

Recursion

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Outline

- 1 Introduction
- 2 Fibonacci sequence
- 3 Space Filling Curve
- 4 Towers of Hanoi
- 5 Generating Permutations

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Recursion

What is recursion?

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- A function calling itself with certain properties.

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What is recursion?

- A function calling itself with certain properties.
- There must be a **base criteria** for which the function does not call itself.
- After each call, the function must be closer to the base criteria.

An Example (Can you recognize the expression?)

$$\begin{aligned}f(n) &= n * f(n - 1) \text{ if } n \geq 1 \\ &= 1 \text{ if } n = 0\end{aligned}$$

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Fibonacci sequence

The Definition

$$\begin{aligned} fib(n) &= fib(n - 1) + fib(n - 2) \text{ if } n \geq 2 \\ &= 1 \text{ if } n = 1 \\ &= 1 \text{ if } n = 0 \end{aligned}$$

Fibonacci sequence

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Some values

$fib(2) = 2$; $fib(3) = 3$; $fib(4) = 5$; $fib(5) = 8$; $fib(6) = 13$;
 $fib(7) = 21$; $fib(8) = 34$; $fib(9) = 55$; $fib(10) = 89$; $fib(11) = 144$;

A recursive implementation

```
#include<stdio.h>
```

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```

```
int main(void){  
    unsigned int m,val,i;  
    printf("\n What is the value of m::> "); scanf("%d",&m);  
    val = f(m);  
    printf("\n Fibonacci(%d) = %d \n",m,val);  
    return 0;  
}
```

A recursive implementation

```
#include<stdio.h>
unsigned int f(unsigned int num)
{

}

int main(void){
    unsigned int m,val,i;
    printf("\n What is the value of m::> "); scanf("%d",&m);
    val = f(m);
    printf("\n Fibonacci(%d) = %d \n",m,val);
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A recursive implementation

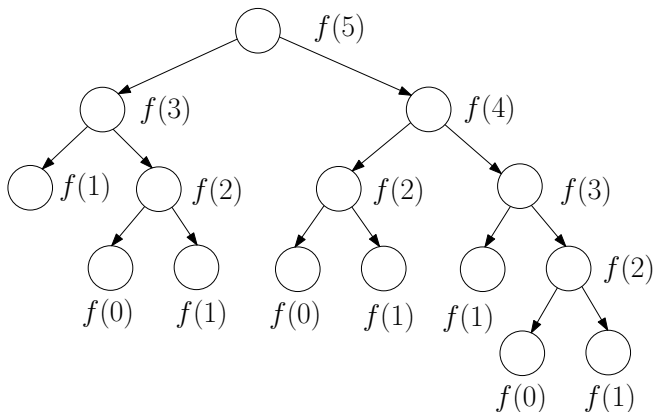
```
#include<stdio.h>
unsigned int f(unsigned int num)
{
    if(num==0 || num==1)    return 1;
}
int main(void){
    unsigned int m,val,i;
    printf("\n What is the value of m::> "); scanf("%d",&m);
    val = f(m);
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A recursive implementation

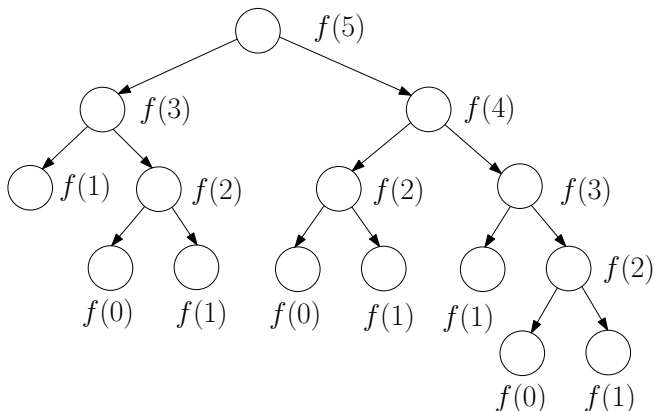
```
#include<stdio.h>
unsigned int f(unsigned int num)
{
    if(num==0 || num==1)    return 1;
    else return(f(num-1)+f(num-2));
}
int main(void){
    unsigned int m,val,i;
    printf("\n What is the value of m::> "); scanf("%d",&m);
    val = f(m);
    printf("\n Fibonacci(%d) = %d \n",m,val);
    return 0;
}
```

The recursion tree

The recursion tree



The recursion tree



How many function calls?

