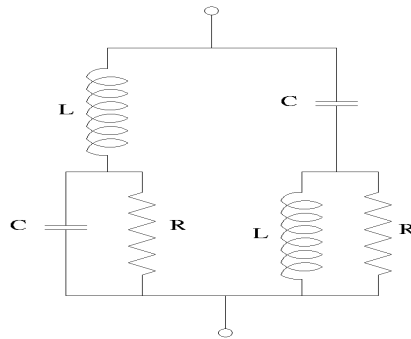
- P10. (a) Calculate the donor concentration of an n-type Germanium specimen having a specific resistivity of 0.1 ohm-metre at 300K, if the electron mobility $\mu_e = 0.25 \text{ metre}^2/\text{Volt-sec}$ at 300K, and the magnitude of the electronic charge is 1.6×10^{-19} Coulomb.
- (b) An n-type Germanium specimen has a donor density of $1.5 \times 10^{15} \text{ cm}^{-3}$. It is arranged in a Hall effect experiment where the magnitude of the magnetic induction field B is $0.5 \text{ Weber/metre}^2$ and current density $J = 480 \text{ amp/metre}^2$. What is the Hall voltage if the specimen is 3 mm thick?
- P11. Two heavy bodies A and B , each having charge $-Q$, are kept rigidly fixed at a distance $2a$ apart. A small particle C of mass m and charge $+q$ ($\ll Q$), is placed at the midpoint of the straight line joining the centers of A and B . C is now displaced slightly along a direction perpendicular to the line joining A and B , and then released. Find the period of the resultant oscillatory motion of C , assuming its displacement $y \ll a$.
- If instead, C is slightly displaced towards A , then find the instantaneous velocity of C , when the distance between A and C is $\frac{a}{2}$.
- P12. An elementary particle called Σ^- , at rest in laboratory frame, decays spontaneously into two other particles according to $\Sigma^- \rightarrow \pi^- + n$. The masses of Σ^- , π^- and n are M_1 , m_1 , and m_2 respectively.
- (a) How much kinetic energy is generated in the decay process?
- (b) What are the ratios of kinetic energies and momenta of π^- and n ?

- P13. Consider the following truth table where A , B and C are Boolean inputs and T is the Boolean output.

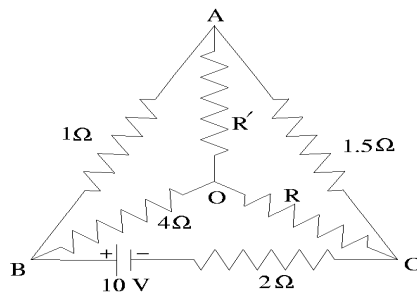
A	B	C	T
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Express T in a product-of-sum form and hence, show how T can be implemented using NOR gates only.

- P14. (a) Find the relationship between L , C and R in the circuit shown in the figure such that the impedance of the circuit is independent of frequency. Find out the impedance.



- (b) Find the value of R and the current flowing through R shown in the figure when the current is zero through R' .



P15. A gas obeys the equation of state $P = \frac{\tau}{V} + \frac{B(\tau)}{V^2}$ where $B(\tau)$ is a function of temperature τ only. The gas is initially at temperature τ and volume V_0 and is expanded isothermally and reversibly to volume $V_1 = 2V_0$.

(a) Find the work done in the expansion.

(b) Find the heat absorbed in the expansion.

(Hint: Use the relation $\left(\frac{\partial S}{\partial V}\right)_\tau = \left(\frac{\partial P}{\partial \tau}\right)_V$ where the symbols have their usual meaning.

P16. (a) From the Earth, an observer sees two very high speed rockets A and B moving in a straight line in the same direction with velocities $\frac{c}{2}$ and $\frac{2c}{3}$ respectively. What is the velocity of B relative to A ? Here, c denotes the speed of light in vacuum.

