# Computing Laboratory

## Searching Techniques

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# The searching problem

Given a data structure $S$ over a domain of data items $D$, verify (search) whether a given data item $s$ belongs to $D$ or not. If it belongs, then return the position (index) of $s$ in $D$ with respect to its organization in $S$.

# Searching on Linear Data Structures

- Linear search
- Jump search
- Binary search
- Interpolation search
- Fibonacci search

# Searching on Linear Data Structures

| Searching Method | On Arrays | On Linked Lists |
|---|---|---|
| Linear | Possible | Possible |
| Binary | Possible | Only possible with alternative implementation (no deletion applied) |
| Jump | Possible | Only possible with alternative implementation (no deletion applied) |
| Interpolation | Possible | Only possible with alternative implementation (no deletion applied) |
| Fibonacci | Possible | Possible with alternative implementation (no deletion applied) |

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| $i = i + 1$ | | $\downarrow$ | | | | |
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Return $i = 3$.

# Linear search

```
int LinearSearch(int *LIST, int SIZE, int s){
    int i;
    for(i=0; i<SIZE; i++)
        if(*(LIST + i) == s)
            return i; // Data is at the index i
    return; // Data is nowhere
}
```

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| $pre\_i = pre\_i + 1$ | | | | ↓ | | |
| Whether $LIST[pre\_i] = s$ | 10 | 20 | 30 | 40 | 50 | TRUE |

**Note:** The block size in jump search is taken as $b = \sqrt{n}$, where $n$ denotes the size of the list.

# Jump search

```
int JumpSearch(int *LIST, int SIZE, int b, int s){
    int pre_i = 0, i = b;
    while(*(LIST + Min_Func(i, SIZE) - 1) < s)
        pre_i = i, i += b;
        if(pre_i >= SIZE)
            return;
    // Linear search between pre_i and i
    for(j=pre_i+1; j<i; j++)
        if(*(LIST + j) == s)
            return i; // Data is at the index j
    return; // Data is nowhere
}
int Min_Func(int x, int y){
    return x < y ? x : y;
}
```

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| | | | | | | |
|---|---|---|---|---|---|---|
| $l = 0, r = 4, i = l + (r - l)/2 = 2$ | | | ↓ | | | |
| Whether $LIST[i] = s$ | 10 | 20 | 30 | 40 | 50 | FALSE |

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| Whether $LIST[i] = s$ | 10 | 20 | 30 | 40 | 50 | FALSE |
| | | | ↓ | | | |
| Whether $LIST[i] < s$ | 10 | 20 | 30 | 40 | 50 | TRUE |
| $l = i+1, i = l+(r-l)/2 = 3$ | | | | ↓ | | |
| Whether $LIST[i] = s$ | 10 | 20 | 30 | 40 | 50 | TRUE |

# Binary search

```
int BinarySearch(int *LIST, int l, int r, int s){
    int i;
    if(r >= l){
        i = l + (r - l)/2;
        if(*(LIST + i) == s)
            return i; // Data is in the middle
        if(*(LIST + i) < s) // Data is in the right
            return BinarySearch(LIST, i+1, r, s);
        // Data is in the left
        return BinarySearch(LIST, l, i-1, s);
   }
    return; // Data is nowhere
}
```

# Comparing linear/jump/binary search

| | **Linear search** | **Jump search** | **Binary search** |
|---|---|---|---|
| Requirements | Nothing | Ordered list | Ordered list |
| Backward scan required | No | Once | Multiple times |
| Time complexity (best case) | $O(1)$ | $O(b)$ | $O(1)$ |
| Time complexity (average case) | $O(n)$ | $O(\sqrt{n})$ | $O(\log n)$ |
| Time complexity (worst case) | $O(n)$ | $O(\sqrt{n})$ | $O(\log n)$ |

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| $i = 0 + (40 - 10) * (4 - 0)/(50 - 10) = 3$ | | | | ↓ | | |
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| Whether $LIST[i] = s$ | 10 | 20 | 30 | 40 | 50 | TRUE |

**Note:** The interpolation search is efficient over binary search for the ordered lists having uniformly distributed data items.

