Comments solicited.

Consumption, Quality of Life and Growth

Dipankar Dasgupta
Economic Research Unit
Indian Statistical Institute
203 B.T. Road, Kolkata 700108
e-mail: ddg@isical.ac.in

and

Sugata Marjit
Centre for Studies in Social Sciences, Calcutta
R 1 Baishnabghata Patuli Township, Kolkata 700094
e-mail: smarjit@hotmail.com

July, 2002

Abstract

The paper investigates the implications of consumption generated improvements in the quality of life of the labour supplying population in a society. A higher quality of life engenders a form of labour augmenting technical progress that affects the rate of growth of the economy. The consumption effect is allowed to be reinforced by a learning by doing type technical progress. Both forms of labour augmentation generate externalities that the market economy fails to internalize, causing thereby a potential divergence between the market rate of growth and the socially optimal rate. It is possible for the latter to fall short of the former as in the case of Schumpeterian growth. A tax-subsidy scheme is shown to attain the social optimum in a decentralised manner.

JEL Classification: D 62, D 90, E 13, E 21, O 30, O 40.

Key Words: Decentralization, Endogenous growth, Externality, Quality of Life, Subsidy, Tax.

Consumption, Quality of Life and Growth

Dipankar Dasgupta & Sugata Marjit[1]

1 Introduction

Growth economics is concerned with the trade off between consumption and saving. Higher growth calls for larger saving, hence smaller consumption. The transformation of saving into capital can be direct (i.e., one to one), as in Solow, or indirect as in all endogenous growth models. But whether direct or indirect, it is more or less universally recognized that an increase in growth rate can be brought about through a sacrifice in present consumption alone. In this paper, we present an alternative possibility, where the realtionship between consumption and growth may be complementary rather than competitive over a specified range of factor inputs.

The manner in which the complementarity arises may be summarized as follows. Growth allows a society to attain higher levels of per capita consumption. However, as the economy expands, the nature of consumption changes gradually from necessities of life, such as food and essential clothing, to commodities that enrich the quality of life as a whole. These commodities are consumption *services* in general, with basic facilities like health care, better schooling for children and so on at one end of the spectrum and entertainment and cultural activities at the other. The upgradation of the consumption basket in this fashion is expected to affect an average individual's attitude towards work. Betterment of lifestyle gives rise to the so called "feel good" factor that returns back to the workhouse in the shape of positively motivated workers. This tends to expand labour supply at the micro level of individual families. But, it is also likely to have an overall macro effect of rise in efficiency, hence labour augmentation, through social intercourse amongst more advantageously provided people. The boost in the

[1]The authors wish to acknowledge comments received from the participants at a workshop organised by the Centre for Studies in Social Sciences, Calcutta. In particular, they are indebted to Maitreesh Ghatak for observations that led to significant improvement in the paper.

effective size of the labour force raises the marginal productivity of capital by the usual neoclassical logic and this in turn has implications for the growth rate of the economy. Thus, while growth helps to attain higher living standards, the latter feeds back on growth also. The element of simultaneity so generated works then to determine the growth rate of the economy as in any standard model of endogenous growth.

In the economic development literature, a somewhat similar idea may be traced to Banerjee & Gupta (1997), Dasgupta & Ray (1986), Ray & Streufert (1993), Ray (1998) and others. A worker's capacity to work depends obviously on the level of nutrition he or she enjoys. Ray (1998) notes "$\cdots$ the relationship that exists between a person's nutritional status and his capacity to do sustained work $\cdots$" and studies "$\cdots$ how this relationship creates a vicious cycle in the labour market: poverty leading to undernutrition, hence inability to work, which feeds back on the incidence of poverty." Ray & Streufert (1993) note further that labour efficiency depends not merely on current consumption, but also its history. Thus, they are able to establish links between nutrition and labour supply over time. The main emphasis in this line of work, however, is on the issue of equilibrium unemployment and efficiency wage. A higher wage rate permits higher nutritional standards and labour supply, whereas below a minimum wage rate, the supply drops to dramatically low levels.

As noted above though, the idea can be pushed forward to produce a consumption led theory of endogenous economic growth. Whether the labour augmenting impact of consumption services is an important and discernible empirical phenomenon is a separate issue altogether. But at the level of pure theory, a macro model of steady state growth cannot afford to ignore the interplay between growth and the standard of living. The objective of economic growth is the attainment of a way of life that goes beyond the satisfaction of primary needs of existence, such as the nutritional value of consumption. However, to the extent that the fulfilment of broader consumption objectives may itself activate the engine of growth, consumption need not be viewed as an end in itself, as the Von Neumann (1937) variety of pure accumulation models would have argued. The difference between the Von Neumann structures and the proposed theory of this paper lies of course in the recognition of technical progress. Even if the rate of accumulation tends to diminish with accumulation, as the pure neoclassical logic dictates, the productivity rise through the different channels of consumption causes to reduce the decline in the rate of growth and may even overpower it sometimes.

In the model below, consumption affects labour productivity multiplicatively, as is the case for the learning by doing models of Arrow (1962), Frankel (1962), Sheshinski (1967) and later on Romer (1986) and d'Autume and Michel (1993). In fact, we allow a hybrid coefficient of labour augmentation, composed of consumption as well as the learning by doing effect. In the standard learning by doing models ushered in by Sheshinski's work, the aggregate *quantity* of the surrgate capital stock is assumed to capture the heterogeneity in capital structure that is normally associated with the growth process as well as the learning by doing form of technical progress. The compromise so introduced is amply compensated by the insight it provides into the manner in which capital accumulation can itself be the vehicle of technical progress. In a similar vein, the present paper treats the *size* of aggregate consumption as the index for quality of life.[2] The abstraction so introduced turns out actually to be an analytical necessity more than a simplification.