(b) Two objects having rest masses m_1 and m_2 move with relativistic speeds. Their total energies are E_1 and E_2 and kinetic energies are K_1 and K_2 respectively. If $2m_2E_1 = 5m_1E_2$ and K_1 is only 5% less than E_1 , find the value of K_2 in terms of E_2 .

P17. Consider the spin angular momentum operator $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ for

which $\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are the two eigen states with eigenvalues $+1$ and -1 respectively. Calculate the expectation value and variance of σ_z for the state $\psi = \sqrt{\frac{2}{3}}\alpha + \frac{1}{\sqrt{3}}\beta$.

Computer Science

C1. (a) A grammar is said to be left recursive if it has a non-terminal A such that there is a derivation $A \Rightarrow^+ A\alpha$ for some sequence of

symbols α . Is the following grammar left-recursive? If so, write an equivalent grammar that is not left-recursive.

$$\begin{array}{ll} A \rightarrow Bb & A \rightarrow a \\ B \rightarrow Cc & B \rightarrow b \\ C \rightarrow Aa & C \rightarrow c \end{array}$$

- (b) An example of a function definition in C language is given below:

```
char fun (int a, float b, int c)
{ /* body */ ... }
```

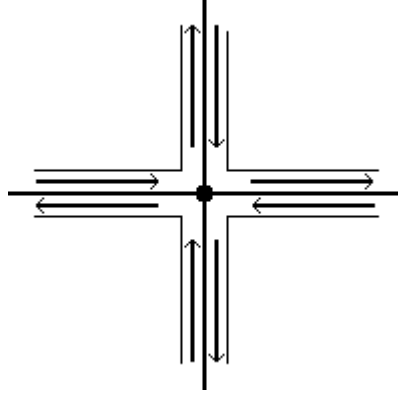
Assuming that the only types allowed are char, int, float (no arrays, no pointers, etc.), write a grammar for function headers, i.e., the portion `char fun(int a, ...)` in the above example.

- (c) Consider the floating point number representation in C programming language.
Give a regular expression for it using the following convention:
 l denotes a letter, d denotes a digit, S denotes sign and p denotes point.
State any assumption that you may need to make.

C2. The following functional dependencies are defined on the relation $\mathcal{R}(A, B, C, D, E, F)$:

- Find the candidate keys for $\mathcal{R}$.
- Is $\mathcal{R}$ normalized? If not, create a set of normalized relations by decomposing $\mathcal{R}$ using only the given set of functional dependencies.
- If a new attribute F is added to $\mathcal{R}$ to create a new relation $\mathcal{R}'(A, B, C, D, E, F)$ without any addition to the set of functional dependencies, what would be the new set of candidate key for $\mathcal{R}'$?
- What is the new set of normalized relations that can be derived by decomposing $\mathcal{R}'$ for the same set of functional dependencies?
- If a new dependency is declared as follows:
For each value of A , attribute F can have two values,
what would be the new set of normalized relations that can be created by decomposing $\mathcal{R}'$?

