TESTING MARKET EFFICIENCY IN THE FRAMEWORK
OF MODEL SPECIFICATION : AN EMPIRICAL
INVESTIGATION WITH INDIAN DATA

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ABSTRACT

The paper suggests a systematic approach towards studying efficiency in stock market with an aim to identify the causes of observed inefficiency in any stock market, and apply the same in the context of one of the most important emerging stock markets viz., Indian stock market. The proposed approach defines inefficiency to include nonlinear dependence in the returns and envisages appropriate specification of both the conditional first and second order moments so that final conclusions on efficiency are free from any probable statistical consequences of misspecification. Towards this end, a number of rigorous tests are applied on the returns based on four major daily indices of Indian stock market. It is found that the Indian stock market is inefficient, and this inefficiency is due to serial correlation, nonlinear dependence, day-of-the week effects, parameter instability, conditional heteroscedasticity (GARCH), daily-level seasonality in volatility, short term interest rate (in some sub-periods of some indices) and some dynamics in the higher order moments.

Key words : Automatic variance ratio test; BDS test; Hansen test; market efficiency; misspecification; nonlinear dependence.

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1. INTRODUCTION

One of the earliest and most enduring questions of financial economics is whether financial asset prices are forecastable. The concept of efficient market hypothesis which asserts that the asset price changes are unforecastable, can be traced back at least as far as the pioneering theoretical contribution of Bachelier (1900) and the empirical research of Cowles (1933). The modern literature on financial market efficiency begins with Samuelson (1965) who in his landmark article tried to prove why properly anticipated prices fluctuate randomly. In an informationally efficient market --- different from an allocationally or Pareto - efficient market -- price changes must be unforecastable if they are properly anticipated, i.e., if they fully incorporate the expectations and information of all market participants. Fama (1970, p. 384) summarizes this idea by stating that "A market in which prices always 'fully reflect' available information is called 'efficient' ".

More recently, Malkiel (1992) has given the definition of informational efficiency more explicitly, the economic implication of which is that it is impossible to make economic profits by trading on the basis of the given information set.

Before the days of nonlinear dynamics, tests for Fama's efficient market hypothesis (EMH) in the context of stock market usually meant testing the null hypothesis that autocorrelation coefficient of different lags are statistically insignificant. For this purpose runs test, Ljung-Box (1978) test of autocorrelation and regression tests used to be normally applied. But since 1980's it is well appreciated that lack of linear dependence (i.e., serial correlation) does not rule out nonlinear dependence which, if present, would contradict the EMH and may aid in forecasting, especially over short time intervals. Specifically, Granger and Anderson (1978) and Sakai and Tokumaru (1980) have shown that simple nonlinear models exhibit no serial correlation while containing strong nonlinear dependence. This has, in fact, led several researchers like Granger and Anderson (1978), Hinich and Patterson (1985) and Scheikman and LeBaron (1989) to look for nonlinear structures in stock returns. It may be noted in this context that one of the most important and useful tests available in the literature for detecting nonlinear patterns i.e., the existence of potentially forecastable structures, is due to Brock et al. (1987, revised 1996), to be henceforth denoted as BDS test. With increasing power of computers coupled with advances in both nonlinear dynamics and chaos, the volume of research into the re-examination of the behaviour of security returns from the standpoint of market-efficiency has increased considerably, and most of these (see Hsieh, 1991; Willey, 1992; Sewell et al., 1993; Opong et al., 1999; among others) have cast doubt on the conclusion of market efficiency based only on the lack of serial correlation in returns.

Apart from complicated nonlinear dependence/dynamics, there are two well-known reasons as to why stock prices may deviate from the random walk model. First, conditional variance of stock returns is not constant over time. This fact has led to the development of autoregressive conditional heteroscedasticity (ARCH) and generalised ARCH (GARCH) models (cf. Engle, 1982; Bollerslev, 1986). Returns based on equity prices/indices are most often found to have time dependent conditional variance and hence ARCH/GARCH models are used to
take care of the volatility observed in the time series of returns. Some of the tests for (linear) autocorrelation mentioned earlier perform poorly in presence of conditional heteroscedasticity in the returns. In fact, Diebold (1986), Lo and MacKinlay (1988), Silvapulle and Evans (1993), and others have noted that in the presence of ARCH, the serial correlation tests, if not corrected, can result in misleading inferences.

Further, Lo and MacKinlay (1988), have suggested a simple volatility-based specification test, called variance ratio test, to test random walk hypothesis. The variance ratios as test statistics are intuitively appealing and are known to have optimal properties under certain circumstances as shown in Faust (1992). But researchers have now noted that the test's performance critically depends on 'lag truncation point' which is often chosen arbitrarily by most researchers. Very recently Choi (1999) has suggested calculating variance ratio test by using Andrews (1991) optimal data dependent method, so that this shortcoming could be overcome. This modified test is now known in the literature as automatic variance ratio test. A few more recent tests for serial correlation are available (see, for instance, Choi, 1999, Kawakatsu and Morey, 1999; for details); but the optimality of some of these tests are not yet clearly established.

The other reason for stock returns to deviate from random walk model is due to what is known as calendar anomalies/effects. Many authors like French (1980), Rogalski (1984), Keim and Stambaugh (1984), Engle et al. (1987) and Fama and French (1988) have found that returns differ by small yet statistically significant amounts during different periods of time i.e., day-of-the-week, week-of-the-month and month-of-the-year. These effects, if present in returns, indicate that stock prices have a predictable pattern in their movements leading to the conclusion that the stock market is inefficient.

Additionally, there may be predictable component in stock returns in the form of significant time-varying risk factor which obviously goes against EMH. In view of this we incorporate risk component in mean part when the model is considered along with conditional heteroscedasticity.

An important point to be noted at this stage is that in all the studies on efficiency the underlying models are assumed to have correctly specified conditional mean. It is now too well-known that inferences based on models suffering from misspecification due to inappropriate conditional mean could very well be misleading and incorrect. It is worth mentioning that in the context of studies on efficiency in the framework of ARCH/GARCH, Lumsdaine and Ng (1999) (see also, Weiss, 1986; Tong, 1990; Giles et al., 1993) have shown that in general the popular Lagrange Multiplier (Rao's score) test for testing the null of homoscedasticity leads to overrejection of the null hypothesis of conditional homoscedasticity if there is misspecification of conditional mean. It thus becomes important to test for ARCH in the general context of a possibly misspecified conditional mean and then take appropriate steps for guarding against misspecification in the mean function in case the test rejects the null hypothesis of no misspecification of conditional mean. As stated by Lumsdaine and Ng, the misspecification problem referred to here can arise if the functional form and/or conditioning information set is misspecified. For linear dynamic models, notable cases of such misspecifications are omitted shifts in the trend
function, selecting a lag length in an autoregression that is lower than the true order, failure to account for parameter instability, residual autocorrelation and omitted variables. They have also proposed a method based on use of recursive residuals for adjusting the standard ARCH test to allow for possible misspecification of unknown form. In this context it is also relevant to note that incorrectly specified conditional mean might as well lead to misspecification of conditional variance. In fact, GARCH model would be correctly specified if only there is no serial correlation. As a way out for this problem in the context of studying serial correlation, Robinson (1991) and Woolridge (1991a, b) have suggested ways of robustifying tests for serial correlation to allow for possible misspecification of conditional variance.

Finally, we would like to point out that the BDS test for testing nonlinear dependence in stock returns might reject the null hypothesis of i.i.d. returns for a variety of reasons like (i) predictibility in the conditional mean --- linear or otherwise, (ii) existence of GARCH as conditional variance and (iii) existence of dynamics in higher order moments. When the i.i.d. null is found to be rejected, the researchers usually stop without much explanations as regards the possible reasons for the rejection, and hence the probable causes of observed inefficiency in the stock market remain unexplained. In case the framework of the efficiency study is so made that it incorporates modelling aspects like proper specification of both the conditional mean and the conditional variance, then rejection of null i.i.d. may be attributed to the existence of some dynamics in higher order moments only.

