Demand Shift Effect Of R&D And The R&D Organization

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Abstract

We assume that R&D investment by a firm improves the quality of the product. This is reflected in an upward shift of the demand function. Firms can do R&D either independently or cooperatively. We show that cooperative research strictly dominates non-cooperative research both in terms of profitability and welfare. Also R&D investment by each firm under cooperative research is larger for a relatively high R&D output elasticity. Higher the degree of product differentiation, and/or, larger the R&D output elasticity, larger is the increase in quality level under cooperative research compared to non-cooperative research.

*Key words:* Cooperative research, product quality, R&D output elasticity.  
*JEL Classifications:* D21, L13, O31.
1. Introduction

Hindustan Motors Ltd., a famous company in motor car industry in India, had been for sometime finding difficulty to market its own brand, ‘contessa’; the reason suspected for low demand was the use of its fuel inefficient engine. Recently, the company had struck a collaborative deal with a Japan-based multinational, Isuzu, and replaced its own brand engine by the latter’s fuel-efficient engine. Thereafter the sale of contessa in the local market has picked up. This is possibly one of many such examples where improvement of the quality of a product or that of some components of a product with the help of a reputed company results in an upward shift of the demand. It is also possible that demand for a product goes up as the firm uses the foreign brand name. Consumers are willing to pay more as they perceive an improvement in the quality of the good.

This idea can perhaps be generalized. We may presume that a final product comprises of a number of various product components over which consumers’ utility is defined. For example, a computer means computer monitor, central processing unit, keyboard, printer etc., and a firm uses different brand or quality of each of these components to complete the final (composite) product. A consumer who consumes such a composite good is willing to pay a higher price for the product if it perceives a quality improvement for any of its components. The present paper seeks to model this observation. In particular, we want to explore the question of why a cooperative agreement between two firms to improve the quality of the product can be mutually profitable.

We construct a model of two firms which produce differentiated (substitute) products and compete non-cooperatively in the final goods market. We define a product to mean assemble of its different components or goods. Firms also engage in research and development (R&D) activity by which they can improve the quality of (some particular component of) the product. If they agree to do cooperative research, they pool their resources together and do research to improve the common components of their products. In any case, demand for final goods goes a rightward shift. However, we assume that firms cannot contract upon the resources to be invested by each. So there is a moral hazard problem in the case of cooperative research. Given this structure, we study whether the firms will have incentives for cooperative research. We derive following results in the paper.

1. Cooperative research unambiguously dominates non-cooperative research in terms of profitability as well as social welfare.
2. Quality of the final goods under cooperative research is always greater than that under non-cooperative research, and higher the degree of product differentiation, and/or higher the elasticity of research output, higher is the increase in quality under cooperative research compared to non-cooperative research.
3. The R&D investment level of each firm is larger under cooperative research for a relatively high R&D output elasticity.

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1 See Choi (1992), in particular, for the analysis. Each participant under cooperative R&D tends to under-provide the non-contractible input.
Let us now briefly outline the existing literature on the above issue. R&D activity involves uncertainty, asymmetric information and R&D spillovers. As a result there is in general a conflict between private and social incentives of R&D. One policy suggested in this context is to promote cooperative research under which competing firms pool their R&D resources and share the research results. Katz (1986) discusses the problem on this perspective. d’Aspremont and Jacquemin (1988), Ghosh (1990) and Suzumura (1992) have studied cooperative and non-cooperative R&D behavior in the presence of spillovers, but without uncertainty. Marjit (1991) and Combs (1992) divert attention to uncertainty without spillovers. Choi (1993) and Silipo (1995) discuss the problem when there are both uncertainty and spillovers. Cooperative R&D along with the possibility of product market collusion is discussed in Ordover and Willig (1985), Yi (1996) and Kabiraj and Mukherjee (2000). Cooperative research in the context of antitrust laws is analyzed in Brodley (1990) and Grossman and Shapiro (1986). Motta (1992) provides an analysis when products are vertically differentiated. The Kamien et al. (1992) paper gives a comparative study of the different forms of cooperative research such as R&D cartelization, RJV-competition and RJV-cartelization.