- C3. (a) A relation $R(\underline{A}, B, C, D)$ has to be accessed under the query $\sigma_{B=10}(R)$. Out of the following possible file structures, which one should be chosen and why?
- R is a heap file.
 - R has a clustered hash index on B .
 - R has an unclustered B^+ tree index on (A, B) .
- (b) If the query is modified as $\pi_{A,B}(\sigma_{B=10}(R))$, which one of the three possible file structures given above should be chosen in this case and why?
- (c) Let the relation have 5000 tuples with 10 tuples/page. In case of a hashed file, each bucket needs 10 pages. In case of B^+ tree, the index structure itself needs 2 pages. If the disk needs 25 msec. to read or write a disk page, what would be the disk access time for answering the above queries?
- C4. Let A and B be two arrays, each of size n . A and B contain numbers in sorted order. Give an $O(\log n)$ algorithm to find the median of the combined set of $2n$ numbers.
- C5. (a) Consider a pipelined processor with m stages. The processing time at every stage is the same. What is the speed-up achieved by the pipelining?
- (b) In a certain computer system with cache memory, 750 ns (nanosec) is the access time for main memory for a cache miss and 50 ns is the access time for a cache hit. Find the percentage decrease in the effective access time if the hit ratio is increased from 80% to 90%.
- C6. (a) A disk has 500 bytes/sector, 100 sectors/track, 20 heads and 1000 cylinders. The speed of rotation of the disk is 6000 rpm. The average seek time is 10 millisecs. A file of size 50 MB is written from the beginning of a cylinder and a new cylinder will be allocated only after the first cylinder is totally occupied.
- Find the maximum transfer rate.
 - How much time will be required to transfer the file of 50 MB written on the disk? Ignore the rotational delay but not the seek time.
- (b) Consider a 4-way traffic crossing as shown in the figure. Suppose that we model the crossing as follows:



- each vehicle is modeled by a process,
- the crossing is modeled as a shared data structure. Assume that the vehicles can only move straight through the intersection (no left or right turns). Using read-write locks (or any standard synchronization primitive), you have to devise a synchronization scheme for the processes. Your scheme should satisfy the following criteria:
 - i) prevent collisions,
 - ii) prevent deadlock, and
 - iii) maximize concurrency but prevent indefinite waiting (starvation).

Write down the algorithm that each vehicle must follow in order to pass through the crossing. Justify that your algorithm satisfies the given criteria.

- C7.
- (a) A computer on a 6 Mbps network is regulated by a token bucket. The bucket is filled at a rate of 2 Mbps. It is initially filled to capacity with 8 Megabits. How long can the computer transmit at the full 6 Mbps?
 - (b) Sketch the Manchester encoding for the bit stream 0001110101.
 - (c) If delays are recorded in 8-bit numbers in a 50-router network, and delay vectors are exchanged twice a second, how much bandwidth per (full-duplex) line is consumed by the distributed routing algorithm? Assume that each router has 3 lines to other routers.

- C8. Consider a binary operation *shuffle* on two strings, that is just like shuffling a deck of cards. For example, the operation *shuffle* on strings ab and cd , denoted by $ab \parallel cd$, gives the set of strings $\{abcd, acbd, acdb, cabd, cadb, cdab\}$.
- (a) Define formally by induction the *shuffle* operation on any two strings $x, y \in \Sigma^*$.
- (b) Let the *shuffle* of two languages A and B , denoted by $A \parallel B$ be the set of all strings obtained by shuffling a string $x \in A$ with a string $y \in B$. Show that if A and B are regular, then so is $A \parallel B$.
- C9. (a) Give a method of encoding the microinstructions (given in the table below) so that the minimum number of control bits are used and maximum parallelism among the microinstructions is achieved.

Microinstructions	Control signals
I_1	$C_1, C_2, C_3, C_4, C_5, C_6,$
I_2	$C_1, C_3, C_4 C_6,$
I_3	$C_2, C_5, C_6,$
I_4	$C_4, C_5, C_8,$
I_5	$C_7, C_8,$
I_6	$C_1, C_8, C_9,$
I_7	$C_3, C_4, C_8,$
I_8	$C_1, C_2, C_9,$

- (b) A certain four-input gate G realizes the switching function $G(a, b, c, d) = abc + bcd$. Assuming that the input variables are available in both complemented and uncomplemented forms:
- (i) Show a realization of the function $f(u, v, w, x) = \Sigma(0, 1, 6, 9, 10, 11, 14, 15)$ with only three G gates and one OR gate.
- (ii) Can all switching functions be realized with $\{G, OR\}$ logic set ?
- C10. Consider a set of n temperature readings stored in an array T . Assume that a temperature is represented by an integer. Design an $O(n + k \log n)$ algorithm for finding the k coldest temperatures.