# Interpolation search

```
int InterpolationSearch(int *LIST, int SIZE, int s){
    int l = 0, r = SIZE - 1, p;
    while (l<=r && s>=*(LIST + l) && s<=*(LIST + r)){
        // Interpolate the position
        p = l + (s-*(LIST + l)) * (r-l) /
                        (*(LIST + r)-*(LIST + l));
        if(*(LIST + p) == s)
            return p;
        if (*(LIST + p) < s) // Data is in the right
            left = p + 1;
        else // Data is in the left
            right = p - 1;
    }
    return; // Data is nowhere
}
```

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| Whether $LIST[i] < s$ | 10 | 20 | 30 | 40 | 50 | TRUE |
| $f = 4-2+1 = 3, i = i + Fibonacci(f-2) = 3$ | | | | ↓ | | |
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**Note:** Fibonacci search is a suitable replacement of binary search where the list is unbounded. Unlike binary search, it does not use the costly division operation (though avoidable with bitwise shift).

# Searching on Nonlinear Data Structures

1. Searching on graphs
   - Breadth-first search
   - Depth-first search
2. Searching on trees
   - Searching in binary trees
   - Searching in heaps
   - Searching in binary search trees
   - Searching in balanced search trees

# Other Searching Problems

- Substring search
- Sublist search
- Range search
- Searching in streams

# Range search

A database query may ask to find out all the students with GPA between $g_1$ and $g_2$ (say 7.5 and 8.5), and salary between $s_1$ and $s_2$ (say 90K and 100K).

| Roll No. | Salary | GPA |
|---|---|---|
| 1 | 90K per mensem | 7.2 |
| 2 | 80K per mensem | 7.7 |
| 3 | 100K per mensem | 7.1 |
| 4 | 90K per mensem | 7.3 |
| 5 | 70K per mensem | 8.2 |

# Different variants of range search

**1-D range query problem:** Given a set of $n$ points (data items) on $\mathbb{R}$ (real line) and an interval (1-D query range), efficiently find the points that stay within the interval.

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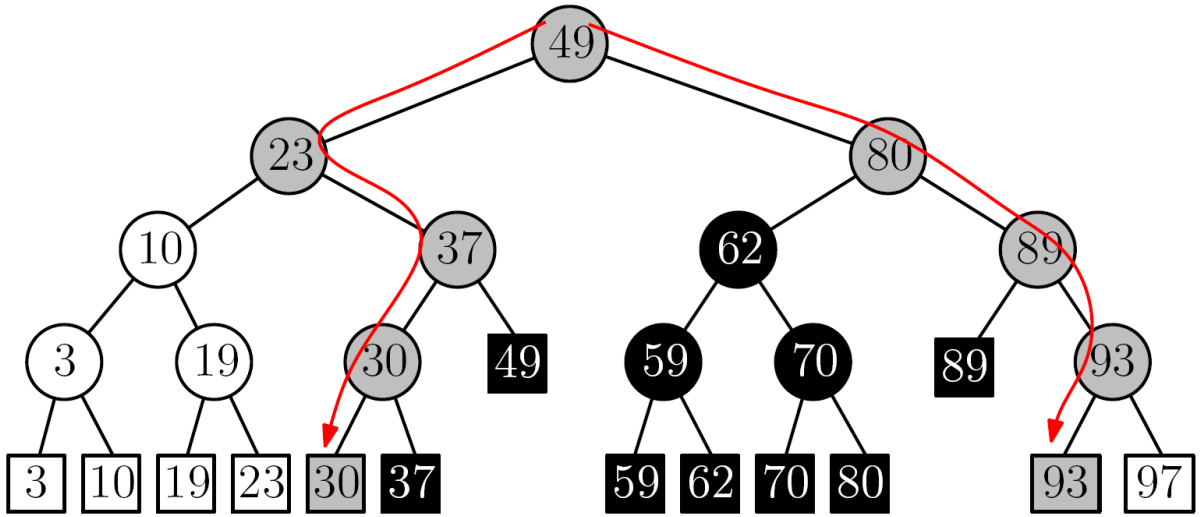


A 1-D range search with query [25, 90] (using search tree)

**Note:** The gray nodes are only visited (traversed).

# Different variants of range search

**2-D range query problem:** Given a set of $n$ points (coordinates) on $\mathbb{R}^2$ (2-D space) and a boundary (2-D query range), efficiently find the points that stay within the boundary.