Now to relate our work with the existing literature we first note that most of the works cited above assume homogeneous goods and cost-reducing R&D. One exception is Motta (1992) which assumes that products are vertically differentiated and that R&D improves the quality of the products. Thus in Motta (1992), product differentiation is endogenous. Contrarily, in our paper final goods are horizontally differentiated and substitutes each other, with the degree of substitutability specified exogenously. Moreover, we focus on the quality improving aspect of R&D in the following sense. An R&D activity improves the quality of the product or its component; this is reflected in the rightward shift of the demand for final goods. No other papers look the problem from this angle. Conceptually, an upward shift of demand may be thought of as equivalent to a downward cost shift. But in our structure we get quite different results. All the papers cited above assume either spillovers or uncertainty or both, and it is the high value of spillovers (or high probability of success) that generates favorable impact. In contrast, in our paper we have neither spillovers nor uncertainty and so no positive externalities therefrom. In our paper cooperative R&D is strictly dominating although there is a moral hazard problem in supplying R&D inputs by the cooperative partners. In other papers the impact of welfare is, in general, ambiguous, while in our paper cooperative R&D always yield higher welfare compared to non-cooperative research. However, like others, total R&D investment can be higher or lower. We show that the total R&D investment is larger if the R&D output elasticity coefficient is relatively large. In no other paper the role of this factor is emphasized.

Plan of the paper is the following. In section 2 we present the basic model for both non-cooperative and cooperative R&D. In section 3 we derive the equilibrium for both cases --- subsection 3.1 discusses the non-cooperative equilibrium and subsection 3.2 discusses the cooperative equilibrium. Section 4 discusses the choice of R&D organization. Finally, the concluding remarks are given in section 5.

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2 Like other papers, both in Ghosh (1990) and Motta (1992) it is mostly the spillover effect that directs their results.
2. Description of the Model

Consider a partial equilibrium model of duopoly with product differentiation. We denote these firms as firm 1 and 2. Their products are differentiated in the following sense. On the one hand goods are horizontally differentiated in the sense that the respective firms’ products are substitute to each other in the consumer’s utility function. On the other hand, the consumer’s willingness to pay for the product of the ith firm depends on the quality of the inputs used in its production. This means we presume that the quality of inputs used determine the quality of the product. This is reflected in the position of the demand curve, as given by the demand intercept of the product.

Let the demand function of the ith firm be linear and in inverse form be given by the equation

\[ p_i = A_i(x_i) - q_i - \theta q_j, \quad i \neq j; \quad i, j = 1, 2 \]

where \( p_i \) is the price and \( q_i \) is quantity demanded for the ith firm’s products. The parameter \( \theta \) captures the degree of product differentiation; \( \theta = 0 \) means products are independent, and \( \theta = 1 \) implies products are homogeneous. Hence in our case \( 0 < \theta < 1 \).

The argument, \( x_i \) in the demand intercept \( A_i() \) denotes the quality of the inputs used in the production of the ith firm.

Without loss of generality, let us assume

\[ A_i(x_i) = x_i \]

With this, the demand function as faced by the ith firm becomes

\[ p_i = x_i - q_i - \theta q_j, \quad i \neq j; \quad i, j = 1, 2 \]  \hspace{1cm} (1)

Now, we assume that each firm plays a two-stage game. In the first stage firms decide their R&D investment which determine the quality of the inputs to be used and hence the quality of each firm’s final good. We consider two cases of R&D organization, viz., the non-cooperative R&D and the cooperative R&D. Then given the quality (and hence the demand intercept), in the second stage firms compete non-cooperating in the final goods market.

Let us assume that to attain a quality \( x_i \), the ith firm will have to invest in an input \( e_i \) (say skilled labor or effort in general). This gives the R&D production function of the ith firm as

\[ x_i = x(e_i) \quad \text{with} \quad x_i' > 0, \quad x_i'' < 0. \]  \hspace{1cm} (2)

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3 Note that we are not making any distinction between the inputs used in the final production and the outputs assembled to final goods.
We have put the usual restrictions of decreasing marginal returns from R&D activity. Also assume that \( x(0) = 0 \). The associated R&D cost associated with the input \( e_i \) is

\[
g_i = g(e_i) \text{ with } g' > 0, \ g'' > 0.
\]  

(3)

Obviously, \( g'' = 0 \) implies marginal cost of effort constant.

Note that in the second (i.e., output) stage, when firms choose their quantities simultaneously, R&D costs incurred in the first stage are sunk and qualities of the goods are considered fixed.