The focus of this paper is to advocate a systematic approach towards studying stock market efficiency with due emphasis on modelling issues as discussed above. In other words, what we mean is that along with nonlinear dependence the approach focusses on appropriate specification of the conditional first and second-ordered moments so that the final inferences are free from any possible consequences of misspecification. Obviously, such an approach which leads to a proper specification of the model on returns also enables one to conclude on the sources of inefficiency which include predictibility in the mean --- linear or otherwise, GARCH and dynamics in higher order moments. We use a battery of tests mentioned earlier to carry out the steps involved. In this paper we apply this procedure to study the efficiency of the Indian stock market. It is worthwhile to note that India is an interesting and important case study because in the wake of liberalisation of the Indian economy since the early 1990's some fundamental changes have taken place at the Indian capital market. These include the formation of the Securities and Exchange Board of India (SEBI) as the regulatory authority of the Indian capital market, the birth of the National Stock Exchange (NSE) as a competitor of the Bombay Stock Exchange (BSE), introduction of screen based trading at all the major exchanges, and dematerialization of shares. It may also be relevant to mention that there have been very few serious studies on the behaviour of Indian equity prices/indices; notables among these are Basu and Morey (1998), Bhowmick (1997) and Thomas (1995). The paper has been organized as follows. Section 2 deals with the modelling approach. The data sets are described in Section 3. Empirical results are discussed in Section 4. The paper concludes with some observations in Section 5.
2. MODELLING APPROACH

In this section we describe the approach to be followed along with the tests to be used in our study. Assuming $p_t$ to be the logarithm of stock price index, $P_t$, return $r_t$ is defined as $r_t = p_t - p_{t-1}$. Fama's efficient market hypothesis (EMH) is very often tested by assuming $p_t$'s to follow a random walk model under the null hypothesis. It is well-known that the simplest version of the random walk model is the independently and identically distributed (i.i.d.) increments in which the dynamics of $\{p_t\}$ are given by the following equation:

$$p_t = \mu + p_{t-1} + \varepsilon_t, \quad t = 1, 2, ..., n, \text{ and } \varepsilon_t \sim \text{IID}(0, \sigma^2)$$

where $\mu$ is the expected price change or drift. Independence of the increments $\{\varepsilon_t\}$ implies not only that increments are linearly uncorrelated, but that any nonlinear functions of the increments are also uncorrelated. The relevant null and alternative hypotheses are thus stated as $H_0: r_t$ is i.i.d. and $H_1: r_t$ is not i.i.d. It may be noted that both linear and nonlinear dependences are allowed under the alternative.

Now, we first test the stationarity of $r_t$ by applying the augmented Dickey-Fuller (ADF) test (1985) and Phillips-Perron (PP) test (1988). Once the stationarity of $r_t$ is established, we carry out tests for serial correlation in the $r_t$. To this end, we first use the automatic variance ratio test by Choi (1999) as described below.

**Automatic Variance Ratio Test:** Lo and MacKinlay (1988) suggested a procedure for testing EMH which was quite different from the usual test of serial correlation. The novelty of their test, known as variance ratio test, is that it is robust to many forms of conditional heteroscedasticity. Though variance ratio test is intuitively appealing and known to have optimal properties under certain conditions (cf. Faust (1992)), the main limitation of this is regarding the choice of lag truncation point which in the absence of any objective criterion, has been chosen arbitrarily by researchers. In view of this shortcoming Choi (1999) suggested that the variance ratio test be calculated by using Andrews (1991) optimal data dependent method. This is what is known as automatic variance ratio test.

For testing the random walk hypothesis, the usual variance ratio estimator of Lo-MacKinlay defined by $VR(l) = \frac{\text{Var}(p_t - p_{t-1})}{l \text{Var}(p_t - p_{t-1})}$ equals one at all possible lag truncation points under $H_0$ of no serial correlation. Hence, Lo and MacKinlay suggested comparing a consistent estimate of $VR(l)$ with one to test for EMH. It is, however, clear that the test crucially depends on the arbitrary choice of the lag truncation points. To take care of this limitation Choi suggested using the quadratic spectral Kernel originally due to Andrews (1991). This is optimal in estimating the spectral density at the zero frequency and hence the lag truncation point is also chosen optimally. Thus, the automatic variance ratio estimator is defined as

$$\hat{VR}(l) = 1 + 2 \sum_{i=1}^{n-1} K(i/l) \hat{\rho}(i)$$

(2.1)
where

\[ \hat{\rho}(i) = \frac{\sum_{i=1}^{n-i} r_i r_{i+i}}{\sum_{i=1}^{n} r_i^2} \]

and

\[ K(x) = \frac{25}{12\pi^2 x^2} \left[ \frac{\sin(6\pi x/5)}{6\pi x/5} - \cos(6\pi x/5) \right]. \]

After proper standardization of the variance ratio estimator \( V\hat{R}(l) \), the standardized statistic becomes \( VR = \sqrt{n/l} \left[ V\hat{R}(l) - 1 \right] / \sqrt{2} \), which is called the automatic variance ratio test statistic. Under the null of no serial correlation, the asymptotic distribution of the statistic has been found to be standard normal (cf. Priestley, 1980; Choi 1999). As this test is a two-sided test, the critical values are taken from both tails of the standard normal distribution.

After we have carried out the automatic variance ratio test, we consider specification of the conditional mean of returns as well as test(s) for detecting possible sources of misspecification of conditional mean; the outcome of the tests would enable us to take steps to ensure that the conditional mean is properly specified. As already discussed in the previous section, returns \( r_t \) might deviate from i.i.d. assumptions for reasons of existence of serial correlations, seasonal effects, time-varying risk factor, conditional heteroscedasticity and other nonlinear dependences. It is also understandable that any possible misspecification of the conditional mean might include, \textit{inter alia}, exclusion of contemporaneous variables. Thus, consideration of proper specification entails that we include such contemporaneous independent variables in the specification of conditional mean of \( r_t \). Researchers like Ang and Bekaert (2001) have found nominal interest rate as the most "popular" predictor of stock returns. Fama and Schwert (1977), Campbell (1987), Lee (1992) and Shiller and Beltratti (1992) have also observed that predictability of excess stock returns could be explained by nominal interest rate.

Taking all these into consideration, we propose the specification of the conditional mean of \( r_t \) to be as follows:

\[ r_t = \sum_{k=1}^{\tilde{m}} \varsigma_k r_{t-k} + \sum_{j=1}^{d} \beta_j D_{jt} + \omega i_t + \epsilon_t, \ t = 1,2, \ldots, n, \]

\[ E(\epsilon_t | \psi_{t-1}) \sim N(0, h_t), \]

where \( h_t \) represents conditional variance at time \( t \), \( D_j \)'s \( (j = 1,2, \ldots, d) \) denote the seasonal 0-1 dummies, \( i_t \) is the nominal interest rate, \( \psi_{t-1} = \{ r_{t-1}, r_{t-2}, \ldots \} \) stands for the information set at time \( t-1 \), and \( \tilde{m} \) is the appropriate lag value of \( r_t \) capturing its autocorrelations and it is determined by Hall's (1994) procedure. (2.2) may be conveniently written in vector notation as

\[ r_t = Z_t^* \gamma + \epsilon_t, \]

(2.3)
where $Z_t' = (r_{t-1},...,r_{t-m}, D_{it},...,D_{di}, i_t)$ and $\gamma' = (\xi_1,...,\xi_m, \beta_1,...,\beta_d, \omega)$.

Now, we test if this conditional mean is correctly specified. Given the specification in (2.2), the most important source of misspecification could be parameter instability. We suggest using Hansen's test (1992) for testing for parameter stability in the model. This test has the advantage of allowing for a somewhat more general specification than the usual linear models. To put it differently, Hansen's test is "robust" enough to allow for possible misspecification of higher order moments.

Based on findings of Hansen's test, the entire period covered by the time series is divided into sub-periods by applying Chow (1960) test. We then again use Hansen test for each sub-period so that parameter stability in each sub-period is confirmed. The approach envisaged by us now proposes to test for any remaining misspecification in the conditional mean for each of these sub-periods. This is done by carrying out a test procedure based on recursive residuals. Such a procedure has been used by many researchers including Lumsdaine and Ng (1999). It may be noted at this stage that any remaining misspecification of the conditional mean may be nonlinear in nature. Our motivation is that any unobserved nonlinearity will be manifested in the recursive residuals, and this nonlinearity may be approximated by functions of the recursive residuals as defined in Brown et al. (1975). Kianifard and Swallow (1996) and others have also demonstrated that among many standard tests for model misspecification, use of recursive residuals (rather than standard OLS residuals) increases the power of such tests.