The analtytical need to capture the quality of life by means of aggregate consumption rather than the increasing variety of consumption goods is driven by a corresponding requirement to keep the growth generating forces in our framework distinct from the Grossman & Helpman (1991) model of brand proliferation or quality ladder. The point we emphasize is that the social benefits of consumption work not only through the utility function as in Grossman & Helpman, but also indirectly through the work capacity of the labourer. Following up this indirect route is justified by the dividend it yields. In particular, we demonstrate that the consumption effect on labour, with or without a concommitant learning by doing effect, can lead to results that are dramatically different from standard predictions of endogenous growth theory. Unlike most endogenous growth models, except for the famous exercise by Aghion & Howitt (1992), the socially optimum growth rate of the economy in our model *might* fall short of the one achieveable by the Market Economy. The reason for this will lie in the fact that the social planner is aware of the zone over which consumption and growth are comple-

[2]To the extent that present consumption depends on its rate of growth over time, there is a sense in which consumption history enters the production function also as in Ray & Streufert (1993).

mentary from the technological point of view and attains an optimum where the trade-off has turned negative. This leads to the choice of a growth rate that is less than the maximum feasible and, possibly, even the market rate of growth.

The Command Economy equilibrium defines the social optimum for the economy under consideration. Consequently, it is important to ask if it can be decentralized by a tax-subsidy programme. In a typical learning by doing framework, the market pays capital less than its social marginal product and labour enjoys a return that is higher than its true contribution. This calls for wage taxation and capital subsidisation for attaining the social optimum. In our augmented model, there could be a reverse effect with capital earning a higher market return on account of a larger effective labour supply brought forth by improved quality of life. Depending on which effect dominates, decentralization of the social optimum may call for a capital tax *or* subsidy. The paper designs a balanced budget scheme of capital income taxation/subsidy backed by consumption subsidy/ tax that sustains the social optimum in an incentive compatible manner. In the case of capital taxation, our paper presents a contrast to Alesina & Rodrik (1994), who predicted low growth rates for poorer economies (ones characterized by inequality of distribution of capital endowment) on account of a democratic choice of high rates of capital taxes. In our model, capital income taxation might raise the market growth rate if the market equilibrium involves a realtively low rate of growth. To the extent that economies with low market driven growth rates are identifiable as developing economies, our result implies that capital income taxation can be a useful tool for improving the growth rate. Needless to say, this does not disprove the Alesina & Rodrik claim, since their framework of analysis was different from ours. We merely assert that low growth rates are not necessarily an outcome of high capital taxes.

The next section discusses the nature of consumption-cum-quality of life effect in greater detail and points out the possible reasons underlying differences between private and social effects of consumption. This is done by comparing the nature of externalities with the ones associated with learning by doing. The ideas are then used to motivate the aggregate production function employed in the remaining part of the paper. This is followed in Section 3 by a presentation of the basic model of the paper and a comparison of this model with other pre-endogenous growth theory exercises. Section 4 goes on to study the market equilibrium growth rate, Section 5 develops the command rate and Section 6 compares the two. Section 7 constructs a tax subsidy based decentralization procedure for sustaining the command solution as a market equilibrium. The paper concludes with Section 8.

2 Learning by Doing vs. Consumption led Labour Augmentation

In order to motivate the consumption driven productivity idea, let us first begin with Sheshinski's (1967) neoclassical extension of the Arrow (1962) learning by doing model. The aggregate production function for this model has the form

$$Y = F(K, K^{\alpha}\ L),\ \ \alpha > 0.$$

In order to interpret the function, suppose there are N identical firms. The production function of each firm i is assumed to have the form

$$Y^i = F(\frac{K}{N}, K^{\alpha}\ \frac{L}{N}).$$

This means that the aggregate capital stock generates an external effect for each firm. But no single firm can internalize the externality. It can happen as follows.

Imagine *each* firm investing an amount ϵ. In any given firm, the workers learn while using the new machines, become more efficient and cause an effective rise in labour supply. An hour of work by the more efficient labourer counts as more than an hour of inefficient labour supply. The outcome is that the marginal product of capital increases through a rise in effective labour supply. Clearly, a firm is aware of this effect.

The increase in marginal productivity of capital outlined above is negligible however when compared with the increase that comes about due to the social effect of learning. To identify the latter, we take into account the fact that *all* firms have been assumed to have invested and the total investment is $N\epsilon$. Workers across the firms are presumed to interact beyond office hours, say in the local pub, and exchange notes about the new machines acquired by the respective firms. In the process, their familiarity with the machines improves, every worker learning from the experience of every other. Put differently, each worker is enriched by the experience of *all* workers in the society. According to the learning by doing hypothesis, the earlier noted effect caused by a worker in a firm learning in isolation is insignificantly small compared to the total effect . Hence, each firm is assumed to ignore the former. On the other hand, it is unable to internalize the total effect. Put differently, when all firms invest, there occurs a labour augmenting technical progress in each and every firm. The marginal productivity of capital in each firm is increased way beyond its private marginal productivity. Nonetheless, no firm pays capital more than the private marginal product, since it cannot internalize the external effect.

Let us now try to extend the above idea to incorporate consumption externalities. Compare a worker who uses an overcrowded public transport system in a tropical country to commute to work with another who uses an air-conditioned private car. The second worker will be less tired at the workplace and hence perform better; in addition, the quality of his leisure hours will also improve. He might be able to give more time to his family and hence improve the overall lifestyle his family maintains. As a result, he will be better rested than the first worker when he arrives for work on the subsequent day. For all these reasons, an hour of labour supply by the second worker would be effectively more than an hour's labour supply by the second. Like the Arrow-Sheshinski model, the marginal productivity of capital will increase if a firm substitutes the first type of worker by the second. Moreover, to the extent that the increase in marginal productivity of capital is caused by this substitution, it might make sense to award that increase to the worker as an incentive wage, as in the development models quoted earlier.

Imagine now, Arrow-Sheshinski like, that there are N identical firms. Also, assume that each firm substitutes the first type of worker by the second. There will then be a general improvement in the quality (i.e., effective quantity) of labour supply. As before the marginal productivity of capital will rise in each firm. However, the rise in this second case will be substantially higher than in the first case. The reason for this is to be found in the fact that in addition to each worker being able to improve the quality of leisure enjoyed, there will be extensive scope for social interaction. Families can meet and plan for cooperative avenues of enjoyment. Such increased social interactions will improve the quality of workers, not only through the Arrow-Sheshinski route, but also because society as a whole attains a higher standard of life. Generally speaking, the manner in which workers spend their time away from work will affect their performance while at work.