- C11. Assume the following characteristics of instruction execution in a given computer:
- ALU/register transfer operations need 1 clock cycle each,
 - each of the load/store instructions needs 3 clock cycles, and
 - branch instructions need 2 clock cycles each.
- (a) Consider a program which consists of 40% ALU/register transfer instructions, 30% load/store instructions, and 30% branch instructions. If the total number of instructions in this program is 10 billion and the clock frequency is 1 GHz, then compute the average number of cycles per instruction (CPI), total execution time for this program, and the corresponding MIPS rate.
- (b) If we now use an optimizing compiler which reduces the total number of ALU/register transfer instructions by a factor of 2, keeping the number of other instruction types unchanged, then compute the average CPI, total time of execution and the corresponding MIPS rate for this modified program.
- C12. A tape S contains n records, each representing a vote in an election. Each candidate for the election has a unique *id*. A vote for a candidate is recorded as his/her *id*.
- (i) Write an $O(n)$ time algorithm to find the candidate who wins the election. Comment on the main memory space required by your algorithm.
- (ii) If the number of candidates k is known *a priori*, can you improve your algorithm to reduce the time and/or space complexity?
- (iii) If the number of candidates k is unknown, modify your algorithm so that it uses only $O(k)$ space. What is the time complexity of your modified algorithm?
- C13. (a) The *order* of a regular language L is the smallest integer k for which $L^k = L^{k+1}$, if there exists such a k , and ∞ otherwise.
- (i) What is the order of the regular language $a + (aa)(aaa)^*$?
- (ii) Show that the order of L is finite if and only if there is an integer k such that $L^k = L^*$, and that in this case the order of L is the smallest k such that $L^k = L^*$.
- (b) Solve for $T(n)$ given by the following recurrence relations:
 $T(1) = 1$;
 $T(n) = 2T(n/2) + n \log n$, where n is a power of 2.

- (c) An A.P. is $\{p + qn | n = 0, 1, \dots\}$ for some $p, q \in \mathbb{N}$. Show that if $L \subseteq \{a\}^*$ and $\{n | a^n \in L\}$ is an A.P., then L is regular.

C14. (a) You are given an unordered sequence of n integers with many duplications, such that the number of distinct integers in the sequence is $O(\log_2 n)$. Design a sorting algorithm and its necessary data structure(s), which can sort the sequence using at most $O(n \log_2(\log_2 n))$ time. (You have to justify the time complexity of your proposed algorithm.)

- (b) Let A be a real-valued matrix of order $n \times n$ already stored in memory. Its (i, j) -th element is denoted by $a[i, j]$. The elements of the matrix A satisfy the following property:
Let the largest element in row i occur in column l_i . Now, for any two rows i_1, i_2 , if $i_1 < i_2$, then $l_{i_1} \leq l_{i_2}$.

2	6	4	5	3
5	3	7	2	4
4	2	10	7	8
6	4	5	9	7
3	7	6	8	12

(a)

Row I	$l(i)$
1	2
2	3
3	3
4	4
5	5

(b)

Figure shows an example of (a) matrix A , and (b) the corresponding values of l_i for each row i .

Write an algorithm for identifying the largest valued element in matrix A which performs at most $O(n \log_2 n)$ comparisons.

C15. (a) You are given the following file abc.h:

```
#include <stdio.h>
#define SQR(x) (x*x)
#define ADD1(x) (x=x+1)
#define BeginProgram int main(int argc,char *argv[]){
#define EndProgram return 1; }
```

For each of the following code fragments, what will be the output?