The application of this procedure calls for a two-step estimation procedure. Starting from $m^* + 1$ observations where $m^* = \hat{m} + 1 + l$ ($< n$), recursive estimation of $r_t$ on the regressors specified in the right-hand side of (2.2) over the remaining $n - m^*$ observations are carried out in the first step. This leads to a set of recursive estimate of parameters, $\hat{\gamma}_t$, based on $t$ observations and a set of recursive residuals $\hat{w}_t$ defined as $\hat{w}_t = r_t - Z_t' \hat{\gamma}_{t-1}$. These recursive residuals contain information for updating $\hat{\gamma}_t$ from $\hat{\gamma}_{t-1}$, and cannot be predicted by the regression model given information at time $t-1$. As noted by Lumsdaine and Ng, the recursive residuals are appealing not just because they are easy to compute, but because $\hat{w}_{t-1}$ is in the econometrician's information set at time $t$. This is the reason behind using $\hat{w}_{t-1}$ in (2.4) at time $t$ rather than $\hat{w}_t$. Obviously, the use of OLS residuals is invalid for the same reason. By construction, these are serially uncorrelated if the model is correctly specified. If, however, the model is misspecified, $\hat{w}_t$ would then contain information about the true conditional mean not captured by the regression function. In the second step we estimate

$$r_t = Z_t' \gamma + f(\hat{w}_{t-1}) + \varepsilon_t$$

(2.4)

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1 We use the same notation $\varepsilon_t$ for the disturbance term in (2.4) with the understanding that $\varepsilon_t$ represents the error associated with the correctly specified mean function.
where \( f(\hat{\omega}_{t-1}) \) is a function (likely to be nonlinear) of the recursive residuals \( \hat{\omega}_{t-1} \). In practice, we often try out \( f(\hat{\omega}_{t-1}) = \tau_1 \hat{\omega}_{t-1}, \tau_2 \hat{\omega}_{t-1}^2, \tau_3 \sum_{i=1}^{t-1} \hat{\omega}_i \), and \( \tau_1 \hat{\omega}_{t-1} + \tau_2 \hat{\omega}_{t-1}^2 + \tau_3 \sum_{i=1}^{t-1} \hat{\omega}_i \). If one or more of the \( \tau \)-coefficients turn out to be statistically significant, we retain the corresponding terms in conditional mean specification of \( r_t \) so that the specification does not suffer from any inappropriateness.

Following the approach discussed so far, we now to specify the conditional mean of \( r_t \) in (2.2) more appropriately i.e.,

\[
 r_t = \sum_{k=1}^{\bar{m}} \zeta_k r_{t-k} + \sum_{j=1}^{d} \beta_j D_{jt} + \omega i_t + \phi \ h_t^\lambda + \epsilon_t, \tag{2.5}
\]

where \( h_t \) is the conditional heteroscedasticity at time point \( t \) representing risk, and \( \lambda \) is a transformation parameter. Although risk may have a more general representation like Box-Cox transformation as suggested by Das and Sarkar (2000) for ARCH-M model, we consider, keeping in mind its limited role in this study, only three functional forms of risk viz., \( h_t \), \( \sqrt{h_t} \) and ln \( h_t \). Now, insofar as proper specification of \( h_t \) is concerned,

\[
 h_t = \alpha_0 + \sum_{j=1}^{d} \beta_j D_{jt} + \alpha_1 e_{t-1}^2 + \ldots + \alpha_q e_{t-q}^2 + \delta_1 h_{t-1} + \ldots + \delta_p h_{t-p} \tag{2.6}
\]

where \( D_j \)'s, \( j=1,2,\ldots,d \), are seasonal dummies on volatility, \( \alpha_0 > 0, \alpha_i \geq 0 \) for all \( i=1,\ldots,q \) and \( \delta_j \geq 0 \forall j = 1,\ldots,p \). As noted by Nelson and Cao (1992), this is a sufficient condition for \( h_t \) to be positive; weaker sufficient conditions also exist. The reason behind including seasonal dummies for the specification of \( h_t \) is the fact that we are here concerned with proper specification of both the first and second order conditional moments. Seasonal dummies have been included in variance in Hsieh (1989). It is conceivable that if returns exhibit seasonal patterns, then so are likely to be the case with observed volatility although the pattern in the later may not be the same.

The issue of appropriate specification of the conditional variance therefore reduces to proper choice of the values of \( p \) and \( q \) of the underlying GARCH \((p,q)\) process as specified in (2.6). This is done by carrying out standard diagnostics on the residuals. It may be noted in this context that usually tests about higher order moments of residuals implicitly assume correct specifications of the lower moments. Since our proposed method of analysis tries to ensure that the conditional mean is properly specified, the routine diagnostic tests should yield appropriate results.
We now suggest detecting nonlinear dependences in the data by using what is known in the literature as BDS test (Brock et al. (1996)). Since it is known that financial data often possess time varying volatilities characterized by GARCH and its variants, BDS would be an appropriate test for testing the null of no serial correlation in \( r_t \) against the alternative of serial correlation. In fact, under the set-up of BDS test, the null hypothesis is specified as \{ \{ r_t \} being i.i.d. and the alternative includes, in addition to serial correlation, higher-order dependences specified by GARCH as well as other unspecified nonlinear forms. The BDS test statistic measures the statistical significance of the correlation dimension calculations. The correlation integral is a measure of the frequency with which temporal patterns are repeated in the data.

The BDS test statistic is defined as:

\[
\sqrt{n-m+1} \frac{T_m(\xi)}{V_m(\xi)}
\]

(2.7)

where \( n \) is the total number of observations, \( T_m(\xi) = C_m(\xi)-C_1(\xi) \), \( C_m(\xi) \) and \( C_1(\xi) \) are the correlation integrals as defined in Brock et al., \( V_m(\xi) \) is the standard error of \( T_m(\xi) \) (ignoring the constant \( \sqrt{n-m+1} \)) and \( \xi \) and \( m \) are the distance and dimension respectively, as defined below. This test statistic converges in distribution to \( N(0,1) \) under \( H_0 \). BDS test has the advantage that no distributional assumption needs to be made in using it as a test statistic for i.i.d. random variables. Two parameters are, however, to be chosen by the user. These are the values of \( \xi \) (the radius of the hypersphere which determines whether two points are 'close' or not) and \( m \) (the value of the embedding dimension). As suggested by Brock et al., Hsieh (1991) and Sewell et al. (1993), in most of the cases the values of \( \xi \) used are 0.5 \( \sigma \) and \( \sigma \), where \( \sigma \) represents the standard deviation of the linearly filtered data, and the value of \( m \) is set in line with the number of observations (e.g., use only \( m \leq 5 \) if \( n \leq 500 \)). Returns are filtered for linear dependence using (2.5). To examine whether the higher order dependence structure can be adequately captured by GARCH, GARCH standardized residuals are then tested for i.i.d. using BDS test statistic. As Brock et al. and Hsieh have pointed out that the asymptotic standard normal distribution of BDS statistic does not apply to GARCH standardized residuals, appropriate critical values (derived from simulation) for the BDS test applied to the standardized residuals of a GARCH (1,1) are taken from Brock et al. (1991) and Brooks and Heravi (1999).

Finally, if the null hypothesis of i.i.d. is not found to be acceptable by the BDS test, we then suspect that it may be due to some dynamics in higher order (greater than second) moments of the residuals. Towards this end, we advocate studying the regressions of higher order residuals, say, \( \hat{\epsilon}_t^3 \) and \( \hat{\epsilon}_t^4 \) on their respective lagged values and test if one or more of the coefficients turn out to be significant. Thus, we would be able to conclude that the observed inefficiency is due, \( \text{inter alia} \), to dependences in moments of residuals of order higher than two. Obviously, in case of such a conclusion we cannot model the returns further by incorporating such dependences in an appropriate manner so that the residuals thus obtained
would turn out to be i.i.d. Anyway, from the standpoint of our study this enables us to attribute dynamics of higher (more than two) order moments being one of the causes of observed inefficiency in returns.

3. DATA

Any stock market index should capture the behaviour of overall equity market, and also reflect the changing expectations of the stock market about future dividends of the country's corporate sector. Measurements of such an index should also represent the returns obtained by "typical" portfolios in the country concerned. Keeping in mind that any single index may not be very representative from all these considerations, and also that stock market indices vary in terms of numbers as well as composition of equities considered in the aggregation, we have considered for our study a number of standard indices representing Indian stock market. Such a choice of several indices would also be useful in assessing the robustness of our findings on the efficiency of the Indian equity market.

We have considered the following daily level data sets for our study: (I) Bombay stock exchange sensitive index (BSESENSEX) at daily level spanning the period January 1986 to December 2000, (ii) Bombay stock exchange national index, currently known as BSE100, at daily level covering the period January 1991 to December 2000, (iii) S and P CNX NIFTY index of National Stock Exchange from November 11, 1994 till December 31, 2000, and (iv) DOLLEX at daily level over the period January 1991 to December 2000.