In case of cooperative R&D we assume that firms will set up an R&D laboratory jointly but each will supply the relevant input (skilled labor) non-cooperatively. The common quality \( x^* \) is given by

\[
x^* = x(e_i + e_j) \text{ with } x' > 0, \ x'' < 0.
\]  

(4)

There is a moral hazard problem in that the firms can not write contracts on the supply of R&D inputs, \( e_i \). Each firm \( i \) bears the cost associated with its supply \( e_i \) as given by (3). Each firm decides its input supply so as to maximize its own profits, not joint profit.

In the second stage of the game firms choose final outputs. The cost of producing final goods for each firm is

\[
c_i = c(q_i) \text{ with } c' > 0, \ c'' > 0.
\]  

(5)

The function states that the cost of production increases at an increasing rate and hence the final good production function has decreasing returns to scale.

3. Derivation of Equilibrium

In subsection 3.1, we characterize the non-cooperative equilibrium of the game described in the previous section, and in subsection 3.2 we portray equilibrium for the case when cooperative agreements are signed between the firms at the pre-competitive stage.

3.1 Non-Cooperative R&D

We solve the model in the usual backward fashion. First we solve equilibrium values of outputs, \( q_1 \) and \( q_2 \), at the second stage for given qualities \( x_1 \) and \( x_2 \), which are functions of \( e_1 \) and \( e_2 \), respectively. The profit function of the \( i \)th firm is

\[
\pi_i = p_i q_i - c(q_i) - g(e_i)
\]

or, \( \pi_i = (x_i - q_i - \theta q_j)q_i - c(q_i) - g(e_i) \)  

(6)
Second stage

The second stage first order condition for profit maximization is
\[
\frac{\partial \pi_i}{\partial q_i} = 0, \quad i = 1, 2.
\]
This gives,
\[
x_i - 2q_i - \theta q_j = c'(q_i)
\]  
(7)
Equilibrium values of \(q_1\) and \(q_2\) are solved from (7) for \(i = 1, 2\). Hence equilibrium \(q_i\) and \(q_2\) will each be function of both \(x_i\) and \(x_2\), that is,
\[
q_i = h_i(x_i, x_j) = q_i(e_i, e_j), \quad i = 1, 2.
\]  
(8)
We assume symmetric equilibrium. The corresponding second order conditions for profit maximization are satisfied, i.e,
\[
\frac{\partial^2 \pi_i}{\partial q_i^2} = -2 - c''(q_i) < 0
\]
Also for stability and uniqueness of the equilibrium we need to satisfy the following condition,
\[
\Delta q = \begin{vmatrix}
\frac{\partial^2 \pi_i}{\partial q_i^2} & \frac{\partial^2 \pi_i}{\partial q_j q_i} \\
\frac{\partial^2 \pi_j}{\partial q_i q_i} & \frac{\partial^2 \pi_j}{\partial q_j^2}
\end{vmatrix} > 0
\]
Note that in our case
\[
\Delta q = \{2 + c''(q_i)\} \{2 + c''(q_j)\} - 0^2 > 0.
\]
We have the following result.

Lemma 1: Given the non-cooperative choice of R&D investment, an increase in firm i’s R&D investment increases firm i’s equilibrium output but decreases the rival firm’s equilibrium output, i.e,
\[
\frac{\partial q_i}{\partial e_i}_{NC} > 0 \quad \text{and} \quad \frac{\partial q_j}{\partial e_i}_{NC} < 0.
\]
Proof: Using the first order, second order and stability conditions we shall get
\[
\frac{\partial q_i}{\partial e_i}_{NC} = \frac{x_i'[2 + c''(q_i)]}{\Delta q} > 0 \quad \text{and} \quad \frac{\partial q_j}{\partial e_i}_{NC} = -\frac{\partial x_i'}{\Delta q} < 0.
\]  
(9)
Lemma 1 is rather intuitive. More R&D spending by firm i increases its output since it increases the quality of its output. This in turn decreases j’s output through strengthening firm i’s competitiveness as \( q_i \) and \( q_j \) are strategic substitutes.

First stage

In this stage, firm i (i = 1, 2) will choose quality \( x_i \) which is the function of \( e_i \). So the problem of this stage is to maximize (6) w.r.t. \( e_i \) subject to (7).