- (i)

```
#include "abc.h"
main() { int y = 4; printf("%d\n", SQR(y+1)); }
```
- (ii)

```
#include "abc.h"
BeginProgram
int y=3; printf("%d\n", SQR(ADD1(y)));
EndProgram
```

(b) Given the following program:

```
#include <iostream.h>
main()
{
cout<<"MTech (CS)\n";
}
```

Without changing the main() in any way, modify the program to produce the following output:

Sample Question
MTech (CS)
Indian Statistical Institute

Engineering and Technology

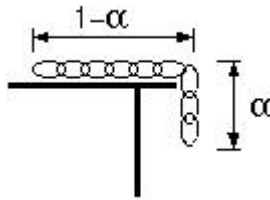
- E1. A bullet of mass M is fired with a velocity of 40 m/s at an angle θ with the horizontal plane. At P, the highest point of its trajectory, the bullet collides with a bob of mass $3M$ suspended freely by a mass-less string of length $\frac{3}{10}$ m. After the collision, the bullet gets

stuck inside the bob and the string deflects with the total mass through an angle of 120° keeping the string taut. Find

- (i) the angle θ , and
- (ii) the height of P from the horizontal plane.

Assume, $g = 10 \text{ m/s}^2$, and friction in air is negligible.

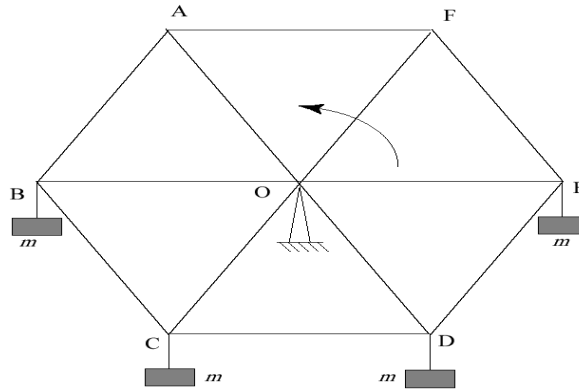
- E2. A rod of length 120 cm is suspended from the ceiling horizontally by two vertical wires of equal length tied to its ends. One of the wires is made of steel and has cross-section 0.2 cm^2 and the other one is of brass having cross-section 0.4 cm^2 . Find out the position along the rod where a weight may be hung to produce equal stress in both wires
- E3. A chain of total length $L = 4$ meter rests on a table top, with a part of the chain hanging over the edge, as shown in the figure below. Let α be the ratio of the length of the overhanging part of the chain to L .



If the coefficient of friction between the chain and the table top is 0.5, find the values of α for which the chain remains stationary. If $\alpha = 0.5$, what is the velocity of the chain when the last link leaves the table?

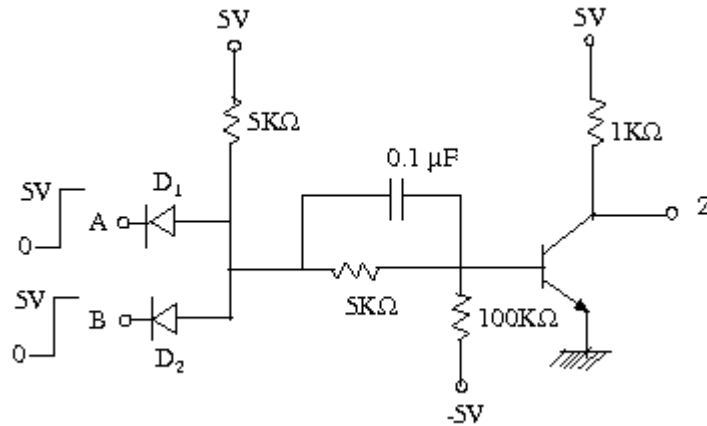
- E4. A flywheel of mass 100 kg and radius of gyration 20 cm is mounted on a light horizontal axle of radius 2 cm, and is free to rotate on bearings whose friction may be neglected. A light string wound on the axle carries at its free end a mass of 5 kg. The system is released from rest with the 5 kg mass hanging freely. If the string slips off the axle after the weight has descended 2 m, prove that a couple of moment $10/\pi^2 \text{ kg.wt.cm}$. must be applied in order to bring the flywheel to rest in 5 revolutions.