The social effect on marginal productivity is assumed by us to be much higher than the effect on each firm when the consumption-cum-life style of workers in other firms is ignored. One way of seeing this would be to compare the quality of labour supply in a developed country, where workers have exposure to higher consumption standards, with that in less developed countries, where this is not the case. Quite obviously, an extra hour of work by the former group improves the marginal productivity of capital far more than an extra hour by the latter. Thus, a society that subscribes to better environmental norms will have *all* households consuming environmentally friendly products. The aggregate effect will be a more healthy labour force compared to the labour force available to a society that is not concerned with enviromental issues.

Another real life example is the mass scale employment of air conditioners in Singapore. It is believed that this has had a salutory impact on labour quality, given the sultry climate of the region. While it is true that a single employer providing an air conditioner to her employees does internalize the productivilty effect, in aggregate such provisions generate better quality of social interactions and have a larger productivity effect quite external to the firm.

Yet another important example of the phenomenon is provided by De Soto (2001) while discussing the social effects of integrated network systems. As he quotes from The Economist, 1st. July, 1995, pp. 4-5:

One telephone is useless: whom do you call? Two telephones are better, but not much. It is only when most of the population has a telephone that the power of the network reaches its full potential to change society.

The fact that an improved network, be it the telephone system or an advanced internet facility, raises productivity is well recognized. Our contention in this paper is that the precise channels through which productivity is affected is not adequately sorted out. A well integrated labour force has a direct impact on capital productivity in the work place itself. But the social integration that a wide spread telephone network achieves cannot be captured by business sector interactions alone. It brings about a quantum change in the quality of life, and that has a strong influence on the work culture also. How the two strands can be separated out in an empirical estimation of productivity effects and what their relatives sizes may be are themselves interesting questions. The present paper, being a preliminary attempt to study the consumption-growth axis, stops short of answering these questions. However, the theoretical conclusions it reaches are interesting enough in our opinion to merit reporting.

No individual firm can be in a position to internalize the positive effect of a consumption enriched labour *force* on the marginal productivity of capital. For notice that social consumption is an activity carried out by the household sector as a whole. It is impossible for any individual firm to account for such activities in its profit-loss calculations, for the same reason that prevents internalization in the Arrow-Sheshinski world. However, a social planner could well think of rewarding the agents causing the productivity increase appropriately. In the absence of internalization of the externality, it is clear that capital is receiving a higher return than otherwise. The appropriate policy should then be to tax capital and provide a consumption subsidy to the household sector, the latter being the agent that gives rise to the improvement. Our decentralization result achieves this, subject to the caveat that the relative strengths of the learning by doing and consumption externalities will determine whether capital is ultimately taxed or subsidised.

Keeping these observations in mind, we shall assume the following form of the aggregate production of the model:

$$Y = F(K, K^{\alpha}C^{1-\alpha}\bar{L}), \quad 0 \leq \alpha < 1,$$

where K and C stand for aggregate capital and consumption respectively and $\bar{L}$ stands for the fixed size of the primary (i.e., non-enriched) labour force, which itself is identically the same as population. It is not difficult to introduce positive growth in L, but this is avoided to highlight the analytical issue of labour augmentation brought about by K and C. For notational ease, we shall choose $\bar{L} = 1$, thus changing the aggregate production function to

$$Y = F(K, K^{\alpha}C^{1-\alpha}), \quad 0 \leq \alpha < 1. \tag{1}$$

The function reduces to the pure learning by doing case when $\alpha = 1$.[3] Under the usual neoclassical assumptions on F, equation (1) reduces to

$$\begin{aligned} Y &= KF(1, (C/K)^{1-\alpha}), \\ &= Kf(\hat{c}), \end{aligned} \tag{3}$$

where $\hat{c}$ stands for $(C/K)^{1-\alpha}$. The function f is assumed to satisfy the Inada conditions, i.e.,

$$f'(\hat{c}) \to \infty \text{ as } \hat{c} \to 0$$
$$f'(\hat{c}) \to 0 \text{ as } \hat{c} \to \infty.$$

In other words, the marginal product of consumption enriched workers is arbitrarily large (resp. small) at low (resp. high) values of consumption per

[3] In the Cobb-Douglas case, the assumed production function boils down to

$$\begin{aligned} Y &= a\ K^{\beta}(K^{\alpha}C^{1-\alpha})^{1-\beta},\ 0 < \beta < 1, a > 0 \\ &= a\ K\ \hat{c}^{(1-\beta)(1-\alpha)} \\ &= a\ K\hat{c}^{\gamma},\ 0 < \gamma < 1. \end{aligned} \tag{2}$$

unit of capital. The second of the Inada conditions accords well with Ray's (1998) discussion of the relationship between nutrition and work capacity. As far as the first condition goes, Banerjee & Gupta (1997), Dasgupta & Ray (1987), (1998) etc. argue that for very low levels of consumption, there may be a stretch over which work capacity is a strictly convex and increasing function of nutrition. Our production function on the other hand admits diminishing returns from the very outset. One way of justifying this fact is to reemphasize what was already noted in the introductory section. Consumption in a macro set up is a broader concept than nutrition generating inputs into the labour process. It includes a whole range of life upgrading pursuits-cum-pastimes-cum-occupations that enrich work capacity. Once all these are taken into account, the macro "C" may appear to be a pretty large object in the representative household's choice set. Further, the representative household being an index of the entire population of the economy, its consumption stands for an average of consumption carried out by the different types of households. Consequently, it should not be entirely meaningless to ignore the convex part of the productivity curve in a predominantly growth oriented macro model.

3 Model

As in any standard growth model, a dynastic household is assumed to maximize

$$\int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} \, e^{-\rho t} dt, \quad 0 < \theta \neq 1 \tag{4}$$

subject to the instantaneous budget constraint

$$C(t) + \dot{K}(t) = rK(t) + wL(t), \tag{5}$$

where $c(t)$ stands for per capita consumption at t and r and w are the steady state interest and wage rates.[4] Given that $\bar{L} = 1$, we have $c(t) = C(t)\ \forall\ t$. The household's problem results in the choice of the demand rate of growth:

$$g^d(r, \tau) = \frac{r - \rho}{\theta}. \tag{6}$$

A profit maximizing competitive entrepreneur equates the (private) marginal productivity of capital to the market rate of interest. Thus,

$$r = f(\hat{c}) - (1 - \alpha)\ \hat{c} f'(\hat{c}), \tag{7}$$

where

$$\frac{\partial r}{\partial \hat{c}} = \alpha f'(\hat{c}) - (1 - \alpha)\hat{c} f''(\hat{c}) > 0. \tag{8}$$

The macro balance equation for the economy is

$$\begin{aligned} C + \dot{K} &= K f(\hat{c}), \\ \text{or,} \qquad g^s &= f(\hat{c}) - (\hat{c})^{1/(1-\alpha)}, \end{aligned} \tag{9}$$

where g^s stands for $\dot{K}/K$ and represents the rate at which the economy is capable of growing in steady state, given $\hat{c}$. We may refer to g^s as the supply rate of growth. The economy is in steady state equilibrium if $g^d = g^s$.