Given (8), rewriting the profit function,

\[
\pi_i = [x(e_i) - q_i(e_i, e_j) - \theta q_j(e_i, e_j)] q_i(e_i, e_j) - c(q_i(e_i, e_j)) - g(e_i) .
\]

The first order condition is \( \frac{\partial \pi_i}{\partial e_i} = 0 \) for i = 1, 2. Using the envelope properties, we shall get

\[
\left[ x'_i - \theta \frac{\partial q_j}{\partial e_i} \right] q_i = g'(e_i).
\]

Equilibrium values of \( x_1 \) and \( x_2 \) are obtained by solving (10) for i = 1, 2. We assume that second order and stability conditions for equilibrium are satisfied.4

3.2 Cooperative R & D

Second stage

Since the second stage of the game is always non-cooperative, the relevant equilibrium conditions are structurally the same as given in the last subsection with the exception that each firm has the same quality, \( x^* \), determined in the first stage. Therefore, the first order condition in this case will be given by

\[
x^* - 2q_i - \theta q_j = c'(q_i).
\]

The equilibrium solutions are

\[
q_i = h_i(x^*(e_i, e_j)) = q_i(E), \quad i = 1, 2
\]

where \( E = e_i + e_j \) is the aggregate cooperative R&D input. We have the following Lemma.

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4 The relevant conditions are: \( \frac{\partial^2 \pi_i}{\partial e_i^2} < 0 \) and \( \frac{\partial^2 \pi_i}{\partial e_i^2} \frac{\partial^2 \pi_j}{\partial e_j^2} > \left( \frac{\partial^2 \pi_i}{\partial e_i \partial e_j} \right)^2 \).
Lemma 2: An increase in firm i’s R&D investment under research joint venture will increase both firms’ equilibrium supply of final goods, i.e., $\frac{\partial q_i}{\partial e_i} > 0$ and $\frac{\partial q_j}{\partial e_i} > 0$.

Proof: Using the first order and stability conditions it is easy to get

$$\frac{\partial q_i}{\partial e_i} = \frac{x^* \{2 - \theta + c''(q_j)\}}{\Delta q^*} > 0 \quad \text{and} \quad \frac{\partial q_j}{\partial e_i} = \frac{x^* \{2 - \theta + c''(q_i)\}}{\Delta q^*} > 0. \quad (13)$$

The intuition of the result is simple. More R&D spending by firm i will increase the quality of the product of both firms. This is equivalent to the shift of the demand intercept for both firms. Hence output of each firm goes up. Note that under the non-cooperative R&D case, an increase in firm i’s investment reduces the rival’s output whereas under the cooperative R&D case rival’s output goes up. The reason is that under the non-cooperative case, as firm i pushes its investment level, its quality level improves. This is reflected in the upward shift of its demand schedule and hence only firm i’s reaction function undergoes a rightward shift. Given quantities are strategic substitutes, the rival is forced to reduce its output. But under the cooperative case if any firm increases its R&D investment, both firms enjoy a symmetric shift of the demand function, and hence both firms have larger output in equilibrium.

First stage

The profit function of firm i is

$$\pi_i = [x^*(E) - q_i(E) - \theta q_j(E)]q_i(E) - c(q_i(E)) - g(e_i)$$

where $E = e_i + e_j$. Firm i will provide input $e_i$ in such a way that its own profit is maximized. Hence firm i will maximize $\pi_i$ w.r.t. $e_i$. The relevant first order condition is $\frac{\partial \pi_i}{\partial e_i} = 0$. This leads to the condition,

$$(x^* - \theta q_j)q_i = g'(e_i). \quad (14)$$

We continue to assume that the second order and stability conditions are satisfied. This completes the first stage solution of the game. To characterize further the equilibrium output of two games as portrayed in subsections 3.1 and 3.2 and to resolve the problem of choice of R&D organization we assume explicit forms of different functions satisfying usual restrictions. These are examined in the following section.
4. The choice of R&D organization

Consider the model as described by the following set of functions. We focus only on the symmetric equilibrium.

The demand for final good is the same as described by (1). To repeat, the function is

\[ p_i = x_i - q_i - \theta q_j. \]

The R&D production function is

\[ x_i = e_i^\alpha \]  \hspace{1cm} (15)\]

where the parameter \( \alpha \) is the R&D output elasticity w.r.t. the input in the R&D production function. The relevant restriction on this parameter is \( 0 < \alpha < 1 \). This ensures that \( x'_i > 0 \) and \( x''_i < 0 \).

