Data and File Structures Laboratory

B-Trees, B^{+}-Trees

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October, 2018

1. Basics
2. B-Trees
3. Implementation
4. B^+-Trees

Motivation

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What if the data items contained in a search tree do not fit into the main memory?

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The population of India: 1,358,856,931 (LIVE!!!)

Source: `http://www.worldometers.info/world-population/india-population`

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The search tree will require more than 20 GB memory (including pointers)!!!

Cycles to access different types of storage

Storage type	Access type	Number of cycles
CPU registers	Random	1
L1 cache	Random	2
L2 cache	Random	30
Main memory	Random	2.5×10^2
Hard disk	Random	3×10^7
	Streamline	5×10^3

Access times (for read or write) on disks are much costlier than the main memory!!!

Search trees on disks

A majority of the tree operations (search, insert, delete, etc.) will require $O(\log_2 n)$ disk accesses where n is the number of data items in the search tree.

The main challenge is to reduce the number of disk accesses.

An m-ary search tree allows m-way branching. As branching increases, the depth decreases. A complete binary tree has a height of $\lceil \log_2 n \rceil$ but a complete m-ary tree has a height of $\lceil log_m n \rceil$.

Characteristics of B-Trees

B-Tree is a low-depth self-balancing tree. The height of a B-Tree is kept low by putting maximum possible keys in a B-Tree node.

Generally, the node size of a B-Tree is kept equal to the disk block size.

B-Trees

Definition (B-Tree)

A B-Tree of order m is an m-ary tree with the following properties:

- The data items are stored at leaves.
- The non-leaf nodes store up to $m-1$ keys to guide the searching; The key i represents the smallest key in subtree $i+1$.
- The root is either a leaf or has between 2 and m children.
- All non-leaf nodes (except the root) have between $\lceil m/2 \rceil$ and m children.
- All leaves are at the same depth and have between $\lceil k/2 \rceil$ and k data items, for some k.

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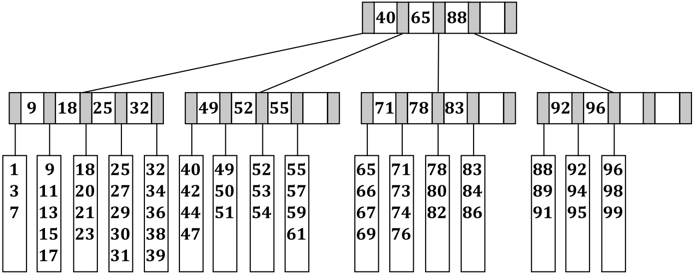
- All leaves are at the same depth and have between $\lceil k/2 \rceil$ and k data items, for some k.

Note: The properties 3 and 5 are relaxed for the first k insertions.

B-Trees

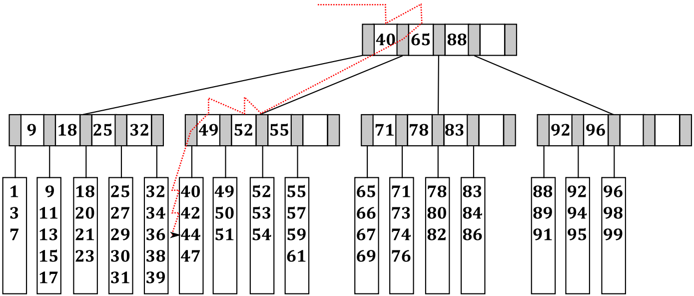
A B-Tree of order 5 and depth 3 that contains 59 data items.



Note: Here, $m = k = 5$.

Searching into B-Trees

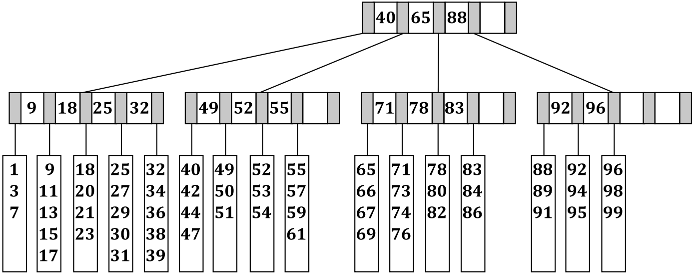
Searching 44 in the following B-Tree:



Note: The lookup (traversal shown in red) is over the disk.

Insertion into B-Trees

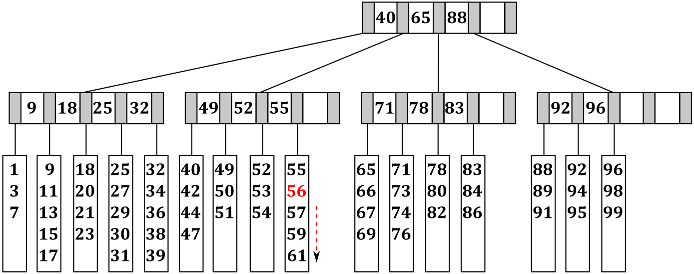
Inserting 56 into the following B-Tree:



Note: Insertion requires shifting of a few data items.