[4] We restrict attention to the case $\theta \neq 1$ for notational ease. When $\theta = 1$, the instantaneous utility function reduces to $ln\ c(t)$ and the different expressions that follow are changed accordingly. Indicating these changes repeatedly does not seem to serve any analytically important purpose.

3.1 Relationship with Traditional Models

Before proceeding further, it is instructive to reflect on the connections between our framework and conventional models of static macro equilibrium and growth. This is best done by considering the Cobb-Douglas specification (2) above. As is easily seen, the function can be reqwritten as

$$\begin{aligned} Y &= a\ K^{\alpha+\beta(1-\alpha)} C^{1-\{\alpha+\beta(1-\alpha)\}} \\ &= a\ K^{1-\mu} C^{\mu},\ 0 < \mu = \alpha + \beta(1-\alpha) < 1. \end{aligned} \tag{10}$$

Assuming a steady state constant rate of saving s along the steady state, the macro balance equation reduces to

$$a\ K^{1-\mu}\ C^{\mu} = C + s\ a\ K^{1-\mu}\ C^{\mu}$$

so that

$$C = \{a\ (1-s)\}^{1/(1-\mu)}\ K. \tag{11}$$

Substituting (11) in (10),

$$Y = \{a\ (1-s)\}^{\mu/(1-\mu)}\ K. \tag{12}$$

Equation (12) shows (as with Barro (1990), d'Autume & Michel (1993) etc.) that along a steady state, the reduced form production function has the familiar AK structure.

It is tempting to note a peculiarly Keynesian (Keynes (1936)) flavour of (12). Given any K, a higher s reduces Y. One could interpret this as a multiplier relationship. On the other hand, the investment-saving equality implies that

$$\dot{K} = s\ \{a\ (1-s)\}^{\mu/(1-\mu)}\ K,$$

so that the rate of growth of the system is

$$g = s\ \{a\ (1-s)\}^{\mu/(1-\mu)}. \tag{13}$$

It is easy to see that g rises as s increases to $1-\mu$, the share of capital in national income, and falls beyond it. In other words, we have here a traditional Sraffa (1960)-Von Neumann (1937) feature: the maximal rate of growth for the system occurs when all profits are invested. Moreover, as (6) indicates, this rate of growth corresponds to the maximal rate of profit also. A similar feature characterized the Barro (1990) model (in the Cobb-Douglas case).[5] It is worth pointing out, moreover, that for a given K, a rise in s in the interval $(1-\mu, 1)$ reduces both the level as well as the growth rate of income Y. In other words, for this range of s, the simple static multiplier story of the negative effect of a rise in s on the *level* of income carries over to a similar effect on its *rate of growth.* Traditional wisdom, however, suggests opposite effects of s on Y and its growth rate.

To end this section, define dK/dY as the Harrodian (Harrod (1939)) accelerator v. Using (12),

$$\frac{dY}{dK} = \{a\ (1-s)\}^{\mu/(1-\mu)}. \tag{14}$$

Consequently, $v = 1/\{a\ (1-s)\}^{\mu/(1-\mu)}$. Thus, Harrod's *warranted* rate of growth for the system is given by

[5]As opposed to the latter case, however, the maximal rate of growth is not socially optimal in our case.

$$\frac{s}{v} = s \left\{a \ (1-s)\right\}^{\mu/(1-\mu)},$$

which is the same as (13). There is nothing surprising in this of course, since any AK model is expected to have Harrodian features. The additional feature of our set up is that, unlike standard AK models, v is not a constant and depends on s itself.

4 Equilibrium: Market Economy

Figure 1 plots (9) on the top panel. The inverted U-shaped curve follows from the standard assumptions on f and the fact that $(\hat{c})^{1/(1-\alpha)}$ is strictly convex and increasing. For any given K, a rise in C has two opposing effects on the growth rate. First, it reduces the growth rate due to reduced savings. Secondly, it increases the growth rate due to increased productivity. The inverted U-shaped curve indicates that upto a certain stage, the second effect dominates. Beyond this, however, the productivity effect becomes two weak on account of diminishing returns and is wiped out by the first effect. The bottom panel of Figure 1 captures equation (7), which is upward rising as per (8). Putting the two curves together, we obtain a relationship between g^s and r. We may call this the supply rate of growth curve. The latter is shown as the backward bending curve OAB in Figure 2. Superimposing (6) on this, we determine the equlibrium endogenous growth rate g^*_ρ. The corresponding value of $\hat{c}$ is denoted by $\hat{c}^*$. [6]

Figure 1 here.

Figure 2 here.

[6]Note a strong feature of our model. Unless ρ is too high, so as to make growth undesirable, existence of a positive endogenous growth rate is guaranteed for all meaningful values of the other parameters.

Proposition 1 *A positive endogenous growth rate exists if the discount rate is not too high.*

A second interesting result is that a rise in the discount rate may *raise* the endogenous growth rate for the model. Compare points C and D in Figure 2 for which the market growth rates are shown as g^*_ρ and $g^*_{\rho'}$ respectively, $\rho' > \rho$.

Proposition 2 *A society with a high discount rate may grow faster than one with a low discount rate.*

The intuition underlying the result is as follows. A rise in ρ implies a rise in preference for current consumption over growth. The improved consumption raises labour productivity and causes the growth rate to rise despite lower propensity to save. Of course, after a certain point, due to diminishing returns, the productivity rise is too weak to compensate for the fall in saving and the growth rate falls.