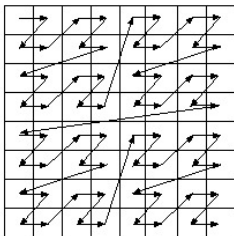
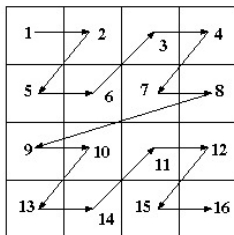
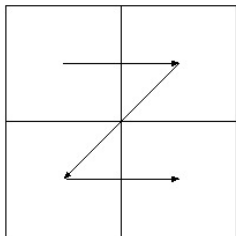
$f(5) : 1, f(4) : 1, f(3) : 2, f(2) : 3, f(1) : 5, f(0) : 3$

Exercise: Find it experimentally and analytically.

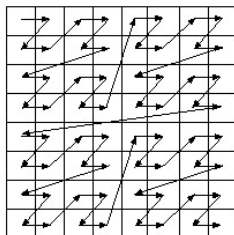
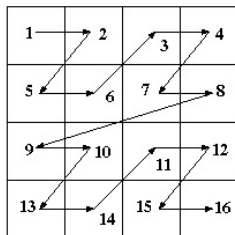
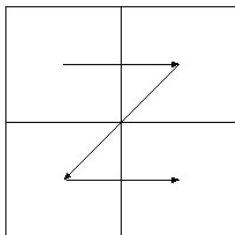
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Space Filling Curve

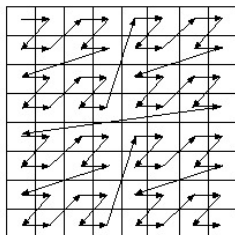
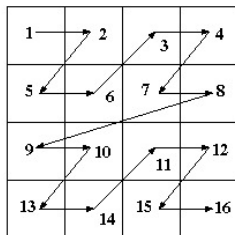
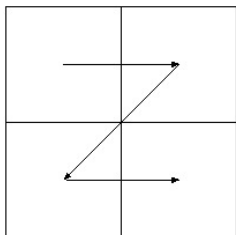


Space Filling Curve



- Z Curve order: 1 2 5 6 3 4 7 8 9 10 13 14 11 12 15 16

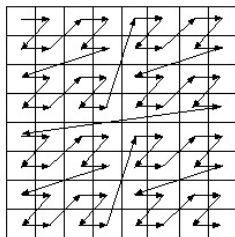
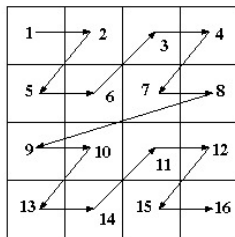
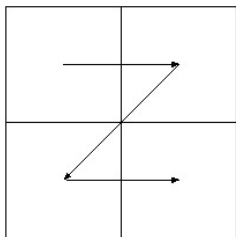
Space Filling Curve



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Problem

Space Filling Curve

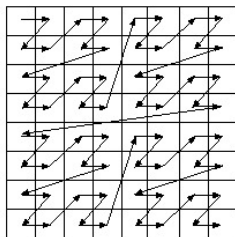
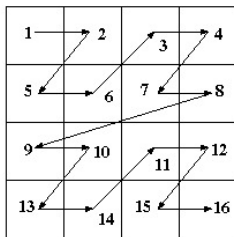
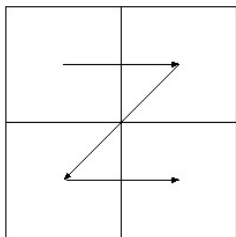


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Problem

- Allocate a 2-D matrix of size $2^m \times 2^m$.