Insertion into B-Trees

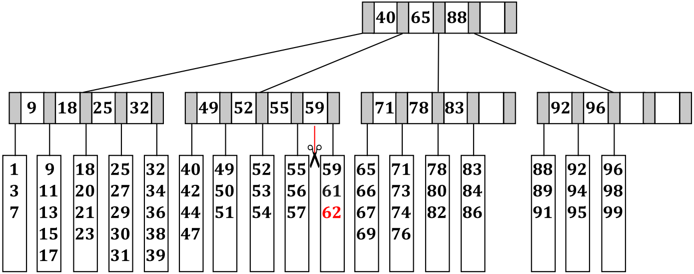
Inserting 62 into the following B-Tree:



Note: Insertion requires breaking a leaf node into a pair of nodes.

Insertion into B-Trees

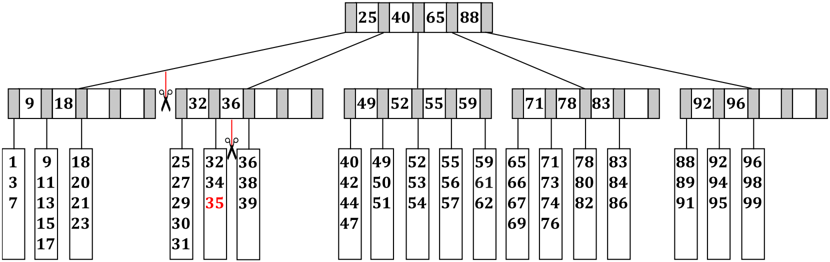
Inserting 35 into the following B-Tree:



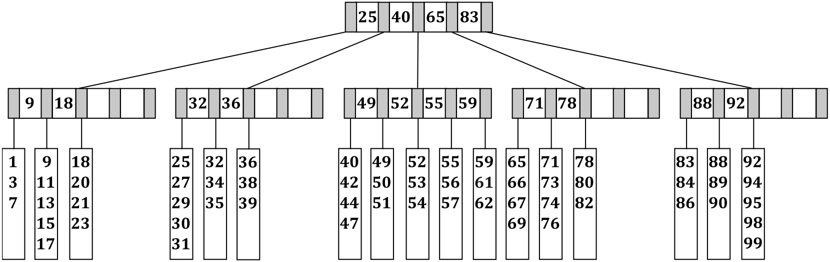
Note: Insertion requires breaking a leaf node into a pair of nodes and the inclusion of a new non-leaf node.

Deletion from B-Trees

Deleting 96 from the following B-Tree:



Deletion from B-Trees



Note: Deletion requires merging of a leaf node with another node.

Conventional implementation of B-Trees

- Individual dynamic memory allocation per node.
- The allocated memories (to the nodes) may be scattered all over the disk making streamline access impossible.

```
typedef struct treeNode{
    int Count;
    DATA d[ORDER];
    struct treeNode *link[ORDER];
    struct treeNode *parent; // This is optional
}BTREENODE;
```

Note: The token `DATA` symbolizes the data type accommodated.

Alternative implementation of B-Trees

- Initial dynamic memory allocation and successive reallocations.
- The allocated memories (to the nodes) are located in a contiguous location on the disk.

```
typedef struct treeNode{
    DATA d[ORDER-1];
    int link[ORDER];
    int parent; // This is optional
}BTREE_NLEAFNODE;
typedef struct treeNode{
    DATA d[K];
    int parent; // This is optional
}BTREE_LEAFNODE;
```

Note: `ORDER` and `K` are not necessarily the same.

Alternative implementation – An example

Initially: root = `NULL` (Say `ORDER` = 3)

	Count	d[0]	d[1]	d[2]	link[0]	link[1]	link[2]
free → 0	–	–	–	–	NULL	NULL	NULL
1	–	–	–	–	NULL	NULL	NULL
2	–	–	–	–	NULL	NULL	NULL
3	–	–	–	–	NULL	NULL	NULL
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$n-1$	–	–	–	–	NULL	NULL	NULL

Note: The value `NULL` is treated as '-1' during implementation.

Characteristics of B^+-Trees

Unlike the B-Trees, a B^+-tree does not have data items in the internal (non-leaf) nodes.

Interestingly, more number of keys can be fit on a page of memory in B^+-Trees (because no data is associated with internal nodes), resulting into fewer cache misses in order to access data that is on a leaf node.

B^+-Trees

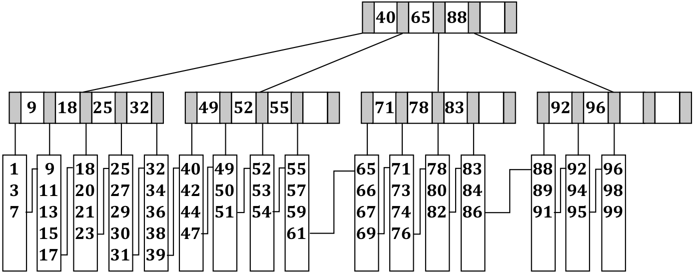
Definition (B^+-Tree)

A B^+-Tree of order m is an m-ary tree with the following properties:

- The data items are stored at leaves.
- The non-leaf nodes store up to $m-1$ keys to guide the searching; The key i represents the smallest key in subtree $i+1$.
- The root is either a leaf or has between 2 and m children.
- All leaves are at the same depth and have up to k data items, for some k.

B$^+$-Trees

A B$^+$-Tree of order 5 and depth 3 that contains 59 data items.



Problems – Day 23

1. Construct a B-Tree of order 7 with alternative implementation.