We end this section by noting an implication of the AK feature pointed out in the immediately preceding one. The model does not call for an out of steady state analysis. Given any initial K_0, the economy chooses the "correct" $C(0)$, i.e. $C(0) = \hat{c}^*\ K_0$, and maintains it forever. The question of instability does not arise.

5 Equilibrium: Command Economy

Using Barro (1990), equation (17), the aggregate utility along a steady state path is given by

$$W = \frac{C_0^{1-\theta}}{(1-\theta)(\rho + (\theta - 1)g)}. \tag{15}$$

For this integral to be well defined, we impose the following restriction:

Assumption: $\rho + (\theta - 1)g > 0$.

For $\theta > 1$, the assumption is automatically satisfied for all $g > 0$. When $\theta < 1$, it amounts to a restriction on the maximum sustainable rate of steady growth, which, in view of equation (9), reduces to a restriction on the technology alone. (For example, for the Cobb-Douglas case, the assumption constrains the range of permissible values of α.)

The social planner is engaged in maximizing (15) subject to (9). As with the Market Economy, we shall restrict attention to steady states alone. Equation (15) is rewritten as

$$\begin{aligned} W &= \frac{K_0^{1-\theta}}{1-\theta} \frac{(C_0/K_0)^{1-\theta}}{\rho + (\theta - 1)g} \\ &= \frac{K_0^{1-\theta}}{1-\theta} \frac{\hat{c}_0^{(1-\theta)/(1-\alpha)}}{\rho + (\theta - 1)g}. \end{aligned} \tag{16}$$

Since $\hat{c}$ is a constant in steady state, (9) reduces to

$$g = f(\hat{c}_0) - (\hat{c}_0)^{1/(1-\alpha)}, \tag{17}$$

where we replace g^s by g, since the planner is merely concerned with finding a feasible rate of growth maximizing welfare and does not need to equate the demand and supply rates of growth as in the Market Economy. His problem reduces then to a static optimization exercise of maximizing (16) subject to (17). Equation (17) generates the same relationship between g and $\hat{c}_0$ as the top panel of Figure 1 and is reproduced in Figure 3.

As far as (16) goes, the level curves in the $\hat{c}_0 - g$ plane are downward sloping, since

$$\frac{dg}{d\hat{c}_0} = -\frac{1}{1-\alpha} \frac{\rho + (\theta - 1)g}{\hat{c}_0} < 0. \tag{18}$$

In general, the level curves need not be convex. This is seen from

$$\frac{d^2g}{d\hat{c}_0^2} = \frac{1}{(1-\alpha)^2} \frac{\theta(\rho + (\theta-1)\ g)}{\hat{c}_0^2} \frac{\theta - \alpha}{1-\alpha}. \tag{19}$$

Equation (19) shows that the level curves will be strictly convex or linear if $\theta \geq \alpha$, i.e., provided the learning by doing effect is not overly strong. Otherwise, they will be strictly concave. In what follows, we shall focus attention on the economically interesting case where the social planner chooses a nonzero rate of growth.[7] Since (17) defines g as a function of $\hat{c}_0$, say $g = \phi(\hat{c}_0)$, the planner will choose a nonzero rate of growth if the slope of the level curve W passing through the point $\hat{c}_{0,max}$ in Figure 3 is less than that of $\phi(\hat{c}_0)$. This leads to the imposition of the condition

$$\rho < (1-\alpha)\hat{c}_{0,max}\ \{\frac{(\hat{c}_{0,max})^{\alpha/(1-\alpha)}}{1-\alpha} - f'(\hat{c}_{0,max})\}, \tag{20}$$

where $[f'(\hat{c}_{0,max}) - (\hat{c}_{0,max})^{\alpha/(1-\alpha)}/(1-\alpha)] < 0$ is the slope of ϕ at $\hat{c}_{0,max}$. The restriction implies that the planner does not employ a discount rate that is so high that positive growth is avoided.

When (20) is satisfied, the socially optimum growth rate is positive. But it may not be unique if the planner's indifference curves are strictly concave, i.e., if $\alpha > \theta$. The planner's equilibrium is depicted by Figure 3 for the case $\theta > \alpha$. Even if this restriction is violated, an optimal growth rate g^{**} chosen by the planner is less than the maximum possible growth rate g_{max}, a result that follows from (18). Moreover, the value of $\hat{c}_0 = \hat{c}_0^{**}$ asociated with g^{**} is strictly larger than that associated with g_{max}. The intuition underlying the result is as follows. First, the planner, being omniscient, is aware of the utility trade off between g and $\hat{c}$. He has full knowledge of the technological

[7] None of the results to follow are affected if the chosen rate of growth is zero.

trade off between the two variables also. Since the social optimum equates the marginal rate of substitution with the rate of technical substitution, the planner will never choose a growth rate in the region where the technological relationship between g and $\hat{c}$ is complementary. In other words, he will exhaust all possibilities of growth improvement through consumption and settle for an equilibrium in a phase where the productivity effect of consumption increase is too weak to counter the negative effect of a savings rate reduction. It is also obvious from (18), that the larger the value of ρ, the smaller is the growth rate for the Command Economy. The findings may be stated as

PROPOSITION **3** *The command economy chooses an optimal growth rate that is smaller than the maximum feasible growth rate for the economy. The associated value of consumption per unit of capital is larger than the one corresponding to the maximal rate of growth. A rise in the discount rate leads to a lower rate of growth and a higher value of consumption per unit of capital.*

Figure 3 here.

As in the case of the Market economy, the Command Economy too can jump on to the equilibrium steady state path at $t = 0$ and continue to be in that state ever afterwards.

6 Market vs. Command Solution

Comparing Figures 2 and 3, it is obvious that, depending on ρ, the Command Economy may grow faster or slower than the Market Economy. The divergence between the two equilibria arises from the fact that while the planner is in a position to utilize all information relevant for his maximization exercise, the Market Economy fails to do so. The market itself drives a wedge so to speak between the entrepreneur and the household. In particular, the entrepreneur has no incentive to reward the household for the externality it generates on the firm through its consumption programme. Added to this is the fact that there is a learning by doing phenomenon at play in the model. The latter too is well-known for non-internalizable externalities. The exact relationship between the two rates depends on the values of the parameters. But one cannot rule out the Command Economy growing at a slower rate. This result parallels the one proved by Aghion & Howitt (1992) and stands in contrast to the usual result in endogenous growth theory that the Command Economy necessarily grows faster than the Market Economy.