The R&D cost function is

\[ g(e_i) = e_i^2. \]  \hspace{1cm} (16)\]

Hence we have \( g' > 0 \) and \( g'' > 0 \).

Finally, the cost of producing final goods is given by

\[ c(q_i) = q_i^2. \]  \hspace{1cm} (17)\]

This also satisfies the restrictions \( c' > 0 \) and \( c'' > 0 \).

**Non-cooperative R & D**

Let \( q_1 = q_2 = q_N \) and \( x_1 = x_2 = x_N \) be the symmetric equilibrium solution for quantities and qualities under the non-cooperative R&D case. Then given the specification of the functions, using (7) the second stage quantity solution is

\[ q_N = \frac{x_N}{4 + \theta} = \frac{e_N^\alpha}{4 + \theta}. \]  \hspace{1cm} (18)\]

In this case,

\[ \Delta q = 16 - \theta^2 \]

Therefore using (9),
\[
\frac{\partial q_j}{\partial e_i} = -\frac{\theta \alpha e_N^{\alpha-1}}{16 - \theta^2}.
\]

To solve the first stage outcome \(x_N\), using (10) we shall get

\[
\left[ \alpha e_N^{\alpha-1} + \frac{\theta \alpha e_N^{\alpha-1}}{16 - \theta^2} \right] \frac{e_N^\alpha}{4 + \theta} = 2e_N
\]

or, \(\alpha e_N^{2(\alpha-1)} = \frac{(4 + \theta)^2 (4 - \theta)}{8}\) \((19)\)

It may be checked that the second order and stability conditions are satisfied (see Appendix A). At this stage, a further restriction on the parameter \(\alpha\) may be noted so that we can ensure a nonnegative profit in equilibrium for each firm. From (6), (18) and (19) we shall get\(^5\)

\[
\pi_N = e_N^2 \left[ \frac{(4 - \theta)}{4\alpha} - 1 \right].
\]

(20)

Then, \(\pi_N \geq 0 \iff 4(1 - \alpha) \geq \theta\).

Therefore, given \(\theta < 0 < 1\), we can define \(\alpha(\theta) = \frac{(4 - \theta)}{4}\), and then we restrict that \(\alpha \in (0, \alpha(\theta))\). Quite obviously, if \(\alpha < 3/4\), then \(\pi_N > 0 \forall \theta \in (0, 1)\). So given \(\theta\), if \(\alpha > \alpha(\theta)\), non-cooperative research is not privately profitable.

**Cooperative R & D**

Let \(q_1 = q_2 = q_C\) and \(e_1 = e_2 = e_C\). Then the common demand intercept is

\(x^* = x(e_1 + e_2) = (2e_C)^\alpha\).

This corresponds to an equilibrium output

\[
q_C = \frac{x^*}{4 + \theta} = \frac{(2e_C)^\alpha}{4 + \theta}.
\]

(21)

Using the results of Lemma 2,

\[
\frac{\partial q_j}{\partial e_i} = \frac{\alpha (2e_C)^{\alpha-1} (4 - \theta)}{16 - \theta^2} = \frac{\alpha (2e_C)^{\alpha-1}}{4 + \theta}.
\]

\(^5\) \(\pi_N = [e_N^\alpha - (1 + \theta)q_N]q_N - q_N^2 - e_N^2 = [(4 + \theta)q_N - (1 + \theta)q_N]q_N - q_N^2 - e_N^2 = 2q_N^2 - e_N^2 = [2e_N^{2\alpha}/(4 + \theta)^2] - e_N^2 = e_N^2[\{2e_N^{2(\alpha-1)}/(4 + \theta)^2\} - 1].\)
From the first stage first order condition (see (14)),
\[
\left( \alpha (2e_C)^{a-1} - \frac{\theta \alpha (2e_C)^{a-1}}{4 + \theta} \right) \frac{2(e_C)^{a}}{4 + \theta} = 2e_C
\]
or,
\[
\alpha (2e_C)^{2(a-1)} = \frac{(4 + \theta)^2}{4}.
\]  
(22)

**Comparison**

Now to compare the results under two forms of research organization, let us divide (19) by (22) to get
\[
\left( \frac{2e_C}{e_N} \right)^{2(1-a)} = 2 - \frac{\theta}{2}.
\]  
(23)

The relation (23) clearly shows that the ratio \( \frac{2e_C}{e_N} \) depends both on R&D output elasticity, \( \alpha \), and degree of product differentiation \( \theta \).