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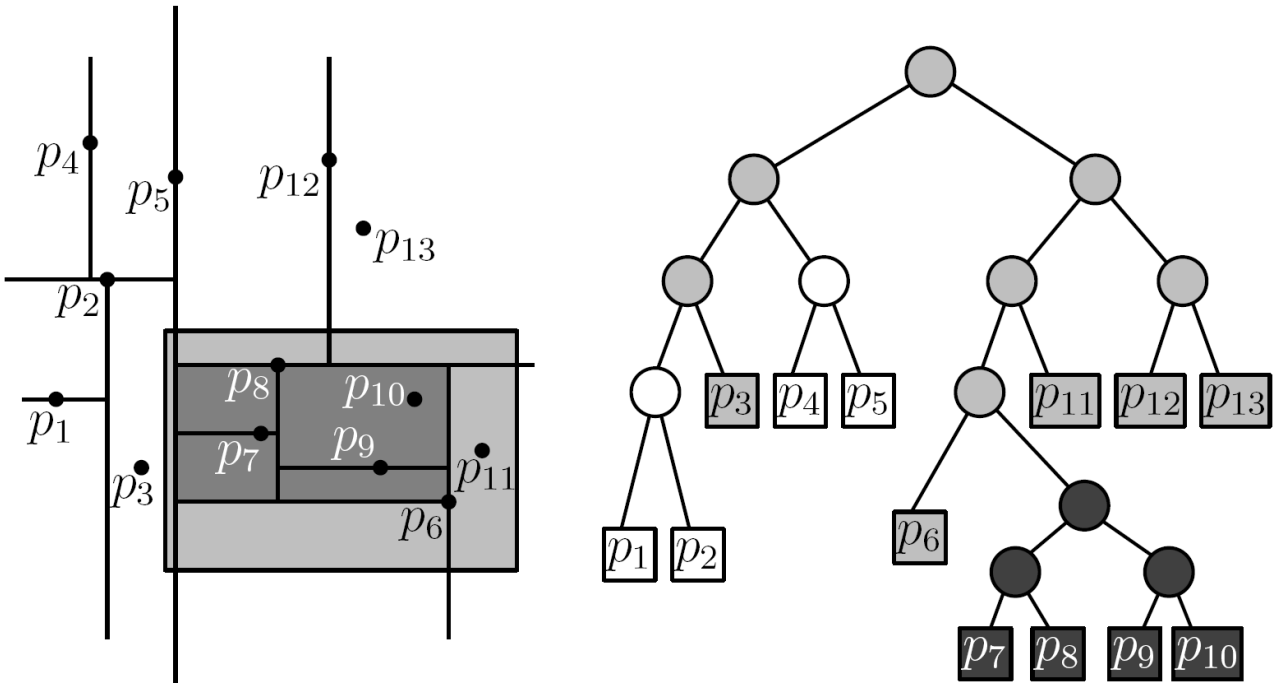
**2-D range query problem:** Given a set of $n$ points (coordinates) on $\mathbb{R}^2$ (2-D space) and a boundary (2-D query range), efficiently find the points that stay within the boundary.




A 2-D range search with query boundary (using $k$-dimensional tree)

# Searching in streams

**Definition (Data stream model)**

A *data stream model* defines an input stream $\mathcal{S} =< s_1, s_2, \cdots >$ arriving sequentially, item by item, and describes an underlying signal $S$, where $S : [1 \ldots N] \rightarrow \mathbb{R}$ is a one-dimensional function.