Space Filling Curve

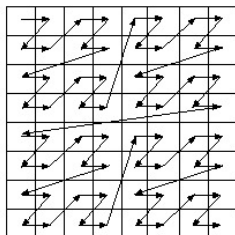
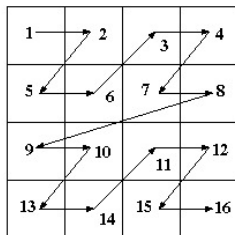
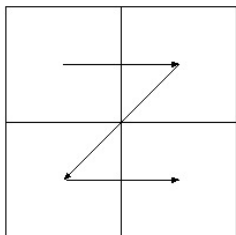


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- Allocate a 2-D matrix of size $2^m \times 2^m$.
- Fill it with integers $1, \dots, 2^{2m}$ in a row-major fashion.

Space Filling Curve



- Z Curve order: 1 2 5 6 3 4 7 8 9 10 13 14 11 12 15 16

Problem

- Allocate a 2-D matrix of size $2^m \times 2^m$.
- Fill it with integers $1, \dots, 2^{2m}$ in a row-major fashion.
- Traverse the matrix in a z-curve order.

The main program

```
int main(void){
int m,N, i,j,k=1,**matrix;
printf(" \n What is the value of m::>"); scanf("%d",&m);
N=(int)(pow(2.0,m));
matrix = allocate2D(N,N);

}
```

The main program

```
int main(void){
int m,N, i,j,k=1,**matrix;
printf(" \n What is the value of m::>"); scanf("%d",&m);
N=(int)(pow(2.0,m));
matrix = allocate2D(N,N);
/*Fill up and print the matrix */
for(i=0;i<N;i++)
for(j=0;j<N;j++){matrix[i][j]=k; k++;}
for(i=0;i<N;i++){ printf("\n");
for(j=0;j<N;j++) printf("%d \t",matrix[i][j]);
}

}
```

The main program

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int main(void){
int m,N, i,j,k=1,**matrix;
printf(" \n What is the value of m::>"); scanf("%d",&m);
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/*Fill up and print the matrix */
for(i=0;i<N;i++)
for(j=0;j<N;j++){matrix[i][j]=k; k++;}
for(i=0;i<N;i++){ printf("\n");
for(j=0;j<N;j++) printf("%d \t",matrix[i][j]);
}
printf("\n \nReading the matrix in z-curve fashion::>");
zee(0,N-1,0,N-1,matrix);
printf("\n"); return 0;
}
```

Allocating the 2-D matrix

```
int** allocate2D(int row,int col){
int **S,k;

S=(int **)calloc(row,sizeof(int *));
if(S==NULL){printf("\n No space \n"); exit(0);}

for(k=0;k<row;k++){
    S[k]=(int *)calloc(col,sizeof(int));
    if(S[k]==NULL){ printf("\n No space \n"); exit(0);}
}
return S;
}
```

The recursive function

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```
void zee(int r_b,int r_e,int c_b,int c_e,  
        int **matrix){  
    if((r_b==r_e)&&(c_b==c_e)){  
        printf(" %2d ",matrix[r_b][c_b]);  
        return;  
    }  
  
    return;  
}
```

```
return;  
}
```

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    }
    zee(r_b,(r_e+r_b)/2,c_b,(c_e+c_b)/2,
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matrix);
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Towers of Hanoi

The Problem

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- Three pegs A, B and C are given and on peg A there are n disks with decreasing size.

Towers of Hanoi

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 - The top disk on any peg can be moved to any other peg.

Towers of Hanoi

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- We have to move disks from peg A to peg C using peg B as an auxiliary peg following some rules.
 - The top disk on any peg can be moved to any other peg.
 - A larger disk can never be placed on a smaller disk.

Towers of Hanoi

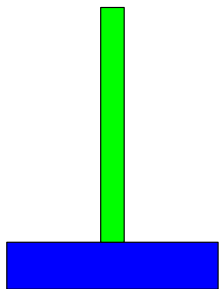
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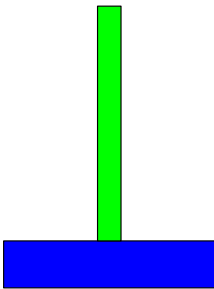
Exercise

Try with $n = 3, 4, \dots$ and start thinking recursively.