- E5. The truss shown in the figure rotates around the pivot O in a vertical plane at a constant angular speed ω . Four equal masses (m) hang from the points B, C, D and E. The members of the truss are rigid, weightless and of equal length. Find a condition on the angular speed ω so that there is compression in the member OE.

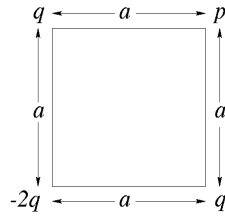


- E6. If the inputs A and B to the circuit shown below can be either 0 volts or 5 volts,
 (i) what would be the corresponding voltages at output Z, and
 (ii) what operation is being performed by this circuit ?

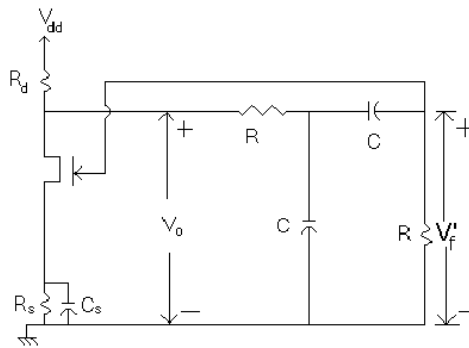
Assume that the transistor and the diodes are ideal and base to emitter saturation voltage = 0.5 volts.



- E7. Two bulbs of 500 cc capacity are connected by a tube of length 20 cm and internal radius 0.15 cm. The whole system is filled with oxygen, the initial pressures in the bulbs before connection being 10 cm and 15 cm of Hg, respectively. Calculate the time taken for the pressures to become 12 cm and 13 cm of Hg, respectively. Assume that the coefficient of viscosity η of oxygen is 0.000199 cgs unit.
- E8. (a) Ice in a cold storage melts at a rate of $\frac{300 \times 3.6}{80 \times 4.2} \text{ kg/hour}$ when the external temperature is 27°C . Find the minimum power output of the refrigerator motor, which just prevents the ice from melting. (Latent heat of fusion of ice = 80 cal/gm.)
- (b) A vertical hollow cylinder contains an ideal gas with a 5 kg piston placed over it. The cross-section of the cylinder is $5 \times 10^{-3} \text{ m}^2$. The gas is heated from 300 K to 350 K and the piston rises by 0.1 m. The piston is now clamped in this position and the gas is cooled back to 300 K. Find the difference between the heat energy added during heating and that released during cooling. (1 atmospheric pressure = 10^5 Nm^{-2} and $g = 10 \text{ ms}^{-2}$.)
- E9. (a) A system receives 10 Kcal of heat from a reservoir to do 15 Kcal of work. How much work must the system do to reach the initial state by an adiabatic process?
- (b) A certain volume of Helium at 15°C is suddenly expanded to 8 times its volume. Calculate the change in temperature (assume that the ratio of specific heats is 5/3).
- E10. A spherical charge distribution has a volume density ρ , which is a function of r , the radial distance from the center of the sphere, as given below.
- $$\rho = \begin{cases} A/r, & A \text{ is constant for } 0 \leq r \leq R \\ 0, & \text{for } r > R \end{cases}$$
- Determine the electric field as a function of r , for $r \geq R$. Also deduce the expression for the electrostatic potential energy $U(r)$, given that $U(\infty) = 0$ in the region $r \geq R$.
- E11. Consider the distribution of charges as shown in the figure below. Determine the potential and field at the point p .

