Recursion

Data and File Structures Laboratory

http://www.isical.ac.in/~dfslab/2018/index.html

Function calls

```
1   void main(void)
2   {  ...
3      u = f(x, y*z);
4      ...
5   }
6
7   int f(int a, int b)
8   {  ...
9      if (a > 0)
10        p = g(b);
11     else
12        p = h(b / 2);
13     return p;
14  }
15
16  int g(int m)
17  { ... }
18
19  int h(int n)
20  { ... }
```

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```

1. Let `a = x`, `b = y*z`.
2. Execute the statements in `f()`.
   (a) If `a` is positive, let `m = b`.
   Execute the statements in `g()`, and store the obtained value in `p`.
   (b) Otherwise, let `n = b/2`.
   Execute the statements in `h()`, and store the obtained value in `p`.
   (c) In either case, return value of `p` to calling function.
3. Store the value returned by `f` in `u`.
4. Continue from line 4.

Terminology

- **Caller**
- **Callee**
- **Formal parameters** (or simply parameters)

  Example: `a`, `b` – formal parameters for `f`
- **Actual parameters** (or actuals / arguments)

  Example: `x`, `y*z` – arguments passed to `f` on line 3

Call by value

- Arguments evaluated, copied into local storage area of called function
- Changes made to parameters in called function **not** reflected in caller
- C — call by value,

Call by value

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- C — call by value, but arrays are interpreted as pointers

Other parameter passing mechanisms

- Call by reference: changes made to argument variables in callee are reflected in caller
- Call by name: parameters are *literally* replaced in body of callee by arguments (like string replacement)
  - C macros use call by name

Passing parameters using pointers in C

- For arrays
- To “simulate” call by reference
- For efficiency

Definition

A recursive function is a function that calls itself.

- The task should be decomposable into sub-tasks that are smaller, but otherwise identical in structure to the original problem.
- The simplest sub-tasks (called the base case) should be (easily) solvable directly, i.e., without decomposing it into similar sub-problems.

Recursive vs. iterative implementations

```
1     int factorial ( int n ) {
2        int prod;
3
4        if (n < 0) return (-1);
5        prod = 1;
6        while (n > 0) {
7           prod *= n;
8           n = n - 1;
9        }
10       return (prod);
11    }
```

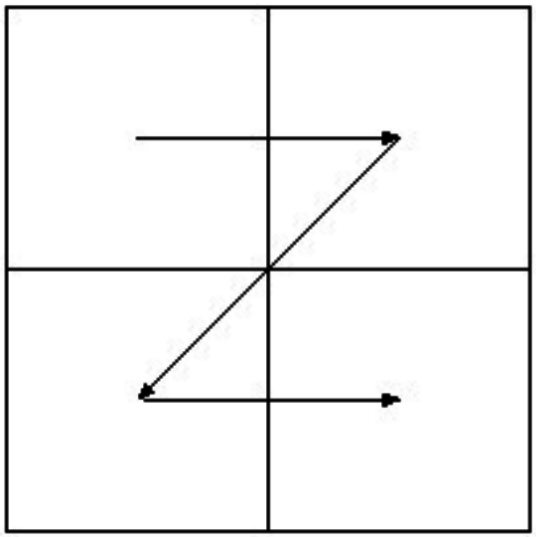
Recursive vs. iterative implementations

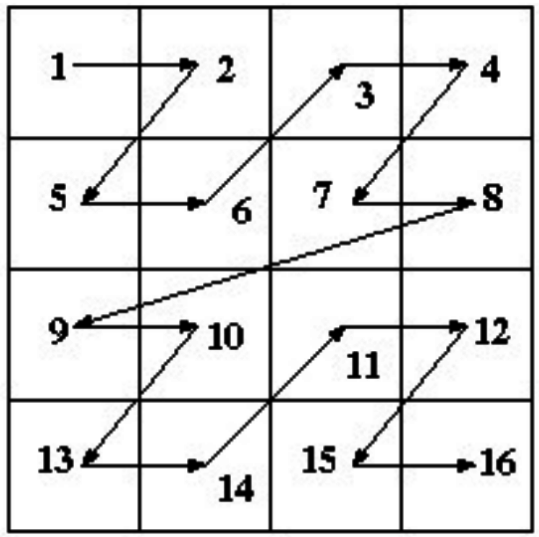
```
1       int factorial ( int n )              /* Recursive implementation */
2       {
3          if (n < 0) return (-1);           /* Error condition */
4          if (n == 0) return (1);           /* Base case */
5          return(n * factorial(n-1));       /* Recursive call */
6       }
```