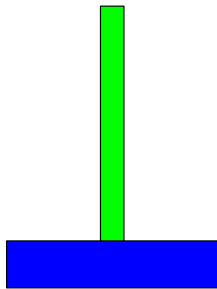
Towers of Hanoi



Peg A

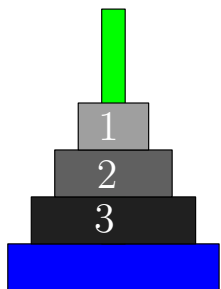


Peg B

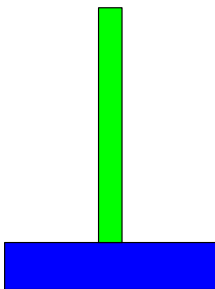


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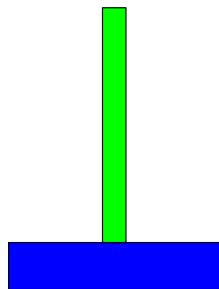
Towers of Hanoi



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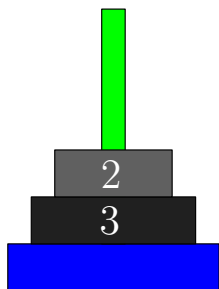


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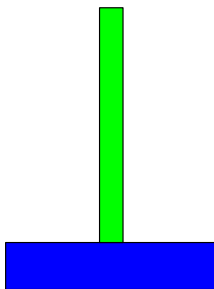


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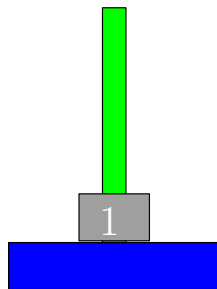
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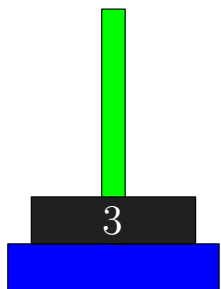


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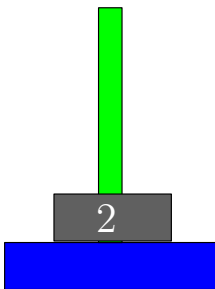


Peg C

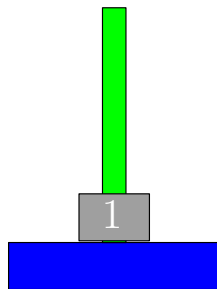
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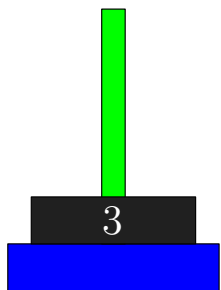


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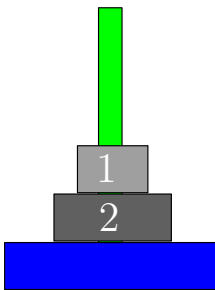


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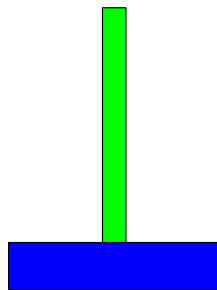
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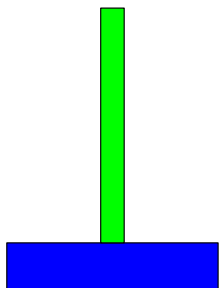


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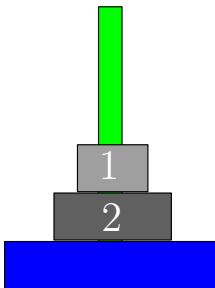


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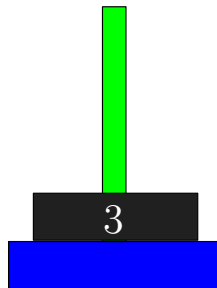
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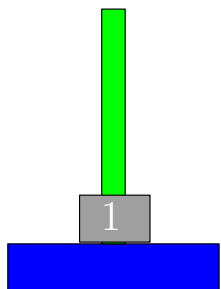


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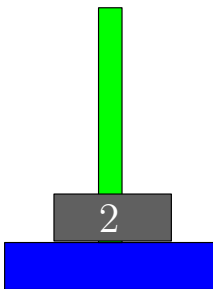