It may be noted that the BSESENSEX (with base at 1978-79 = 100) has, to a considerable extent, been serving the purpose of quantifying the price movements as also reflecting the sensitivity of the market in an effective manner. It is the oldest index in the country consisting of 30 major chips. The daily closing price has been quoted as the value of the index at daily level. The BSE national has the wide coverage (100 companies) and an up-to-date adjustment mechanism in the index construction. It may be noted that the National Stock Exchange (NSE) which came into being in 1994, is based on a sound settlement and trading system, and now represents about 45 percent of the total market capitalisation. In this study we have considered the prime index of the NSE viz., S and P CNX NIFTY. The fourth index i.e., DOLLEX is the dollar conversion of BSE-200. In the present context of increased willingness shown by foreign investors to participate in trading activities of Indian stock markets and also because of growth in the number of foreign financial institutions in the country, the analysis of such an index (i.e., DOLLEX) should be very useful. All these daily data sets have been collected from the official websites of the Bombay Stock Exchange and the National Stock Exchange. Since different index series started at different points of time, the spans of the data sets are different for the different indices.

Insofar as the data on short term interest rate is concerned, we have used the call money rate at daily level. Since call money rate is market determined and available at the daily level we have chosen a short term interest rate. The data on call money rate have been collected from the Reserve Bank of India since it is available i.e., from January 1991 till December, 2000.
4. Empirical Findings with Daily Index

In this section we report the results of our analysis with daily level data. As already stated, the return at period \( t \) is defined by \( r_t = \ln P_t - \ln P_{t-1}, \) where \( P_t \) is the stock price index at period \( t \). Hence the analysed data represent the continuously compounded rates of return for holding the (aggregate) securities for one day.

The usual statistical descriptions of the data sets viz., the values of standard deviation, skewness and kurtosis as well as the values of Ljung-Box test statistic for testing ARCH in the daily returns of the four series BSE SENSEX, BSE 100, NIFTY and the DOLLEX are given in Table 1. The means and standard deviations of the returns based on the full sample period values of the four indices show that means are not significantly different from zero. All the values of coefficient of skewness show that all the distributions except possibly that of NIFTY are skewed indicating departure from normality of the return distributions from the standpoint of the property of symmetry. Also, all the kurtosis values except that of NIFTY are much larger than 3. This shows that these three return series have fat tails compared to normal distribution. From Table 1 we also find that in all the four series the Ljung-Box test statistic values for the squared residuals are significant indicating the presence of second order dependence in returns.

Figures 1 through 4 also indicate clearly that volatility is present in all the four daily return series. Since return series are generally known to be highly volatile, this finding is consistent with the nature of financial data. We then carried out two standard tests of stationarity of the return series viz., ADF and PP tests, as earlier mentioned. The values of the test statistics for all the four series are presented in Table 2. It is evident that all the four return series are stationary at 1 percent level of significance. Once stationarity of returns has been confirmed, we analysed the returns for the four data sets by following the procedure described in the previous section. Except automatic variance ratio test and Hansen test all the computations have been carried out with TSP 4.3. The computation of automatic variance ratio test has been done with GAUSS and that of Hansen test with SHAZAM 7.0. The seven stages of computations (in order) involved are as follows.

I. Testing for no serial correlation by Choi's automatic variance ratio test;
II. Testing for parameter stability by Hansen's test;
III. Consequent partitioning of the entire time period in sub-periods of stable parameters each by applying Chow's test for parameter stability;
IV. Testing for misspecification of conditional mean based on recursive residuals;
V. Testing for adequacy of specification of GARCH model by standard diagnostic tests based on residuals and squared values of standardised residuals;

VI. Testing for nonlinearities by BDS test;

VII. Testing for dynamics of higher order moments in residuals.

The results of automatic variance ratio test of Choi are presented in Table 3. We find from this table that the values of the test statistic are 2.929, 5.158, 1.677 and 6.554 for BSESENSEX, BSE 100, NIFTY and DOLLEX respectively. On comparing these values with the critical values (two-sided) of N (0, 1) distribution we obviously find that the null of no serial correlation is rejected for all the four returns data series. We may thus conclude on the basis of the findings of this test that all the four stock indices are inefficient.

We now report the findings on Hansen's test for parameter instability in the model specified in (2.2). As already discussed, Hansen test involves fitting linear equation to the entire set of observations. Using information-based criteria like AIC and BIC, we determined the value of $m$, the number of lags of $r_t$ appearing as explanatory variables for $r_t$, and taking five 0-1 dummies representing day-of-the-week effects, and taking the daily series of call money rate as the short-term interest rate, the model was estimated, and Hansen test statistic computed. These values for the four series, given in Table 4, clearly show that the computed values are greater than the corresponding critical values for the Hansen instability statistic for each of the four data sets, and hence we conclude that conditional means specified in the four models are misspecified in the sense of parameter(s) being unstable. We give below the estimated models for the returns based on BSESENSEX, BSE 100, NIFTY and DOLLEX.

**BSESENSEX**

$r_t = 0.08 r_{t-1} + 0.053 r_{t-17} + 0.038 r_{t-20} - 0.044 r_{t-22} - 0.034 r_{t-25} + 0.001 D5$

(4.650)** (2.909)** (2.222)* (2.567)* (1.988)* (1.960)*

(4.1)

**BSE100**

$r_t = 0.120 r_{t-1} + 0.062 r_{t-3} + 0.088 r_{t-9} + 0.051 r_{t-10} + 0.02 r_{t-19} + 0.002 D1$

(5.795)** (3.030)** (4.242)** (2.445)* (3.506)* (2.814)**

(4.2)

---

2In the computations with daily returns on BSESENSEX, call money rate variable could not be included in the model since this data set is available from January, 1991 only.
\( r_t = 0.072 \ r_{t-1} - 0.067 \ r_{t-6} - 0.052 \ r_{t-19} - 0.052 \ r_{t-26} - 0.003 \ D_1 - 0.002 \ D_2 - 0.006 \ D_3 \)

\( (2.828)** \ (2.617)** \ (2.056)* \ (2.056)* \ (3.140)** \ (2.519)* \ (6.577)** \)

(4.3)

**DOLLEX** :

\( r_t = 0.153 \ r_{t-1} + 0.055 \ r_{t-3} + 0.054 \ r_{t-5} - 0.046 \ r_{t-6} + 0.099 \ r_{t-9} + 0.049 \ r_{t-17} \)

\( (7.430)** \ (2.656)** \ (2.617)** \ (2.214)* \ (4.790)** \ (2.429)* \)

\(- 0.052 \ r_{t-19} - 0.051 \ r_{t-30} + 0.002 \ D_1 - 0.002 \ D_2 \)

\( (2.517)* \ (2.485)* \ (2.317)* \ (2.283)* \)

(4.4)

[The values in parentheses indicate corresponding absolute values of \( t \)-statistic; *indicates significance at 5% level and ** indicates significance at 1% level of significance.]

If we consider the BSESENSEX given in equation (4.1), for instance, we find that \( r_{t-1}, r_{t-17}, r_{t-20}, r_{t-22} \) and \( r_{t-25} \) are significant; so is \( D_5 \) dummy. The later finding suggests that Friday effect i.e., the weekend effect is highly significant. The value of Hansen test statistic is 3.161, which is highly significant as the critical value of the test statistic is 2.35 for 7 degrees of freedom. Similar conclusions can be drawn for BSE100, NIFTY and DOLLEX series.

But these empirical findings are likely to be erroneous, since these models are misspecified in the sense of instability of parameters by Hansen's test. Suitable partitioning of the time period of each of the four series was done by applying Chow's test. As already discussed, Hansen's test was once again carried out to reconfirm that parameters are indeed stable in each of the sub-periods thus partitioned. The sub-periods for the four series have thus been obtained as follows:


\(^3\) Due to the infamous stock market scam in the early nineties, Indian capital market went through a very turbulent period for about a year. Also due to introduction of various reform processes in financial as well as other sectors of the Indian economy we found no sub-period during June 1990-February 1994 within which parameters were found to be stable. Hence this sub-period has been excluded from our analysis. Similar sub-periods were also obtained for BSE100, NIFTY and DOLLEX.

These findings on partitioning of time periods are consistent with the major developments in the Indian capital market and other sectors of the economy during 1990's. As pointed out by Kulkarni (1997) the Indian market was in the midst of the bear phase since the early 1990's due to political instability and foreign exchange crisis. Moreover, the biggest scam in the history of the Indian capital market was unearthed in April 1992, and as a consequence the government bestowed more power on the autonomous regulator, Securities and Exchange Board of India (SEBI), which, in turn, immediately swung into action. Further, to curb speculation in the BSE, SEBI scrapped the forward trading system (badla) mechanism in December 1993. In this context it may also be pointed out that since 1996 Indian stock market has been experiencing some fundamental changes in the form of reintroduction of badla in January 1996, changes in settlement cycle, introduction of computerized screen based trading at both BSE and NSE and dematerialization of shares.