From the equilibrium solutions we shall get
\[
\frac{x_C}{x_N} = \left[ \frac{2e_C}{e_N} \right]^a.
\]  
(24)

From (23) and (24) we have the following results.

**Proposition 1**: Assume \( 0 < \theta < 1 \) and \( 0 < \alpha < \alpha(\theta) \). The quality of final goods under cooperative R&D is always greater than that under non-cooperative R&D.

**Proof**: Quality under non-cooperative R&D is \( x_N \) which depends on \( e_N \) whereas that under cooperative R&D is \( x_C \) which depends on \( 2e_N \). The R.H.S. of (23) is greater than 1. Since \( \alpha < 1 \), we therefore must have \( 2e_C > e_N \). Finally, from (24), we get \( x_C > x_N \).

Economic intuition of the result is the following. The cooperative R&D investment by a firm can be more than, less than, or equal to its R&D investment in the non-cooperative case. But the sum of their cooperative R&D is always greater than the individual investment under non-cooperative case. Obviously, the quality of final goods under cooperative R&D will be greater than the non-cooperative R&D case.

Comparative static results, also followed from (23) and (24), are summarized in the following proposition.
Proposition 2:
(a) More the degree of product differentiation, higher is the increase in quality level in case of cooperative R&D compared to non-cooperative R&D.

(b) Higher the elasticity of R&D output, the more will be the increase in quality in case of cooperative R&D compared to non-cooperative case.

The intuition behind proposition 2(a) is the following. Higher the degree of product differentiation, less would be the competition in product market; so higher will be the return by increasing quality. Firms will invest more for increasing quality. Hence cooperative research would be preferred to non-cooperative R&D (by Proposition 1). Similarly, the intuition behind Proposition 2(b) is that higher the elasticity of R&D output, higher would be the quality for given investment. Hence increase in quality would be more in case of joint research compared to independent research.

A further inspection of equilibrium (23) reveals that the cooperative R&D investments of a firm might even fall below the non-cooperative level (i.e., $e_C < e_N$), although we have unambiguously $x_C > x_N$ (because $2e_C > e_N$). Hence we have the following proposition.

Proposition 3: For a relatively high R&D output elasticity, the R&D investment of a firm will go up under cooperative research, but for a lower R&D output elasticity non-cooperative R&D investment of each firm will be larger.

Proof: Let us first restrict to $\alpha \in (0, \alpha(\theta))$. Then for every $\theta$, given equilibrium (23), we can define an $\alpha^*(\theta)$ such that $e_C \geq (<) e_N \Leftrightarrow \alpha \geq (<) \alpha^*$. \hfill $\square$

The result is shown in Figure 1. We have drawn two curves. First we draw $\alpha(\theta) = \frac{4 - \theta}{4}$ which is a downward sloping straight line. Then we restrict our attention to all combinations of $(\alpha, \theta)$ lying below the curve. Then using (23) we draw another locus of $(\alpha, \theta)$ such that $e_C = e_N$. This locus, denoted by $\alpha^*(\theta)$, is upward sloping with $\alpha = 1/2$ at $\theta = 0$ and $\alpha < 3/4$ at $\theta = 1$. Hence for all points bounded by these two curves we have $e_C > e_N$ and all points below the $2^{nd}$ relation reflects $e_C < e_N$.

Finally, to compare profits under these two forms of research organization we first derive the profit expression as

$$
\pi_N = \frac{2e_N^{2\alpha}}{(4+\theta)^2} - e_N^2
$$

(25) and

$$
\pi_C = \frac{2(2e_C)^{2\alpha}}{(4+\theta)^2} - e_C^2.
$$

(26)
Then from (25) and (26)

\[
\frac{\pi_C + e_C^2}{\pi_N + e_N^2} = \left(\frac{2e_C}{e_N}\right)^{2\alpha} > 1.
\]

Hence,

\[
\pi_C - \pi_N > e_N^2 - e_C^2.
\]  (28)

Quite obviously, we have \( \pi_C > \pi_N \) if \( e_C \leq e_N \). Also we can show that \( \pi_C > \pi_N \) whenever \( e_C > e_N \) (see Appendix B). This leads to the following proposition.