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The data stream models can be of the following three types:

- Time Series Model
- Cash Register Model
- Turnstile Model

# Different data stream models

The models are classified based on the information about how the input data elements stream in.

1. Time Series Model
   - Each $s_i$ equals $S[i]$ and they appear in increasing order of $i$.
   - Observing the traffic at an IP link for each 5 min, or volume estimation of share trading in every 10 min, etc.
2. Cash Register Model
   - Perhaps the most popular data model.
   - Here $s_i$'s are increments to $S[j]$'s.
   - Monitoring IP addresses that access a web server.
3. Turnstile Model
   - This is the most general model.
   - Here $s_i$'s are updates to $S[j]$'s.
   - Monitoring the stock values of a company.

# Processing streaming data

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A useful model for processing streaming data is to work on a window of the most recent $N$ elements received. Alternatively, we consider the elements received within a fixed time interval $T$.

*Consider a stream of integers. Further suppose, we want to find out the average of the integers in a window of size $N$.*

For the first $N$ inputs, we can sum and count the integers and calculate the average. After that, for each of the new input $i$ received, add $\frac{(i-j)}{N}$ to the previous average value, where $j$ is the oldest integer in the window.

# Streaming algorithms

In streaming algorithms, the data is available as a stream and we would like –

- the per-item processing time
- storage and
- overall computing time

to be simultaneously $O(N, t)$, preferably $O(polylog(N, t))$, at any time instant $t$ in the data stream.

**Note:** $O(polylog(N, t))$, often written as $polylog(N, t)$, means $O((\log n)^k)$ or $O(\log^k n)$ for some $k$.

# Problems – Day 14

1. Suppose a pair of linked lists in which the elements are sorted in ascending order are given. Write a program to efficiently find out whether one of them is a sublist of the other.
   **Input format:**

   ```
   1 3 5 7 9 11 13 15 17 19 21 23 # List 1
   5 7 9 11 # List 2
   ```

2. You are given a robotic hammer and some metal sheets. You can manually set the force (in Newton) imparted by the hammer as integral values. If a force cannot break the metal sheet cannot even make it weaker. Write a program to explore the maximum tolerable force (in Newton) of the metal sheet in minimum trials. The input (treat it as a black box) is the maximum tolerance level of the metal sheet and the outputs are the forces imparted by the robotic arm in successive trials.

# Problems – Day 14

3. Let us define a sequence $a_0, a_1, a_2, \ldots, a_{n-1}$ as $\Lambda$-*bitonic* if there exists a $j$, $0 \leq j < n$, such that $a_0 < a_1 < \cdots < a_j > a_{j+1} > \cdots > a_{n-1}$. Consider an $m \times n$ matrix $A$ consisting of integer entries, such that each row and each column of the matrix forms a $\Lambda$-bitonic sequence. Write a program to efficiently find the largest element of the matrix.
4. Suppose you are given a pair of ordered lists one of which has the salaries of ISI MTech students of 2016-18 batch in descending order and the other of 2017-19 in ascending order. Write a program to find out the median salary of all the students taken together involving time logarithmic to the (maximum) size of those lists.
5. Write a program to implement Fibonacci search on a generic array having ordered list of data items. Consider that the comparator() function works on various data types in a usual sense.

# Problems – Day 14

6. Suppose the marks of your Assignment 1 are available in ascending order. You have to figure out the students whose marks are same as their ranks (1 to $n$). Write a program to achieve this in no more than $\lfloor \log_2 n + 1 \rfloor$ trials.
7. Let $\pi$ be a permutation of $\{1, \ldots, n\}$ and $\pi_{-1}$ symbolizes $\pi$ with one element missing. Consider that the elements from the set $\pi_{-1}[i]$ appears as a stream in the increasing order of $i$, one by one. Write a program to determine the missing integer at the end. You are allowed to store only $O(\log n)$ bits of memory to determine the missing integer. Try to further restrict it to exactly $\log n$ bits of memory.

   **Input format:**

```
8 # The value of n
1 4 6 2 8 5 7 # The set of values streaming in
```