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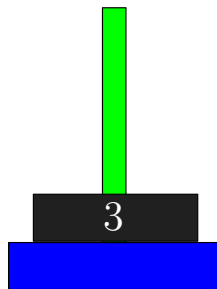
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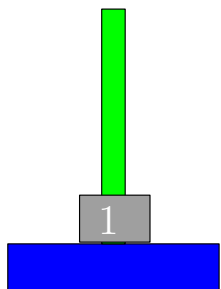


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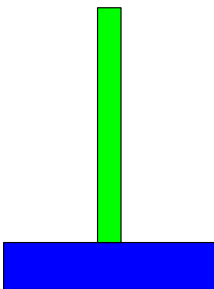


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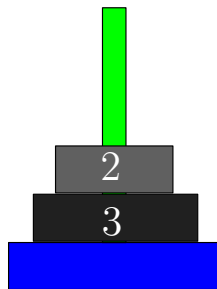
Towers of Hanoi



Peg A

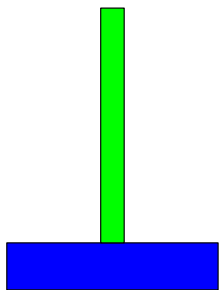


Peg B

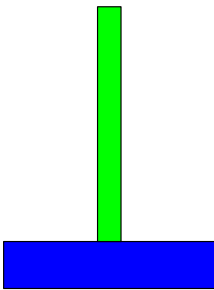


Peg C

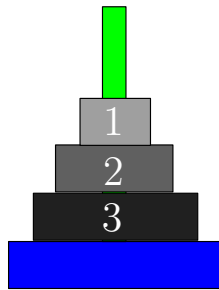
Towers of Hanoi



Peg A



Peg B



Peg C

The algorithm

The recursive method

The algorithm

The recursive method

- Move the top $n - 1$ disks from peg A to peg B using peg C as auxiliary.

The algorithm

The recursive method

- Move the top $n - 1$ disks from peg A to peg B using peg C as auxiliary.
- Move the top disk from peg A to peg C.

The algorithm

The recursive method

- Move the top $n - 1$ disks from peg A to peg B using peg C as auxiliary.
- Move the top disk from peg A to peg C.
- Move the top $n - 1$ disks from peg B to peg C using peg A as auxiliary.

The main program

```
int main(void) {
    int n;
    printf("Enter the number of disks : ");
    scanf("%d",&n);
    printf("The Tower of Hanoi involves the moves :\n \n");

    return 0;
}
```

The main program

```
int main(void) {
    int n;
    printf("Enter the number of disks : ");
    scanf("%d",&n);
    printf("The Tower of Hanoi involves the moves :\n \n");
    towers(n,'A','C','B');

    return 0;
}
```

The main program

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int main(void) {
    int n;
    printf("Enter the number of disks : ");
    scanf("%d",&n);
    printf("The Tower of Hanoi involves the moves :\n \n");
    towers(n,'A','C','B');
    printf("\n \n");
    return 0;
}
```

The recursive function

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```
void towers(int n,char frompeg,char topeg,char auxpeg){
```

```
}
```

The recursive function

```
void towers(int n,char frompeg,char topeg,char auxpeg){  
/* If only 1 disk, make the move and return */
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void towers(int n,char frompeg,char topeg,char auxpeg){  
/* If only 1 disk, make the move and return */  
  
  
/* Move top n-1 disks from A to B, using C as auxiliary */  
  
  
  
  
  
  
  
  
  
}
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The recursive function

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void towers(int n,char frompeg,char topeg,char auxpeg){  
/* If only 1 disk, make the move and return */  
  
/* Move top n-1 disks from A to B, using C as auxiliary */  
  
/* Move remaining disks from A to C*/  
  
}
```

The recursive function

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void towers(int n,char frompeg,char topeg,char auxpeg){  
/* If only 1 disk, make the move and return */  
  
/* Move top n-1 disks from A to B, using C as auxiliary */  
  
/* Move remaining disks from A to C*/  
  
/* Move n-1 disks from B to C using A as auxiliary */  
}
```

The recursive function