Two additional and strong features of our result are that, unlike Aghion & Howitt, the rate of interest is determined endogenously by the model. Moreover, once again contrary to Aghion & Howitt, the solution to the Command Economy problem is independent of the rate of interest for the Market Economy. Needless to say, however, the problem addressed by this paper being totally different from the one studied by Aghion & Howitt, the strengths and weaknesses of the models are not strictly comparable.

PROPOSITION **4** *The Command Economy growth rate will in general differ from the Market Economy growth rate(s). Depending on parameter values, but not the rate of interest which is endogenously determined in the Market Economy, the Command Economy can grow at a slower rate than the Market Economy.*

7 Public Policy: Decentralizing the Command Solution

It is of interest to ask if the government can design a tax-subsidy programme to sustain the Command Economy steady state within the market structure. This section outlines such a scheme.

The government is assumed to levy a proportional income tax on capital income or offer a subsidy. Let (τ) denote the rate of tax (> 0) or subsidy (< 0). It uses the tax proceeds to purchase consumer goods from the market and freely distribute them back to the household in a lump sum manner. Alternatively, it gives a proportional subsidy to capital by imposing a lump sum consumption tax on the household. The decentralization procedure has the following features:

1. At time point $t = 0$, the government announces a rate of interest r_m, a wage rate w_m and a tax or subsidy rate τ_m to the household and asks it to maximize (4) subject to

$$C_p(t) + \dot{K}(t) = (1 - \tau_m)\ r_m\ K(t) + w_m \tag{21}$$

in the class of steady growth paths, where $C_p(t)$ represents the household's private consumption choice.

2. The government supplements the household's chosen consumption path $C_p(t)$ by a lump sum subsidy or imposes a lump sum tax on $C_p(t)$.

3. The values of r_m, w_m and τ_m can be so chosen that the household's choice of the growth rate will be g^{**} of Section 5. Further, the procedure will lead, after netting in/out the subsidy/tax, to the same aggregate consumption path for the household as the one followed by the Command Economy.

4. It will not be in the household's interest to make a false declaration about the growth rate to the government.

We now proceed to prove the following

PROPOSITION **5** *The Command Economy steady state path is decentralizable in an incentive compatible manner through proportional capital taxation (resp. subsidisation) and lump sum consumption subsidy (resp. tax).*

Proof: Given r_m, τ_m, the household's demand rate of growth is

$$g^d(r_m, \tau_m) = \frac{(1 - \tau_m)r_m - \rho}{\theta}, \tag{22}$$

which, we may write for convenience as

$$r_m = \frac{\theta\ g^d}{1-\tau_m} + \frac{\rho}{1-\tau_m}. \tag{23}$$

Consider now the marginal productivity of capital corresponding to the C/K ratio chosen by the planner, viz. $\hat{c}^{**}$, which equals $\hat{c}_0^{**}$ of Section 5. If this ratio were to prevail in the free market, then the the rate of interest would equal $f(\hat{c}^{**}) - (1-\alpha)\hat{c}^{**} f'(\hat{c}^{**})$, the private marginal productivity of capital. Choose r_m equal to this value. Similarly, choose w_m to be the marginal product of labour at $\hat{c}^{**}$. Let τ_m be the tax (resp. subsidy) rate (See Figure 4 for tax and Figure 5 for subsidy) such that

$$r_m = \frac{\theta\ g^{**}}{1-\tau_m} + \frac{\rho}{1-\tau_m}, \tag{24}$$

where g^{**} is the rate of growth of the Command Economy. It is obvious from the figures that a nonzero τ_m will exist (unless the market solution is fortuitously the same as the command solution).

Figure 4 here.

Figure 5 here.

The household's demand rate of growth corresponding to r_m will therefore be g^{**}. Using (21), we may write

$$\begin{aligned} C_p(t) + \dot{K}(t) &= (1-\tau_m)\ r_m K(t) + w_m, \\ &= r_m K(t) + w_m - \tau_m\ r_m K(t), \\ \text{or,} \qquad C_p(t) + \dot{K}(t) + \tau_m r_m K(t) &= r_m K(t) + w_m. \end{aligned} \tag{25}$$

For $t = 0$ in particular, this reduces to

$$C_p(0) + \dot{K}_0 + \tau_m r_m K_0 = r_m K_0 + w_m, \tag{26}$$

Let $C_g(0) = C(0)^{**} - C_p(0)$, where $C(0)^{**}$ stands for aggregate consumption at $t = 0$ in the Command Economy. We break up the analysis into two cases depending on the sign of $C_g(0)$.

Case 1: $C_g(0) > 0$. In this case, the government subsidises the household by augmenting $C_p(0)$. Two questions arise here. First, is the government's budget balanced? Second, is the aggregate consumption $C_p(0) + C_g(0)$ technologically feasible? To answer the first question, observe that an aggregate consumption $C(0)^{**}$ at $t = 0$ entails an aggregate output equal to $Y(0)^{**} = F(K_0, K_0^{\alpha}(C(0)^{**})^{1-\alpha}) = K_0 f(\hat{c}^{**})$. Moreover, by choice of r_m and w_m and using linear homogeneity, we have $r_m K_0 + w_m = Y^{**}$. Dividing out (26) by K_0 and substituting from (24),

$$\frac{C_p(0)}{K_0} + \tau_m r_m + g^{**} = f(\hat{c}^{**}). \tag{27}$$

Next, the fact that $\hat{c}^{**}$ satisfies (17) implies

$$\begin{aligned} \tau_m r_m &= (\hat{c}^{*})^{1/(1-\alpha)} - \frac{C_p(0)}{K_0} \\ &= \frac{C(0)^{**} - C_p(0)}{K_0}, \end{aligned}$$

since, by definition, $\hat{c}^{*} = ((C/K)^{*})^{1-\alpha}$. Hence, the choice of $C_g(0) = C(0)^{**} - C_p(0)$ balances the government's budget. Since the economy is in steady state, the same argument applies for all $t > 0$ also.