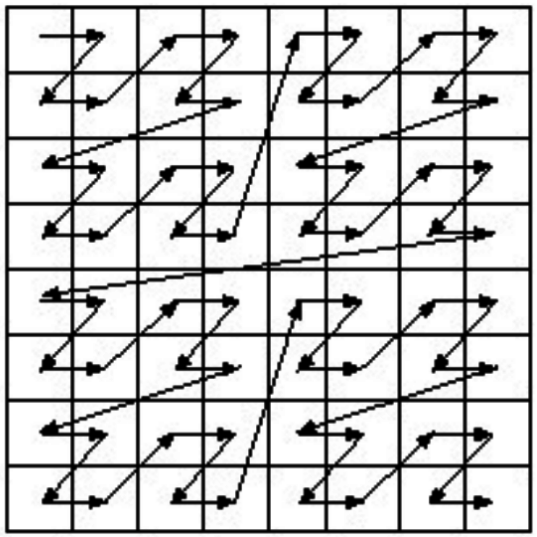
Z curve

Problem statement

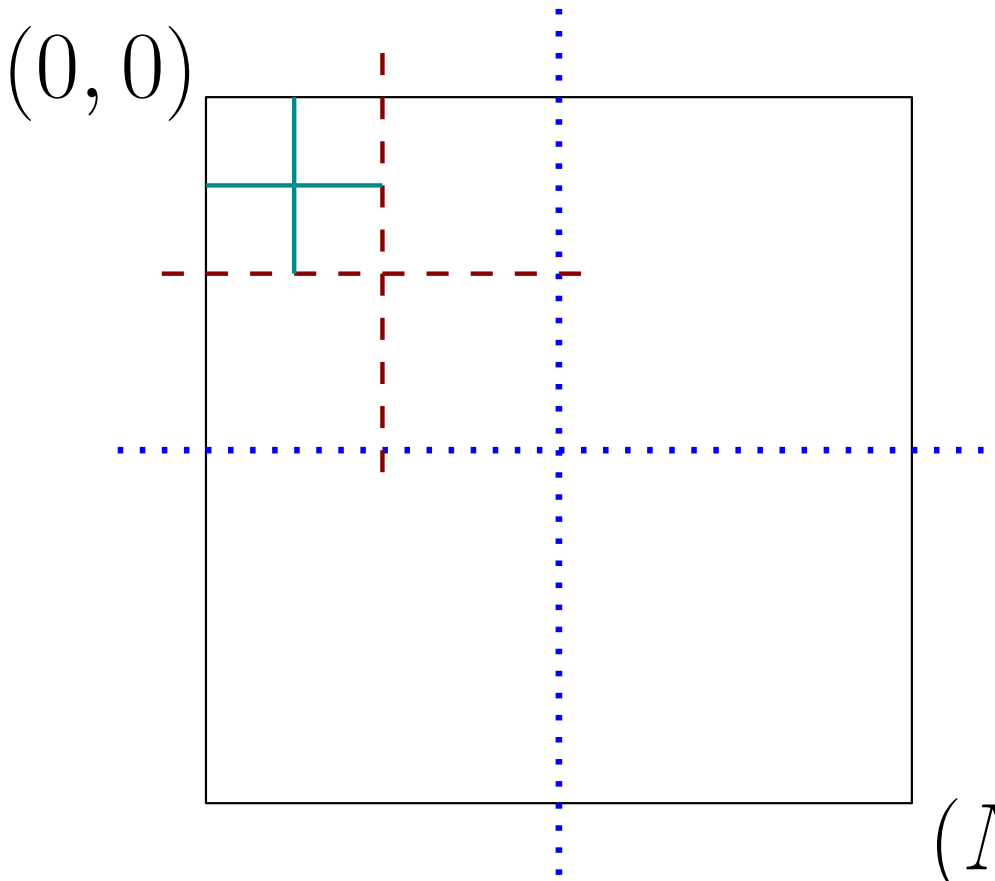
Consider a 2-D matrix of size $2^m \times 2^m$. The entries of the matrix are, in row-major order, $1, 2, 3, \ldots, 2^{2m}$. Print the entries of the matrix in Z-curve order (as shown in the picture below).