We now report the results of the recursive residual based test of misspecification of conditional mean. This test has been carried out to find if the conditional mean is still misspecified. After obtaining recursive residual $\hat{w}_t$'s as discussed in Section 2, and then including terms like $\hat{w}_{t-1}$, $\hat{w}_{t-1}^2$, $\hat{w}_{t-1}^3$, $\sum_{i=1}^{t-1} \hat{w}_i$ etc., we obtain the estimated models for each of the sub-periods of all the four data sets. For the sake of brevity of space, we here report results concerning BSESENSEX only.

**Sub-period I (January 1986 - May 1990):**

\[
\begin{align*}
    r_t &= -0.037 r_{t-1} + 0.065 r_{t-2} + 0.090 r_{t-7} + 0.091 r_{t-8} + 0.166 \hat{w}_{t-1} + 1.727 \hat{w}_{t-1}^2 \\
     &\quad - 31.267 \hat{w}_{t-1}^3 - 0.044 \sum_{i=1}^{t-1} \hat{w}_i + \hat{\epsilon}_t \\
     &\quad \text{(4.5)}
\end{align*}
\]

**Sub-period II (March 1994 - December 1995):**

\[
\begin{align*}
    r_t &= 0.255 r_{t-1} - 0.128 r_{t-2} + 0.148 r_{t-27} - 0.09 r_{t-28} + 0.005 D5 - 0.00013 \hat{t}_i \\
     &\quad + 0.192 \hat{w}_{t-1} + 2.536 \hat{w}_{t-1}^2 - 170.635 \hat{w}_{t-1}^3 - 0.001 \sum_{i=1}^{t-1} w_i + \hat{\epsilon}_t \\
     &\quad \text{(4.6)}
\end{align*}
\]
Sub-period III (January 1996 - February 2000) :

\[ r_t = 0.037 r_{t-1} + 0.069 r_{t-3} - 0.091 r_{t-6} + 0.054 r_{t-9} + 0.095 r_{t-18} + 0.073 r_{t-39} \]

\( (0.327) \quad (2.219) \quad (2.900) \quad (1.715) \quad (3.021) \quad (2.316) \)

\[ + 2.013 r_{t-1}^2 - 41.631 r_{t-1}^3 + 0.005 D1 + 0.003 D3 - 0.0001 \hat{t}_t \]

\( (0.762) \quad (0.970) \quad (3.315) \quad (2.520) \quad (2.105) \)

\[ + 0.053 \hat{w}_{t-1} - 0.329 \hat{w}_{t-1}^2 + 16.882 \hat{w}_{t-1}^3 + \hat{\nu}_t \]

\( (0.452) \quad (0.133) \quad (0.332) \)

Sub-period IV (March 2000 - December 2000) :

\[ r_t = 0.459 r_{t-1} - 0.187 r_{t-19} - 0.062 \hat{w}_{t-1} - 0.327 \hat{w}_{t-1}^2 - 161.041 \hat{w}_{t-1}^3 \]

\( (2.314) \quad (2.724) \quad (0.279) \quad (0.171) \quad (3.030) \)

\[ - 0.001 \sum_{i=1}^{t-1} \hat{w}_i + \hat{\nu}_t \]

\( (0.119) \)

[The values in the parentheses represent corresponding absolute values of t-statistics].

It is evident from the above estimated equations that none of the coefficients associated with \( \hat{w}_{t-1}, \hat{w}_{t-1}^2, \hat{w}_{t-1}^3, \sum_{i=1}^{t-1} \hat{w}_i \) is significant even at 5 percent level of significance in any of the four sub-periods except the coefficient of \( \hat{w}_{t-1}^3 \) in Sub-period IV.

Now that the conditional mean of returns has been properly specified, we estimate this model along with the GARCH assumption for the conditional heteroscedasticity \( h_t \) as specified in (2.6). It may be recalled that for reasons stated in the previous section, the specification of \( h_t \) involves dummies representing the day-of-the-week effects. The estimated models for \( r_t \) and \( h_t \) for the four sets of return data are given below for respective sub-periods.

**BSESENSEX**

Sub-period I :

\[ r_t = 0.099 r_{t-1} - 0.078 r_{t-2} + 0.076 r_{t-7} + 0.062 r_{t-8} + \hat{\nu}_t \]

\( (2.806) \quad (2.266) \quad (2.198) \quad (1.763) \)

\[ \hat{h}_t = 0.000004 + 0.044 \hat{\nu}_{t-1}^2 + 0.888 \hat{h}_{t-1} + 0.00008 D5 \]

\( (0.489) \quad (3.234) \quad (23.586) \quad (2.797) \)
Sub-period II:

\[ r_t = 0.347 r_{t-1} - 0.154 r_{t-2} + 0.139 r_{t-27} - 0.096 r_{t-28} + 0.005 D5 - 0.0001 i_t + \hat{\epsilon}_t \]
\[ \hat{h}_t = 0.000025 + 0.194 \hat{\epsilon}_{t-1}^2 + 0.592 \hat{h}_{t-1} \]

Sub-period III:

\[ r_t = 0.073 r_{t-3} - 0.105 r_{t-6} - 0.089 r_{t-18} + 0.004 D1 + 0.003 D3 - 0.001 i_t + \hat{\epsilon}_t \]
\[ \hat{h}_t = 0.0002 + 0.092 \hat{\epsilon}_{t-1}^2 + 0.099 \hat{h}_{t-1} + 0.0003 D1 + 0.0001 D3 \]

Sub-period IV:

\[ r_t = 0.166 r_{t-1} - 0.134 r_{t-19} + \hat{\epsilon}_t \]
\[ \hat{h}_t = 0.00003 + 0.138 \hat{\epsilon}_{t-1}^2 + 0.779 \hat{h}_{t-1} \]

BSE100

Sub-period I:

\[ r_t = 0.493 r_{t-1} - 0.174 r_{t-2} - 0.003 D1 + 0.0002 D5 + \hat{\epsilon}_t \]
\[ \hat{h}_t = 0.143 \hat{\epsilon}_{t-1}^2 + 0.759 \hat{h}_{t-1} + 0.00035 D4 \]

Sub-period II:

\[ r_t = 0.185 r_{t-1} - 0.094 r_{t-6} + 0.059 r_{t-8} - 0.053 r_{t-11} - 0.074 r_{t-18} + 0.006 D1 + 0.003 D3 - 0.0001 i_t - 41.719 r_{t-3}^3 + \hat{\epsilon}_t \]
\[ \hat{h}_t = 0.016 \hat{\epsilon}_{t-1}^2 + 0.163 \hat{h}_{t-1} + 0.435 \hat{h}_{t-2} + 0.0003 D1 + 0.00006 D4 \]
Sub-period III :

\[ r_t = 0.480 \, r_{t-1} - 0.242 \, r_{t-19} - 220.625 \, \hat{w}_{t-1}^3 + \hat{\epsilon}_t \]  
\[ h_t = 0.0001 + 0.279 \, \hat{\epsilon}_{t-1}^2 + 0.540 \, h_{t-1} \]

\[ (5.314) \quad (4.757) \quad (4.497) \]

\[ (2.576) \quad (2.617) \quad (4.422) \]

**NIFTY**

Sub-period I :

\[ r_t = 0.412 \, r_{t-1} - 0.147 \, r_{t-2} - 0.005 \, D1 + \hat{\epsilon}_t \]  
\[ h_t = 0.0001 + 0.130 \, \hat{\epsilon}_{t-1}^2 + 0.797 \, h_{t-1} \]

\[ (5.964) \quad (2.291) \quad (3.559) \]

\[ (1.620) \quad (2.365) \quad (10.122) \]

Sub-period II :

\[ r_t = 0.214 \, r_{t-1} - 0.099 \, r_{t-6} - 115.50 \, r_{t-1}^3 - 0.003 \, D2 + 0.008 \, D3 + \hat{\epsilon}_t \]  
\[ h_t = 0.0001 + 0.151 \, \hat{\epsilon}_{t-1}^2 + 0.191 \, h_{t-1} + 0.0001 \, D1 + 0.00005 \, D3 + 0.00005 \, D4 \]

\[ (4.757) \quad (3.554) \quad (4.054) \quad (2.302) \quad (6.491) \]

\[ (5.447) \quad (5.371) \quad (1.816) \quad (4.053) \quad (1.719) \quad (1.755) \]

Sub-period III :

\[ r_t = 0.170 \, r_{t-17} - 0.155 \, r_{t-19} - 0.157 \, r_{t-26} + 0.006 \, D3 + \hat{\epsilon}_t \]  
\[ h_t = 0.000009 + 0.191 \, \hat{\epsilon}_{t-1}^2 + 0.688 \, h_{t-1} + 0.0002 \, D1 \]

\[ (3.277) \quad (2.895) \quad (3.122) \quad (2.884) \]

\[ (0.314) \quad (2.409) \quad (5.917) \quad (2.311) \]