```
void towers(int n,char frompeg,char topeg,char auxpeg){
/* If only 1 disk, make the move and return */
  if(n==1) {

    }
/* Move top n-1 disks from A to B, using C as auxiliary */

/* Move remaining disks from A to C*/

/* Move n-1 disks from B to C using A as auxiliary */

}
```

The recursive function

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void towers(int n,char frompeg,char topeg,char auxpeg){
/* If only 1 disk, make the move and return */
  if(n==1) { printf("\n Move disk 1 from peg %c
                  to peg %c", frompeg,topeg);
              return;
            }
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/* Move remaining disks from A to C*/

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                  to peg %c", frompeg,topeg);
              return;
            }
/* Move top n-1 disks from A to B, using C as auxiliary */
  towers(n-1,frompeg,auxpeg,topeg);
/* Move remaining disks from A to C*/

/* Move n-1 disks from B to C using A as auxiliary */

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/* If only 1 disk, make the move and return */
  if(n==1) { printf("\n Move disk 1 from peg %c
                  to peg %c", frompeg,topeg);
              return;
            }
/* Move top n-1 disks from A to B, using C as auxiliary */
  towers(n-1,frompeg,auxpeg,topeg);
/* Move remaining disks from A to C*/
  printf("\n Move disk %d from peg %c to peg %c",
         n,frompeg,topeg);
/* Move n-1 disks from B to C using A as auxiliary */
  towers(n-1,auxpeg,topeg,frompeg);
}
```

Outline

- 1 Introduction
- 2 Fibonacci sequence
- 3 Space Filling Curve
- 4 Towers of Hanoi
- 5 Generating Permutations**

Generating Permutations

- Suppose we can generate permutations of all $n - 1$ numbers.

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- To generate permutations of n numbers, do the following:

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- Generate permutations of all the numbers $1, 3, 4, \dots, n$ and add 2 to the beginning.

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- ...
- Generate permutations of all the numbers $1, 2, \dots, n - 1$ and add n to the beginning.

The Basic Function

```
void Permute(int *matrix, int k,int n, int *number_per)
```

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```
void Permute(int *matrix, int k,int n, int *number_per)
{

}

}
```

The Basic Function

```
void Permute(int *matrix, int k,int n, int *number_per)
{
int i;

}
}
```

The Basic Function

```
void Permute(int *matrix, int k,int n, int *number_per)
{
int i;
if(k==n) {

}

}
```

The Basic Function

```
void Permute(int *matrix, int k,int n, int *number_per)
{
int i;
if(k==n) {
    PrintPermutation(matrix,n); (*number_per)++;
}

}
```

The Basic Function

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void Permute(int *matrix, int k,int n, int *number_per)
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int i;
if(k==n) {
    PrintPermutation(matrix,n); (*number_per)++;
}
else {

}

}
```

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void Permute(int *matrix, int k,int n, int *number_per)
{
int i;
if(k==n) {
    PrintPermutation(matrix,n); (*number_per)++;
}
else {
    for(i=k;i<n;i++){

}
}
}
```

The Basic Function

```
void Permute(int *matrix, int k,int n, int *number_per)
{
int i;
if(k==n) {
    PrintPermutation(matrix,n); (*number_per)++;
}
else {
    for(i=k;i<n;i++){
        swap(matrix,i,k);

        swap(matrix,k,i);
    }
}

}
```

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void Permute(int *matrix, int k,int n, int *number_per)
{
int i;
if(k==n) {
    PrintPermutation(matrix,n); (*number_per)++;
}
else {
    for(i=k;i<n;i++){
        swap(matrix,i,k);
        Permute(matrix,k+1,n,number_per);
        swap(matrix,k,i);
    }
}
}
```

The Basic Function

```
void Permute(int *matrix, int k,int n, int *number_per)
{
int i;
if(k==n) {
    PrintPermutation(matrix,n); (*number_per)++;
}
else {
    for(i=k;i<n;i++){
        swap(matrix,i,k);
        Permute(matrix,k+1,n,number_per);
        swap(matrix,k,i);
    }
}
return;
}
```