**Proposition 4**: Profits under cooperative R&D are always larger compared to the case of non-cooperative R&D.

Since both profits and quality (and hence the demand shift) are larger under cooperative research, the overall welfare must also be larger under cooperative research compared to non-cooperative research.

Finally, consider the comparative static effects of the change of \( \alpha \) and \( \theta \) on profits. We have following results (see Appendix B):

\[
\frac{d}{d\alpha} \left( \frac{\pi_C}{\pi_N} \right) > 0 \quad \text{and} \quad \frac{d}{d\theta} \left( \frac{\pi_C}{\pi_N} \right) > 0.
\]

This states that as the elasticity of R&D output goes up, cooperative profit increases more than non-cooperative profit, but as the degree of product differentiation goes up, profits under cooperative R&D increases less compared to the case of non-cooperative R&D. Intuition of the result is the following. Change of \( \alpha \) affects the respective payoffs only through an indirect effect. An increase in \( \alpha \) will lead to an increase in R&D investment (and hence quality of the product) more under cooperative research than under non-cooperative research. Therefore, profit increase under cooperative research is larger compared to the non-cooperative case (see (25) and (26)). But change of \( \theta \) has both direct and indirect effects. An increase in product differentiation (i.e., fall in \( \theta \)) lessens product market competition, and increases profits under both under cooperative and non-cooperative research (profits under cooperative research in fact increases more). Again, as \( \theta \) falls, R&D investment goes up due to higher profitability, but because of the moral hazard problem increase in cooperative R&D investment is not large compared to the non-cooperative level. This indirect effect dominates the direct effect, and hence increase in profit under cooperative research will be less compared to non-cooperative case.
5. Conclusion

In this paper we have considered demand shift effects of R&D. Consumers are willing to pay a higher price for a better quality of the product. We have constructed a two-stage model of duopolistic quantity competitive structure where in the first stage firms decide their R&D investment either cooperatively or non-cooperatively. Although there is moral hazard in supplying R&D inputs, in our structure cooperative research strictly dominates non-cooperative research both in terms of profits and consumers’ welfare. Moreover, this happens without having any uncertainty or spillovers of R&D. This provides a new argument in favor of promotion of R&D cooperative agreement. Also R&D investment by each firm under cooperative research is larger for a relatively high R&D output elasticity. Higher the degree of product differentiation and/or larger the R&D output elasticity, larger is the increase in both quality and quantity of final goods under cooperative research compared to non-cooperative research. We have also studied effects of their changes on the respective profit levels. Thus the paper focuses on the importance of the parameters like the degree of product differentiation and the elasticity of R&D output in the choice of R&D organization.
Appendix

Appendix A

\[
\frac{\partial \pi_i}{\partial e_i} = (x_i - 2q_i - \theta q_j - c'(q_i)) \frac{\partial q_j}{\partial e_i} + x_i' - \theta \frac{\partial q_j}{\partial e_i} q_i - g'(e_i)
\]

\[
= \frac{\partial \pi_i}{\partial q_i} \frac{\partial q_i}{\partial e_i} + \left( x_i' - \theta \frac{\partial q_j}{\partial e_i} \right) q_i - g'(e_i)
\]

\[
\frac{\partial^2 \pi_i}{\partial e_i^2} = \frac{\partial^2 \pi_i}{\partial q_i^2} \left( \frac{\partial q_i}{\partial e_i} \right)^2 + \frac{\partial \pi_i}{\partial q_i} \frac{\partial^2 q_i}{\partial e_i^2} + \left( x_i' - \theta \frac{\partial q_j}{\partial e_i} \right) \frac{\partial q_i}{\partial e_i} + \left( x_i'^\prime - \theta \frac{\partial^2 q_j}{\partial e_i^2} \right) q_i - g''(e_i)
\]

\[
= \frac{\partial^2 \pi_i}{\partial q_i^2} \left( \frac{\partial q_i}{\partial e_i} \right)^2 + 0 + \frac{g'(e_i)}{q_i} \frac{\partial q_i}{\partial e_i} + \left( x_i'^\prime - \theta \frac{\partial^2 q_j}{\partial e_i^2} \right) q_i - g''(e_i)
\]