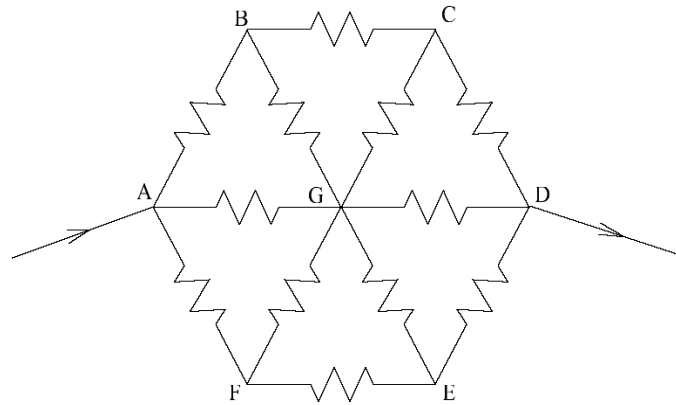


- E12. A proton of velocity 10^7 m/s is projected at right angles to a uniform magnetic induction field of 0.1 w/m^2 . How much is the path of the particle deflected from a straight line after it has traversed a distance of 1 cm? How long does it take for the proton to traverse a 90° arc?
- E13. (a) State the two necessary conditions under which a feedback amplifier circuit becomes an oscillator.
 (b) A two-stage FET phase shift oscillator is shown in the diagram below.



- (i) Derive an expression for the feedback factor β .
 (ii) Find the frequency of oscillation.
 (iii) Establish that the gain A must exceed 3.
- E14. A circular disc of radius 10cm is rotated about its own axis in a uniform magnetic field of 100 weber/m^2 , the magnetic field being perpendicular to the plane of the disc. Will there be any voltage developed across the disc? If so, then find the magnitude of this voltage when the speed of rotation of the disc is 1200 rpm.
- E15. A 3-phase, 50-Hz, 500-volt, 6-pole induction motor gives an output of 50 HP at 900 rpm. The frictional and windage losses total 4 HP and the stator losses amount to 5 HP. Determine the slip, rotor copper loss, and efficiency for this load.

- E16. A shunt D.C. generator supplies a load of two motors each drawing 46 Amps and a lighting load consisting of twenty-two 60 watt lamps at 220V. The resistance of shunt field, series field and armature are 110 ohms, 0.06 ohms and 0.05 ohms respectively.
- Find the electrical efficiency of the generator.
 - If the overall efficiency of the generator at the above load is 77%, find the total constant (iron and mechanical) loss.
- E17. An alternator on open-circuit generates 360 V at 60 Hz when the field current is 3.6 A. Neglecting saturation, determine the open-circuit e.m.f. when the frequency is 40 Hz and the field-current is 24A.
- E18. A single phase two-winding 20 KVA transformer has 5000 primary and 500 secondary turns. It is converted to an autotransformer employing additive polarity mechanism. Suppose the transformer always operates with an input voltage of 2000 V.
- Calculate the percentage increase in KVA capacity.
 - Calculate the common current in the autotransformer.
 - At full load of 0.9 power factor, if the efficiency of the two-winding transformer be 90%, what will be the efficiency of the autotransformer at the same load ?
- E19. The hybrid parameters of a $p-n-p$ junction transistor used as an amplifier in the common-emitter configuration are: $h_{ie} = 800\Omega$, $h_{fe} = 46$, $h_{oe} = 8 \times 10^{-5}$ mho, $h_{re} = 55.4 \times 10^{-4}$. If the load resistance is 5 k Ω and the effective source resistance is 500 Ω , calculate the voltage and current gains and the output resistance.
- E20. Find the equivalent resistance between the points A and D of the circuit shown in the diagram.



Each resistance = $r\Omega$

- E21. (a) Design a special purpose counter to count from 6 to 15 using a decade counter. Inverter gates may be used if required.
 (b) For a 5 variable Boolean function the following minterms are true: (0, 2, 3, 8, 10, 11, 16, 17, 18, 24, 25 and 26). Find a minimized Boolean expression using Karnaugh map.
- E22. In the figure, consider that *FF1* and *FF2* cannot be set to a desired value by reset/preset line. The initial states of the flip-flops are unknown. Determine a sequence of inputs (x_1 , x_2) such that the output is zero at the end of the sequence.