Z curve: structure



Problem structure:

- *Base case:* single element
- Recursive structure:
  - break given square into 4 sub-squares
  - process squares in Z-curve order

Z curve: code I

```
void z_curve(int top_left_row, int top_left_column,
             int bottom_right_row, int bottom_right_column,
             int **matrix)
{
    /* Base case */
    if (top_left_row == bottom_right_row &&
        top_left_column == bottom_right_column) {
        printf("%d ", matrix[top_left_row][top_left_column]);
        return;
    }


    /* Recurse */

    /* upper-left sub-square */
    z_curve(top_left_row,
            top_left_column,
            (top_left_row + bottom_right_row)/2,
            (top_left_column + bottom_right_column)/2,
            matrix);
```

Z curve: code II

```
    /* upper-right sub-square */
    z_curve(top_left_row,
            (top_left_column + bottom_right_column)/2 + 1
            (top_left_row + bottom_right_row)/2,
            bottom_right_column,
            matrix);

    /* lower-left sub-square */
    z_curve((top_left_row + bottom_right_row)/2 + 1,
            top_left_column,
            bottom_right_row,
            (top_left_column + bottom_right_column)/2,
            matrix);

    /* lower-right sub-square */
    z_curve((top_left_row + bottom_right_row)/2 + 1,
            (top_left_column + bottom_right_column)/2 + 1,
            bottom_right_row, bottom_right_column,
            matrix);
    return;
}
```

Permutations

Algorithm

To generate all permutations of $1, 2, 3, \ldots, n$, do the following:

1. Generate all permutations of $2, 3, \ldots, n$, and add 1 to the beginning.
2. Generate all permutations of $1, 3, 4, \ldots, n$ and add 2 to the beginning.

   $\ldots$

n. Generate all permutations of $1, 2, \ldots, n-1$ and add n to the beginning.

Permutations: code

```
void permute(int *A, int k, int n)
{
    int i;

    if(k==n) {
        for (i = 0; i < n; i++) {
            printf("%d ", A[i]);
        }
        putchar('\n');
        return;
    }

    for(i = k; i < n; i++){
        SWAP(A, i, k);
        permute(A, k+1, n);
        SWAP(A, k, i);
    }

    return;
}
```

Review question – slide I

Which of `read_data1`, `read_data2`, `read_data3` is best?

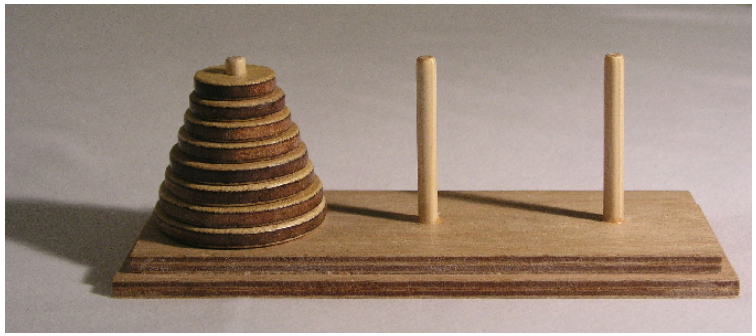
```
1  typedef struct {
2      char name[64];
3      int roll, rank;
4      float percent;
5  } STUDENT;
6
7  STUDENT *read_data1(void)
8  { STUDENT s;
9    scanf("%s %d %d %f",
10     &(s.name[0]), &(s.roll),
11     &(s.rank), &(s.percent));
12   return &s;
13 }
```

Review question – slide II

```
14
15  STUDENT read_data2(void)
16  { STUDENT s;
17    scanf("%s %d %d %f",
18      &(s.name[0]), &(s.roll),
19      &(s.rank), &(s.percent));
20    return s;
21  }
22
23  STUDENT *read_data3(STUDENT *s)
24  { scanf("%s %d %d %f",
25      &(s->name[0]), &(s->roll),
26      &(s->rank), &(s->percent));
27    return s;
28  }
```

Problems – I

1. Towers of Hanoi: see
   `https://en.wikipedia.org/wiki/Tower_of_Hanoi`



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2. Write a recursive function with prototype `int C(int n, int r);` to compute the binomial coefficient using the following definition:

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$$

   Supply appropriate boundary conditions.

Problems – II

3. Define a function $G(n)$ as:

$$G(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ G(n-1) + G(n-2) + G(n-3) & \text{if } n \geq 3 \end{cases}$$

Write recursive **and** iterative (i.e., non-recursive) functions to compute $G(n)$.

Problems – III

4. What does the following function compute?

```
int f ( int n )
{
    int s = 0;
    while (n--) s += 1 + f(n);
    return s;
}
```

5. http://cse.iitkgp.ac.in/~abhij/course/lab/PDS/Spring15/A4.pdf