**DOLLEX**

Sub-period I :

\[ r_t = 0.201 \, r_{t-1} - 0.152 \, r_{t-14} + \hat{\epsilon}_t \]

\[ h_t = 0.096 \, \hat{\epsilon}_{t-1}^2 + 0.394 \, h_{t-1} + 0.0003 \, D1 + 0.0002 \, D2 + 0.0002 \, D4 + 0.0005 \, D5 \]

\[ (2.355) \quad (2.382) \]

\[ (2.558) \quad (4.160) \quad (3.018) \quad (4.803) \quad (3.225) \quad (2.235) \]
Sub-period II:

\[ r_t = 0.441 r_{t-1} - 0.108 r_{t-2} - 0.004 D1 - 0.214 D2 + \hat{\epsilon}_t \]
\[ (8.264) \quad (2.021) \quad (3.855) \quad (1.979) \]

\[ \hat{h}_t = 0.046 \hat{\epsilon}_{t-1}^2 + 0.856 \hat{h}_{t-1} + 0.00004 D4 \]
\[ (1.891) \quad (21.569) \quad (3.254) \]

Sub-period III:

\[ r_t = 0.173 r_{t-1} - 0.107 r_{t-6} + 0.066 r_{t-8} - 0.079 r_{t-18} - 47.766 r_{t-1}^3 \]
\[ (4.042) \quad (4.080) \quad (2.163) \quad (2.799) \quad (2.087) \]

\[ + 0.005 D1 + 0.002 D3 - 0.002 D5 + \hat{\epsilon}_t \]
\[ (3.781) \quad (2.269) \quad (2.352) \]

\[ \hat{h}_t = 0.186 \hat{\epsilon}_{t-1}^2 + 0.414 \hat{h}_{t-1} + 0.0003 D1 + 0.0001 D3 + 0.0001 D4 + 0.00004 D5 \]
\[ (5.717) \quad (8.803) \quad (12.913) \quad (4.729) \quad (4.742) \quad (2.392) \]

Sub-period IV:

\[ r_t = 0.217 r_{t-1} + 0.187 r_{t-9} - 0.238 r_{t-19} + \hat{\epsilon}_t \]
\[ (2.318) \quad (2.311) \quad (2.786) \]

\[ \hat{h}_t = 0.00005 + 0.128 \hat{\epsilon}_{t-1}^2 + 0.780 \hat{h}_{t-1} \]
\[ (1.213) \quad (1.415) \quad (5.580) \]

Sub-period V:

\[ r_t = 0.573 r_{t-1} - 450.71 r_{t-1}^2 + \hat{\epsilon}_t \]
\[ (3.116) \quad (2.508) \]

\[ \hat{h}_t = 0.00007 + 0.263 \hat{\epsilon}_{t-1}^2 + 0.542 \hat{h}_{t-1} \]
\[ (1.072) \quad (1.358) \quad (1.919) \]

[The values in parentheses indicate corresponding absolute values of t-statistics.]

For each of these estimated models, the adequacy of the estimated GARCH model was examined so that there remained no misspecification in the specification of the second order conditional moment. This was done by studying the behaviour of estimated residuals, standardized residuals and squared standardized residuals. The Ljung-Box Q-statistic values pertaining to these residuals are given in Table 5. It is evident from these computations that none of these test statistic values is significant for all the four data sets and for all the sub-periods indicating thus the adequacy of the estimated GARCH specifications. In most of the cases
we have found GARCH (1, 1) to be the most appropriate specification along with some day-of-the-week effects. If we look at the estimated relations and volatility we note that

(i) Call money rate is found to be not significant in most of the cases. In particular for NIFTY and DOLLEX, call money rate is nowhere significant, but for BSESENSEX it is significant for Sub-periods II and III and for BSE 100 it is significant only for Sub-period II. Since in Indian capital market BSESENSEX and BSE100 are the most representative indices of the economy, these findings on the significance of call money rate pertaining, in particular, to the second half of the last decade suggest that the present policy of the government of India of reducing interest rate to encourage investment in the stock market is likely to be effective in due course.

(ii) Day-of-the-week effects are present in most of the cases both in mean as well as in volatility. These findings indicate $r_i$'s are often predictable from consideration of daily pattern. However, there is no uniform day-of-the-week effect across all sub-periods of any index. For instance, in BSESENSEX Friday is significant for Sub-period II, and Monday and Wednesday are significant for Sub-period III. Again for BSE 100 while Monday and Friday are significant for Sub-period I, Monday and Wednesday are significant for Sub-period II. Similarly for NIFTY Monday is significant for Sub-period I, and Tuesday and Wednesday are significant for Sub-period II. For DOLLEX Monday and Tuesday are significant for Sub-period II, but Monday, Wednesday and Friday are significant for Sub-period III.

(iii) Some lagged values of $r_i$ are significant in each of these estimated models. While computing we allowed for a large number of lags to be incorporated so that in some cases as far off a lagged value as 27 was found to be significant. In others, for instance in Sub-period I of BSESENSEX, $7^{th}$ and $8^{th}$ lags of $r_i$ were found to be significant.

(iv) The daily level seasonality was found to be present in volatility in most of the sub-periods of all the four data sets indicating thus the presence of day-of-week effects in volatility as well. It may further be noted that in most of the cases the day effects in $h_t$ were not the same as in $r_i$, the exception being Sub-period III of BSESENSEX.

(v) It may also be observed that in one case viz., Sub-period III of BSE 100, some polynomial of recursive residuals (i.e., $\hat{\nu}_{t-1}^3$) was found to be significant in the mean part of the model. This means that there are some nonlinearities yet to be captured in the actual model.

(vi) In none of the sub-periods, risk factor has been found to be significant indicating that time-varying risk premia has no role in explaining inefficiency in the Indian capital market.
By following the procedure proposed in Section 2, we have now reached the step where we are required to detect nonlinear dependences in the residuals. This detection is indeed important since there are growing evidences that stock prices often show complex regularities. We have, therefore, applied BDS test for detecting i.i.d. distributed system from nonlinear dependences. The results of this exercise have been presented in Table 6.

The first column of the Table 6 gives the values of the distance, $\xi$, measured in terms of half (0.5) and one (1) times the standard deviation of linearly filtered data used in the study; the values of the number of embedding dimensions $m$ are given in column 2. The values of BDS test statistics for all the sub-periods are shown in rest of the columns. The series are examined up to ten dimensions when the number of observations exceed 500. For the observations less than or equal to 500 we use embedding dimensions upto 5. It is known that acceptance of the null hypothesis of i.i.d. observations indicates that the observations are purely random meaning thereby that there are no nonlinear dependences of any kind. In that event we could conclude that the behaviour of the series have no exploitative value. As for computations, we have used the standardized residuals $\hat{\varepsilon}_t / \sqrt{h_t}$, obtained in equation (2.6). Obviously, these are filtered off all linear dependences of $r_t$. Also, the appropriate critical values for BDS test have been obtained from Brock et al. (1991). As regards the findings, we have found that the BDS test statistic values for all the four stock indices are significant as compared to the nonstandard critical values at standard levels of significance for almost all the values of $\xi / \sigma$ and $m$ considered for this study. These computations indicate, as displayed in Table 6, that in most cases, particularly for higher values of $m$ ($m > 5$), the null of i.i.d. observations is rejected. Thus we can conclude on the basis of our analysis that GARCH has been found to be inadequate in capturing all the nonlinear dependences in the series.

Finally, we have carried out the last step of our proposed method of analysis. What we attempt at here is to find out whether there is dynamics in the higher order moments (say, third or fourth) so that the remaining dependences could be explained. Towards this end, we have considered dynamic relations involving $\hat{\varepsilon}_t^3$ and $\hat{\varepsilon}_t^4$, and found the following estimated relations for the different indices.