The main function call

```
int main(void){
```

```
    return 0;  
}
```

The main function call

```
int main(void){  
int i, m, *matrix, number_per=0;
```

```
    return 0;  
}
```

The main function call

```
int main(void){
    int i, m, *matrix, number_per=0;
    printf("\n How many numbers you want to permute::~>");
    scanf("%d",&m);

    return 0;
}
```

The main function call

```
int main(void){
    int i, m, *matrix, number_per=0;
    printf("\n How many numbers you want to permute::~>");
    scanf("%d",&m);
    matrix = allocate1D(m);

    return 0;
}
```

The main function call

```
int main(void){
int i, m, *matrix, number_per=0;
printf("\n How many numbers you want to permute::~>");
scanf("%d",&m);
matrix = allocate1D(m);
/*Fill the matrix with numbers from 1 to m */
for(i=0;i<m;i++)
    matrix[i] = i+1;

return 0;
}
```

The main function call

```
int main(void){
int i, m, *matrix, number_per=0;
printf("\n How many numbers you want to permute::~>");
scanf("%d",&m);
matrix = allocate1D(m);
/*Fill the matrix with numbers from 1 to m */
for(i=0;i<m;i++)
    matrix[i] = i+1;

Permute(matrix,0,m,&number_per);

return 0;
}
```

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int main(void){
int i, m, *matrix, number_per=0;
printf("\n How many numbers you want to permute::~>");
scanf("%d",&m);
matrix = allocate1D(m);
/*Fill the matrix with numbers from 1 to m */
for(i=0;i<m;i++)
    matrix[i] = i+1;

Permute(matrix,0,m,&number_per);
printf("\n Total permutations::~>%d\n",number_per);
return 0;
}
```

Generating Permutations – Another way

- Suppose we can generate permutations of all $n - 1$ numbers.

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Generating Permutations – Another way

- Suppose we can generate permutations of all $n - 1$ numbers.
- To generate permutations of n numbers, do the following:
- Generate permutations of all the numbers $1, 2, \dots, n - 1$ and add n to the 1st position.

Generating Permutations – Another way

- Suppose we can generate permutations of all $n - 1$ numbers.
- To generate permutations of n numbers, do the following:
- Generate permutations of all the numbers $1, 2, \dots, n - 1$ and add n to the 1st position.
- Generate permutations of all the numbers $1, 2, \dots, n - 1$ and add n to the 2nd position.

Generating Permutations – Another way

- Suppose we can generate permutations of all $n - 1$ numbers.
- To generate permutations of n numbers, do the following:
- Generate permutations of all the numbers $1, 2, \dots, n - 1$ and add n to the 1st position.
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- ...

Generating Permutations – Another way

- Suppose we can generate permutations of all $n - 1$ numbers.
- To generate permutations of n numbers, do the following:
- Generate permutations of all the numbers $1, 2, \dots, n - 1$ and add n to the 1st position.
- Generate permutations of all the numbers $1, 2, \dots, n - 1$ and add n to the 2nd position.
- ...
- Generate permutations of all the numbers $1, 2, \dots, n - 1$ and add n to the last position.

The Basic Function

```
void Permute(int *matrix, int k,int n, int *number_per){  
int i;
```

```
return;  
}
```

The Basic Function

```
void Permute(int *matrix, int k,int n, int *number_per){  
    int i;  
    if(k==0) {  
        PrintPermutation(matrix,n); (*number_per)++;  
    }  
  
    return;  
}
```

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void Permute(int *matrix, int k,int n, int *number_per){
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else {
    for(i=0;i<n;i++){

    }
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void Permute(int *matrix, int k,int n, int *number_per){
int i;
if(k==0) {
    PrintPermutation(matrix,n); (*number_per)++;
}
else {
    for(i=0;i<n;i++){
        if(matrix[i]==0) {
            matrix[i]=k;
            Permute(matrix,k-1,n,number_per);
            matrix[i]=0;
        }
    }
}
return;
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scanf("%d",&m);
matrix = allocate1D(m);
Permute(matrix,m,m,&number_per);
printf("\n Total permutations::~>%d\n",number_per);
return 0;
}
```