In our example

\[
\frac{\partial^2 \pi_i}{\partial e_i^2} = -\frac{64\alpha^2}{e_{2(1-\alpha)}(16-\theta^2)} + \frac{8\alpha}{4-\theta} - \frac{16\alpha(1-\alpha)}{e_{2(1-\alpha)}(16-\theta^2)(4-\theta)} - 2.
\]

Then using (19), i.e., \( \alpha = \frac{(4+\theta)^2(4-\theta)}{8} \), we have

\[
\frac{\partial^2 \pi_i}{\partial e_i^2} = -\frac{8\alpha}{4-\theta} + \frac{8\alpha}{4-\theta} - \frac{16\alpha(1-\alpha)}{e_{2(1-\alpha)}(16-\theta^2)(4-\theta)} - 2
\]

\[
= -\frac{16\alpha(1-\alpha)}{e_{2(1-\alpha)}(16-\theta^2)(4-\theta)} - 2 < 0.
\]

Appendix B

\[
\pi_C = 2 \left( \frac{2e_C}{(4+\theta)^2} \right)^{2a} - e_C^2 = \frac{2(2e_C)^{2a} - e_C^2(4+\theta)^2}{(4+\theta)^2}
\]

\[
\pi_N = 2 \left( \frac{e_N}{(4+\theta)^2} \right)^{2a} - e_N^2 = \frac{2(e_N)^{2a} - e_N^2(4+\theta)^2}{(4+\theta)^2}.
\]

Therefore,

\[
\frac{\pi_C}{\pi_N} = \frac{2(2e_C)^{2a} - e_C^2(4+\theta)^2}{2(e_N)^{2a} - e_N^2(4+\theta)^2} = \frac{2(2e_C)^{2a} - (2e_C)^2(1/4)(4+\theta)^2}{2(e_N)^{2a} - e_N^2(4+\theta)^2}
\]

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\[ \frac{\pi_C}{\pi_N} = 2^{2a} \left( \frac{e_C}{e_N} \right)^{2a} \left[ \frac{1 - (4\theta^2) \left( \frac{4}{2} \right) (4 \pm \theta)^2}{1 - (e_N^{2(1-\alpha)}) \frac{1}{2} (4 + \theta)^2} \right] \]  

Hence,

\[ \frac{\pi_C}{\pi_N} = 2^{2a} \left( \frac{e_C}{e_N} \right)^{2a} \left[ \frac{1 - (4\theta^2) (e_N^{2(1-\alpha)}) \frac{1}{2} (4 + \theta)^2}{1 - (e_N^{2(1-\alpha)}) \frac{1}{2} (4 + \theta)^2} \right]. \]  

(B1)

Since \( \frac{4-\theta}{8} < 1 \), the term in the square bracket is positive and greater than one. Hence \( \pi_C > \pi_N \) when \( e_C > e_N \).

From (B1) we can note that, as \( \alpha \) increases, \( \frac{e_C}{e_N} \) increases (see Proposition 3). Also, the first term of the RHS of (B1) goes up. This gives \( \frac{d}{d\alpha} \left( \frac{\pi_C}{\pi_N} \right) > 0 \).

Further using (19) and (23) we can rewrite the expression (B1) as the following,

\[ \frac{\pi_C}{\pi_N} = \left( \frac{4-\theta}{2} \right)^{\alpha/(1-\alpha)} \left[ \frac{1 - \frac{4-\theta}{8} (4 + \theta)^2 (4 - \theta)^2}{1 - \frac{8\alpha}{(4 + \theta)^2 (4 - \theta)^2} \frac{1}{2} (4 + \theta)^2} \right] \]  

\[ = \left( \frac{4-\theta}{2} \right)^{\alpha/(1-\alpha)} \left[ \frac{1 - \frac{\alpha}{2}}{\frac{4\alpha}{2}} \right] = \frac{2 - \alpha}{1 - \frac{4\alpha}{4(1-\alpha)}} \frac{1}{4(1-\alpha) - \theta} \]  

From this it can be shown easily that, \( \frac{d}{d\theta} \left( \frac{\pi_C}{\pi_N} \right) > 0 \).
References