Given that the government's budget is balanced, (26) yields

$$Y^{**} = C_p(0) + C_g(0) + \dot{K}_0$$

$$= C_0^{**} + \dot{K}_0$$
$$= C_0^{**} + g^{**}K_0. \quad (28)$$

By assumption, however, $g^{**}K_0$ equals the investment carried out by the social planner at $t = 0$. Hence, $C_p(0) + C_g(0)$ as defined above is feasible.

Case 2: $C_g(0) < 0$. The same argument applies *mutatis mutandis*. The algebra is identical though the interpretation is different. In the present case, capital will be subsidised, thus raising $C_p(0)$ to a technologically infeasible level. The lump sum consumption tax will restore feasibilty while satisfying the government's budget constraint exactly.

We may end the proof by noting that it is always in the household's interest to announce the true demand rate of growth. Let W^* denote the level of welfare associated with the social optimum. By making a false announcement, the household can achieve at best the utility that falls short of W^*. Hence, the suggested decentralization scheme is incentive compatible for the household. ■

8 Conclusion

This paper has proposed a consumption led model of growth, which, unlike the existing growth models based on variety or brand proliferation, does not act through the utility channel directly. Instead, it works indirectly by enriching the labour force, thus giving rise to a form of labour augmenting technical progress. The effect of consumption on the labour force is contrasted with the standard neoclassical version of learning by doing models and the results for the model are worked out for the general case where both forms of technical change are present. The model argues that both types of technical change bring about an increase in the marginal product of capital that individual firms cannot internalize, thus leading to possible divergences between the socially optimum growth rate and the one attained by the market equilibrium. An interesting result that emerges is that the former can be smaller than the latter, as in the Aghion & Howitt exercise on Schumpeterian growth.

At the policy level, the paper advocates providing government support for consumption and better life styles when the consumption effect is strong. A poor quality of life weighs down on work ability, which manifests in the form of a low productivity of the capital equipment labour works with and finally a low rate of accumulation and growth. This final observation helps build a bridge between growth and development models. The latter have worked out the implications of consumption for work capacity and hence labour supply. A growth model, as we have argued, can take off from here by analysing the impact of increased labour supply on the marginal product of capital and hence the growth rate of the system as a whole.

References

Aghion, P. & P. Howitt (1992) A Model of Growth through Creative Destruction, Econometrica 60, 323-51.

Alesina, A. & D. Rodrik (1994) Distributive Politics and Economic Growth, Quarterly Journal of Economics, 109, 465-90.

Arrow, K.J. (1962) The Economic Implications of Learning by Doing, Review of Economic Studies, 29, 155-73.

Banerjee, S. and M.R. Gupta (1997) The Efficiency Wage Given Long-run Employment and Concave labour Constraint, Journal of Development Economics, 53, 185-95.

Barro, R. J. (1990) Government Spending in a Simple Model of Economic Growth, Journal of Political Economy 98 (5), 103-25.

Barro, R. & X. Sala-i-Martin (1999) Economic Growth, Cambridge, Mass., The MIT Press. (Original McGraw-Hill edition, 1995).

Dasgupta, P. and D. Ray (1986) Inequality as a Determinant of

Malnutrition and Underemployment: Theory, The Economic Journal, 96, 1-11-34.

d'Autume, A and P. Michel (1993) Endogenous Growth in Arrow's Learning by Doing Model, European Economic Review, 37, 1175-84.

De Soto, Hernando (2001) The Mystery of Capital, London, Black Swan.

Frankel, M. (1962) The Production Function in Allocation and Growth: A Synthesis, American Economic Review, 52, 995-1022.

Grossman, G. M. & E. Helpman (1991) Innovation and Growth in the Global Economy, Cambridge, Mass., The MIT Press.

Harrod, R. (1939) An Essay in Dynamic Theory, Economic Journal, 49, 14-33.

Keynes, J.M. (1936) The General Theory of Employment, Interest and Money, London, Macmillan.

Ray, Debraj & P. Streufert (1993) Dynamic Equilibria with Unemployment due to Undernourishment, Economic Theory, 3, 61-85.

Ray, Debraj (1998) Development Economics, New Delhi, Oxford University Press.

Romer, P.M. (1986) Increasing Returns and Long-Run Growth, Journal of Political Economy, 94, 1002-37.

Sheshinski, E. (1967) Optimal Accumulation with Learning by Doing in Carl Shell (ed) Essays on the Theory of Optimal Economic Growth, Cambridge, Mass., MIT Press.

Solow, Robert (1956) A Contribution to the Theory of Economic Growth, Quarterly Journal of Economics, 70, 65-94.

Sraffa, P. (1960) Production of Commodities by Means of Commodities, Cambridge, Cambridge University Press.

Von Neumann, J. (1937) A Model of General Equilibrium, Review of Economic Studies, 13, 1-9, translated by Karl Menger (1945).