**BSESENSEX**

**Sub-period I :**

\[
\hat{\varepsilon}_t^3 = 0.0000013 - 0.077 \hat{\varepsilon}_{t-1}^3 \\
(1.199) \quad (2.333)^* 
\]
Sub-period II:
\[
\hat{e}_t^3 = -0.115 \hat{e}_{t-2}^3
\]
(3.072)
\[
\hat{e}_t^4 = 0.000000046 + 0.071 \hat{e}_{t-2}^4
\]
(4.064) (2.191)

Sub-period III:
\[
\hat{e}_t^3 = 0.00000154 - 0.058 \hat{e}_{t-2}^3
\]
(0.969) (1.853)

Sub-period IV:
\[
\hat{e}_t^3 = -0.0000059 - 0.280 \hat{e}_{t-3}^3
\]
(1.684) (3.971)
\[
\hat{e}_t^4 = 0.00000053 + 0.190 \hat{e}_{t-4}^4
\]
(2.138) (3.527)

BSE100

Sub-period I:
\[
\hat{e}_t^3 = 0.00000016 - 0.236 \hat{e}_{t-2}^3 + 0.124 \hat{e}_{t-3}^3
\]
(0.867) (5.279) (2.785)
\[
\hat{e}_t^4 = 0.00000016 + 0.176 \hat{e}_{t-2}^4 + 0.129 \hat{e}_{t-3}^4
\]
(3.114) (3.900) (2.866)

Sub-period II:
\[
\hat{e}_t^3 = -0.115 \hat{e}_{t-2}^3
\]
(3.072)

Sub-period III:
\[
\hat{e}_t^3 = 0.192 \hat{e}_{t-1}^3
\]
(3.026)
\[
\hat{e}_t^4 = 0.0000053 + 0.190 \hat{e}_{t-1}^4 + 0.121 \hat{e}_{t-3}^4 + 0.223 \hat{e}_{t-4}^4
\]
(2.138) (3.079) (1.935) (3.527)
NIFTY

Sub-period I :

\[ \hat{\varepsilon}_t^3 = -0.00000026 - 0.101 \hat{\varepsilon}_{t-1}^3 - 0.197 \hat{\varepsilon}_{t-2}^3 + 0.128 \hat{\varepsilon}_{t-3}^3 \]
\[
\begin{array}{cccc}
(0.512) & (1.616) & (3.205) & (2.061)
\end{array}
\]

\[ \hat{\varepsilon}_t^4 = 0.00000046 + 0.159 \hat{\varepsilon}_{t-3}^4 + 0.173 \hat{\varepsilon}_{t-3}^3 \]
\[
\begin{array}{cccc}
(2.416) & (2.620) & (2.841)
\end{array}
\]

Sub-period II :

\[ \hat{\varepsilon}_t^3 = 0.117 \hat{\varepsilon}_{t-1}^3 - 0.088 \hat{\varepsilon}_{t-2}^3 \]
\[
\begin{array}{cc}
(3.667) & (2.758)
\end{array}
\]

\[ \hat{\varepsilon}_t^4 = 0.00000042 + 0.090 \hat{\varepsilon}_{t-1}^4 \]
\[
\begin{array}{cc}
(4.164) & (2.832)
\end{array}
\]

Sub-period III :

\[ \hat{\varepsilon}_t^3 = -0.0000045 - 0.274 \hat{\varepsilon}_{t-3}^3 - 0.151 \hat{\varepsilon}_{t-5}^3 \]
\[
\begin{array}{ccc}
(2.122) & (4.513) & (2.872)
\end{array}
\]

\[ \hat{\varepsilon}_t^4 = 0.0000024 + 0.295 \hat{\varepsilon}_{t-3}^4 + 0.144 \hat{\varepsilon}_{t-5}^4 \]
\[
\begin{array}{ccc}
(1.978) & (4.883) & (2.899)
\end{array}
\]

DOLLEX

Sub-period I :

\[ \hat{\varepsilon}_t^3 = 0.421 \hat{\varepsilon}_{t-1}^3 - 0.203 \hat{\varepsilon}_{t-2}^3 \]
\[
\begin{array}{cc}
(6.421) & (3.098)
\end{array}
\]

\[ \hat{\varepsilon}_t^4 = 0.00000061 + 0.536 \hat{\varepsilon}_{t-1}^4 - 0.288 \hat{\varepsilon}_{t-2}^4 + 0.175 \hat{\varepsilon}_{t-3}^4 \]
\[
\begin{array}{ccc}
(1.828) & (7.963) & (3.812)
\end{array}
\]

Sub-period II :

\[ \hat{\varepsilon}_t^3 = 0.092 \hat{\varepsilon}_{t-3}^3 - 0.080 \hat{\varepsilon}_{t-5}^3 \]
\[
\begin{array}{cc}
(1.910) & (2.102)
\end{array}
\]
Sub-period III:

\[ e_i^3 = 0.072 \hat{e}_{i-1}^3 - 0.093 \hat{e}_{i-2}^3 \]
\[ (2.288) \quad (2.950) \]
\[ e_i^4 = 0.00000048 - 0.075 \hat{e}_{i-1}^4 \]
\[ (3.704) \quad (2.400) \]

Sub-period IV:

\[ e_i^3 = -0.0000091 - 0.144 \hat{e}_{i-2}^3 - 0.214 \hat{e}_{i-3}^3 - 0.169 \hat{e}_{i-5}^3 \]
\[ (1.745) \quad (1.475) \quad (2.218) \quad (1.728) \]
\[ e_i^4 = 0.0000004 + 0.223 \hat{e}_{i-3}^4 \]
\[ (3.003) \quad (2.333) \]

[The absolute values of t-statistic are indicated within parentheses].

Thus we find that the residuals have third and/or fourth order dependences on their own lagged values, and this implies that dynamics in higher order moments are significant in explaining inefficiency in the Indian stock market. Although this finding cannot be incorporated for any further improvement in the specification of the model leading possibly to residuals finally becoming i.i.d., what is noteworthy is that insofar as Indian stock returns are concerned, in addition to second order dependence in the returns, higher order (third and/or fourth) dependences are also significant.

5. Concluding Remarks

In this paper we have proposed a systematic approach towards studying stock market efficiency with an aim to identifying causes of any observed market inefficiency after ensuring appropriate specification of first and second order conditional moments. In this approach inefficiency has been defined to include nonlinear dependence in the returns as well. The consideration for appropriate specification is due to the fact that standard tests for conditional heteroscedasticity presume proper specification of the conditional mean, and hence any misspecification of the conditional mean as also of conditional variance could lead to misleading inferences about the model on returns and consequently on the efficiency or otherwise of the stock market.

This modelling approach has been applied for the Indian stock market. Incorporating short-term interest rate through call money rate, risk by conditional heteroscedasticity, 0-1 dummies for the day-of-the week effects in addition to
lagged values of return in the conditional mean, and assuming GARCH specification and 0-1 dummies to represent daily level seasonality in conditional heteroscedasticity, we have applied tests like automatic variance ratio test, Hansen's stability test, Chow's test, test based on recursive residuals and BDS test, and concluded that the inefficiency in the Indian stock market represented by four standard daily indices viz., BSESENSEX, BSE100, NIFTY and DOLLEX, is due to serial correlation, nonlinear dependence, day-of-the week effects, parameter instability, conditional heteroscedasticity (GARCH), daily level seasonality in volatility and call money rate (in some sub-periods of some indices only). The findings thus establish that insofar as Indian stock market is concerned it has passed through several stable and unstable periods during the 1990's (the period of on-going reforms in financial as well as in other sectors of the Indian economy), as observed in terms of parameter instability during this period, and that proper specification of both the conditional mean and variance are relevant and important. Further, we have found that the null of i.i.d. residuals was rejected by the BDS test even in the case of standardized residuals of the properly specified models, leading us to conclude that incorporating second order dependence through GARCH is not adequate to capture all potential nonlinearities in the returns for all indices. We have also observed that the remaining nonlinearities could be attributed to the existence of some dynamics in the higher order moments.
Table 1: Summary Statistics of Returns Based on BSESENSEX, BSE100, NIFTY and DOLLEX

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean</th>
<th>Sd</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>$Q^2$ (12)</th>
<th>$Q^2$ (22)</th>
<th>$Q^2$ (32)</th>
<th>$Q^2$ (42)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSESENSEX</td>
<td>0.000595</td>
<td>0.0196</td>
<td>0.275</td>
<td>6.186</td>
<td>697.754</td>
<td>1040.100</td>
<td>1435.140</td>
<td>1489.743</td>
</tr>
<tr>
<td>BSE100</td>
<td>0.000608</td>
<td>0.0186</td>
<td>0.411</td>
<td>7.704</td>
<td>366.030</td>
<td>600.318</td>
<td>699.443</td>
<td>735.753</td>
</tr>
<tr>
<td>NIFTY</td>
<td>-0.000011</td>
<td>0.0171</td>
<td>0.0642</td>
<td>2.508</td>
<td>87.270</td>
<td>127.645</td>
<td>131.902</td>
<td>136.965</td>
</tr>
<tr>
<td>DOLLEX</td>
<td>0.000113</td>
<td>0.0192</td>
<td>-0.309</td>
<td>14.107</td>
<td>273.829</td>
<td>369.571</td>
<td>508.173</td>
<td>516.445</td>
</tr>
</tbody>
</table>

Note: $Q^2(k)$ represents the values of Ljung-Box statistics of squared returns with k degrees of freedom. Further, all the $Q^2(k)$ values are significant at 1% level of significance.

Table 2: Unit Root Tests of Stock Returns

<table>
<thead>
<tr>
<th>Index</th>
<th>ADF</th>
<th>PP</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSESENSEX</td>
<td>-10.825</td>
<td>-3216.381</td>
<td>3323</td>
</tr>
<tr>
<td>BSE100</td>
<td>-9.851</td>
<td>-2233.127</td>
<td>2285</td>
</tr>
<tr>
<td>NIFTY</td>
<td>-10.559</td>
<td>-1388.236</td>
<td>1506</td>
</tr>
<tr>
<td>DOLLEX</td>
<td>-9.226</td>
<td>-2300.339</td>
<td>2283</td>
</tr>
</tbody>
</table>

Note: All the values of the two test statistics are significant at 1% level of significance. 

n: Number of observations.
Table 3: Results of Automatic Variance Ratio (AVR) Test

<table>
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<tr>
<th>Index</th>
<th>AVR Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSESENSEX</td>
<td>2.929**</td>
</tr>
<tr>
<td>BSE100</td>
<td>5.158**</td>
</tr>
<tr>
<td>NIFTY</td>
<td>1.677*</td>
</tr>
<tr>
<td>DOLLEX</td>
<td>6.554**</td>
</tr>
</tbody>
</table>

*Note: *indicates significant value of the test statistic at 10% level of significance and **indicates the same at 1% level of significance.*

Table 4: Results of Hansen Test

<table>
<thead>
<tr>
<th>Index</th>
<th>Full Period</th>
<th>Sub-period I</th>
<th>Sub-period II</th>
<th>Sub-period III</th>
<th>Sub-period IV</th>
<th>Sub-period V</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSESENSEX</td>
<td>3.161**</td>
<td>1.248</td>
<td>0.642</td>
<td>2.633</td>
<td>1.744**</td>
<td>-</td>
</tr>
<tr>
<td>BSE100</td>
<td>2.561**</td>
<td>1.765</td>
<td>2.329</td>
<td>1.236</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NIFTY</td>
<td>3.673**</td>
<td>0.795</td>
<td>1.138</td>
<td>1.775</td>
<td>-</td>
<td>-</td>
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<tr>
<td>DOLLEX</td>
<td>3.429**</td>
<td>0.227</td>
<td>1.385</td>
<td>2.624*</td>
<td>1.317</td>
<td>0.317</td>
</tr>
</tbody>
</table>

*Note: *indicates significant value of Hansen test statistic at 5% level of significance and **indicates the same at 1% level of significance.*
<table>
<thead>
<tr>
<th>Index</th>
<th>Non-standardized residuals ( \hat{\epsilon}_t )</th>
<th>( Q (12) )</th>
<th>( Q (22) )</th>
<th>( Q (32) )</th>
<th>( Q (42) )</th>
<th>LM-statistic$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSESENSEX</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub-period I</td>
<td>5.37</td>
<td>8.79</td>
<td>20.46</td>
<td>28.20</td>
<td>6.062</td>
<td>*</td>
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<tr>
<td>Sub-period II</td>
<td>5.28</td>
<td>14.55</td>
<td>17.13</td>
<td>27.78</td>
<td>5.653</td>
<td>*</td>
</tr>
<tr>
<td>Sub-period III</td>
<td>18.14</td>
<td>26.40</td>
<td>27.83</td>
<td>50.10</td>
<td>3.513</td>
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</tr>
<tr>
<td>Sub-period IV</td>
<td>9.41</td>
<td>17.93</td>
<td>28.92</td>
<td>37.82</td>
<td>9.291**</td>
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<td>BSE100</td>
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<td></td>
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<tr>
<td>Sub-period I</td>
<td>14.40</td>
<td>29.08</td>
<td>34.56</td>
<td>38.94</td>
<td>8.112**</td>
<td></td>
</tr>
<tr>
<td>Sub-period II</td>
<td>10.16</td>
<td>17.55</td>
<td>27.66</td>
<td>44.40</td>
<td>18.784**</td>
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<tr>
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<td>14.24</td>
<td>21.76</td>
<td>29.71</td>
<td>34.97</td>
<td>23.041**</td>
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</tr>
<tr>
<td>NIFTY</td>
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<td>Sub-period I</td>
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<td>2.043</td>
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<tr>
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<td>17.09</td>
<td>22.66</td>
<td>32.49</td>
<td>19.822**</td>
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<td>12.24</td>
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<tr>
<td>Sub-period I</td>
<td>4.21</td>
<td>10.81</td>
<td>14.28</td>
<td>19.65</td>
<td>20.932**</td>
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<td>18.91</td>
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<td>38.97</td>
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<td>30.66</td>
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<td>24.141**</td>
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<tr>
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<td>26.78</td>
<td>1.844</td>
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<td>25.35</td>
<td>39.53</td>
<td>1.921</td>
<td></td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Index</th>
<th>Standardized residuals ( \hat{\epsilon}_t / \sqrt{\hat{h}_t} )</th>
<th>( Q (12) )</th>
<th>( Q (22) )</th>
<th>( Q (32) )</th>
<th>( Q (42) )</th>
<th>LM-statistic$^3$</th>
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<tr>
<td>BSESENSEX</td>
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<td>17.16</td>
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<td>0.231</td>
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<td>Sub-period I</td>
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<td>34.88</td>
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<td>22.58</td>
<td>31.22</td>
<td>0.011</td>
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<tr>
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<td>43.75</td>
<td>0.767</td>
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<td>DOLLEX</td>
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<td>39.12</td>
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Table 5 (continued)

<table>
<thead>
<tr>
<th>Index</th>
<th>Squared standardized residuals ($\hat{\varepsilon}_t^2 / \hat{h}_t$)</th>
<th>$Q(12)$</th>
<th>$Q(22)$</th>
<th>$Q(32)$</th>
<th>$Q(42)$</th>
<th>LM-statistic$^*$</th>
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</thead>
<tbody>
<tr>
<td>BSESENSEX</td>
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<tr>
<td>Sub-period I</td>
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<td>9.75</td>
<td>10.97</td>
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<tr>
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<tr>
<td>Sub-period I</td>
<td>16.63</td>
<td>26.57</td>
<td>30.82</td>
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<td>0.085</td>
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<tr>
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<td>24.76</td>
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<td>37.50</td>
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<td>31.56</td>
<td>47.50</td>
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<td>Sub-period I</td>
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<td>36.14</td>
<td>43.99</td>
<td>0.100</td>
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<td>25.55</td>
<td>27.50</td>
<td>34.09</td>
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<td>Sub-period III</td>
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<td>32.68</td>
<td>38.43</td>
<td>48.62</td>
<td>0.105</td>
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<td>DOLLEX</td>
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</tr>
<tr>
<td>Sub-period I</td>
<td>16.28</td>
<td>20.35</td>
<td>23.28</td>
<td>25.62</td>
<td>1.417</td>
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<td>21.37</td>
<td>27.89</td>
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<tr>
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<td>24.56</td>
<td>34.76</td>
<td>38.43</td>
<td>0.037</td>
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<td>16.48</td>
<td>27.02</td>
<td>34.37</td>
<td>0.027</td>
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</tr>
</tbody>
</table>

Notes: *indicates significance at 5% level and **indicates significance at 1% level.

$Q(k)$ represents the Ljung-Box statistic value with $k$ degrees of freedom.

$^*$LM-statistic stands for the usual LM test for GARCH model.
### Table 6: GARCH Adjusted BDS Test Statistic Values

<table>
<thead>
<tr>
<th>$\xi / \sigma$</th>
<th>$m$</th>
<th>Sub-period I</th>
<th>Sub-period II</th>
<th>Sub-period III</th>
<th>Sub-period IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2</td>
<td>-0.087*</td>
<td>-6.391</td>
<td>0.753*</td>
<td>-3.480*</td>
</tr>
<tr>
<td>0.5</td>
<td>3</td>
<td>0.557*</td>
<td>-1.727*</td>
<td>8.743</td>
<td>-9.349</td>
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<tr>
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<td>4</td>
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<td>-7.693</td>
<td>-22.746</td>
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<td>-11.235</td>
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<td>-7.814</td>
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<tr>
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<td>-6.019</td>
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<tr>
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<td>9</td>
<td>-3.533*</td>
<td>-4.761*</td>
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Note: The values of BDS test statistic from standardized residuals are compared with the simulated values given in Brock et al. (1991). Values with superscript 'a' indicate non-significance at 5% level. While the values with *indicate significant at 5% level, all others are significant at 1% level of significance. $\xi$, $m$ and $\sigma$ stand for distance, embedding dimension and the standard deviation of the linearly filtered data, respectively.
References