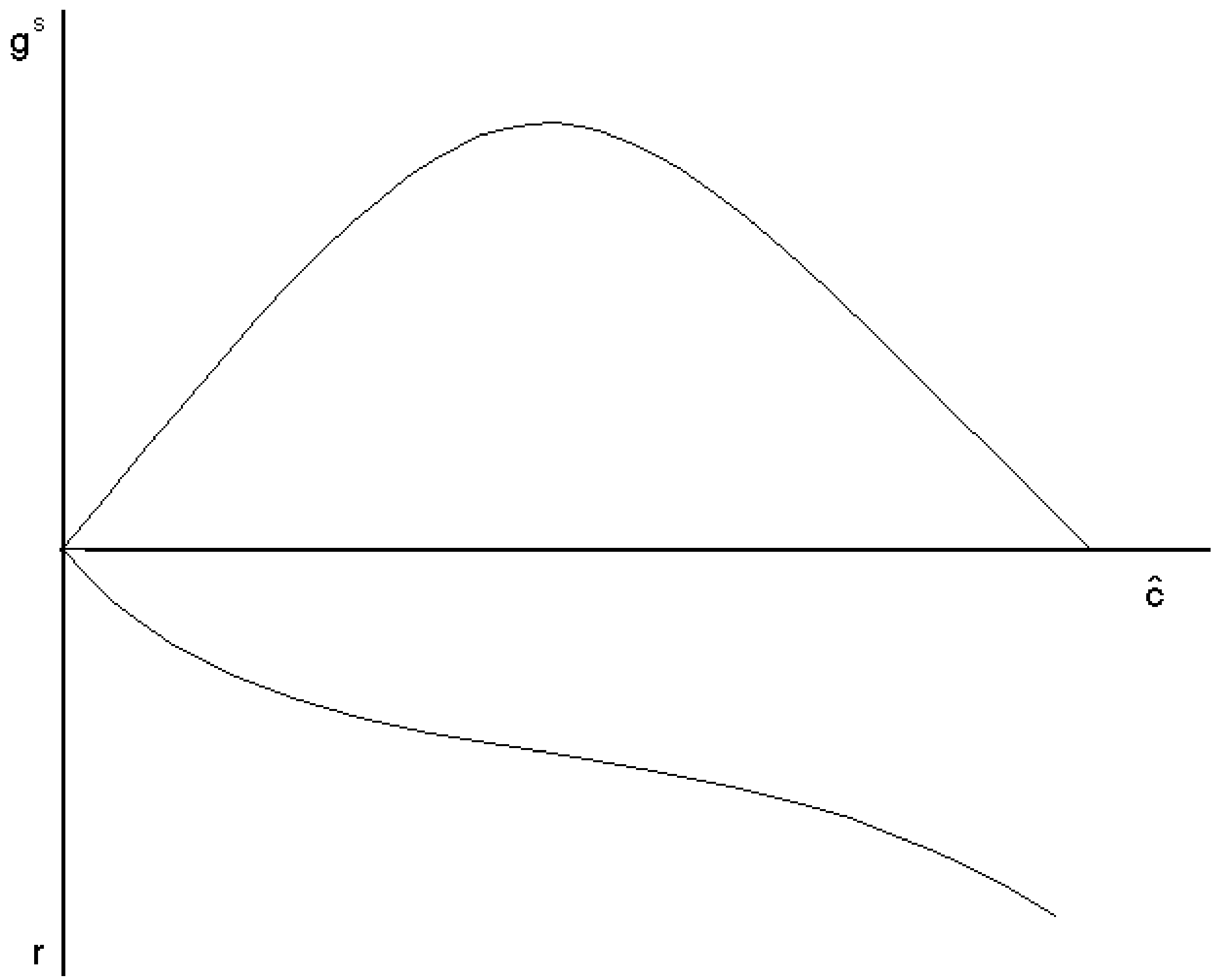

Figure 1

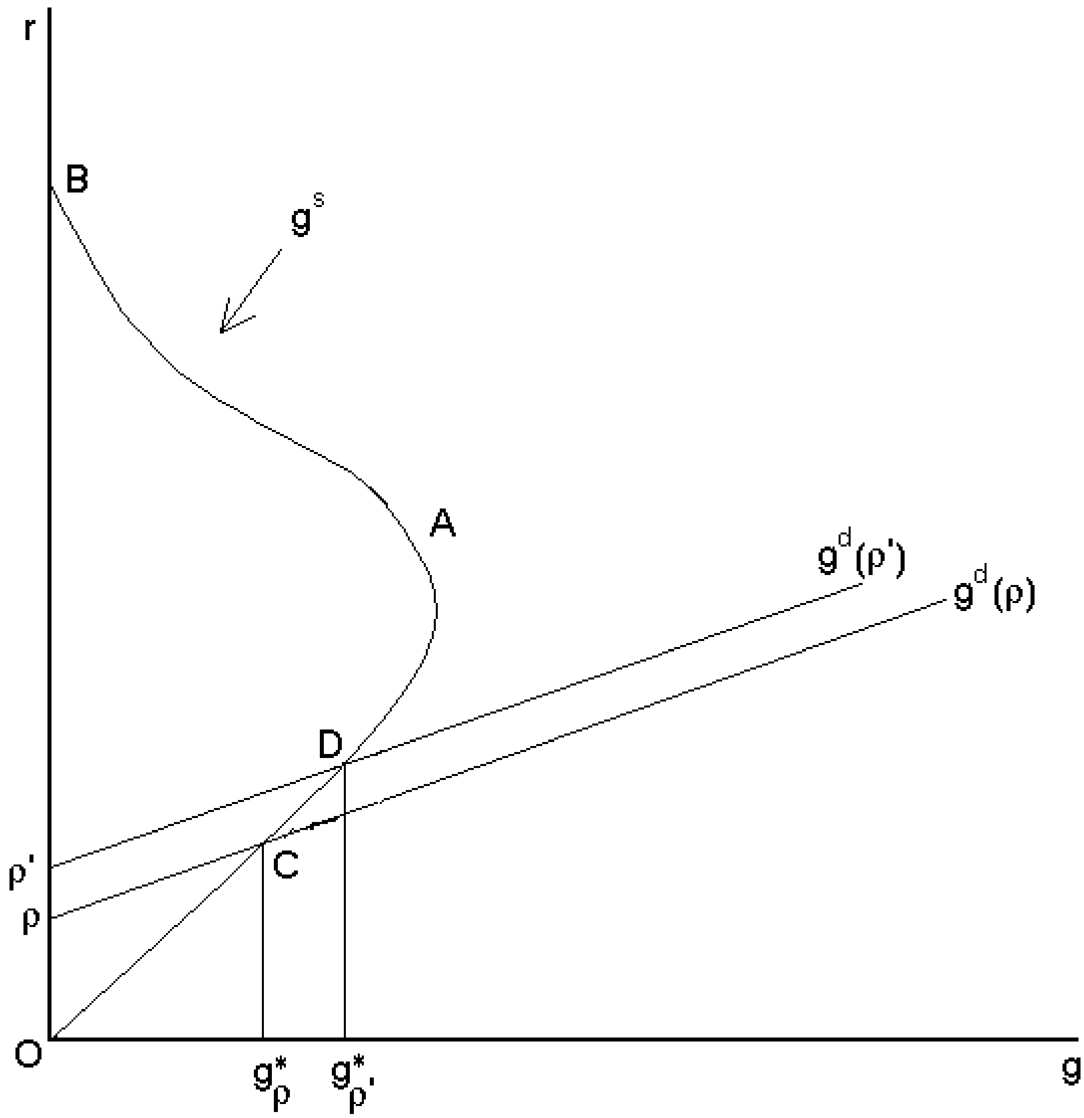

Figure 2

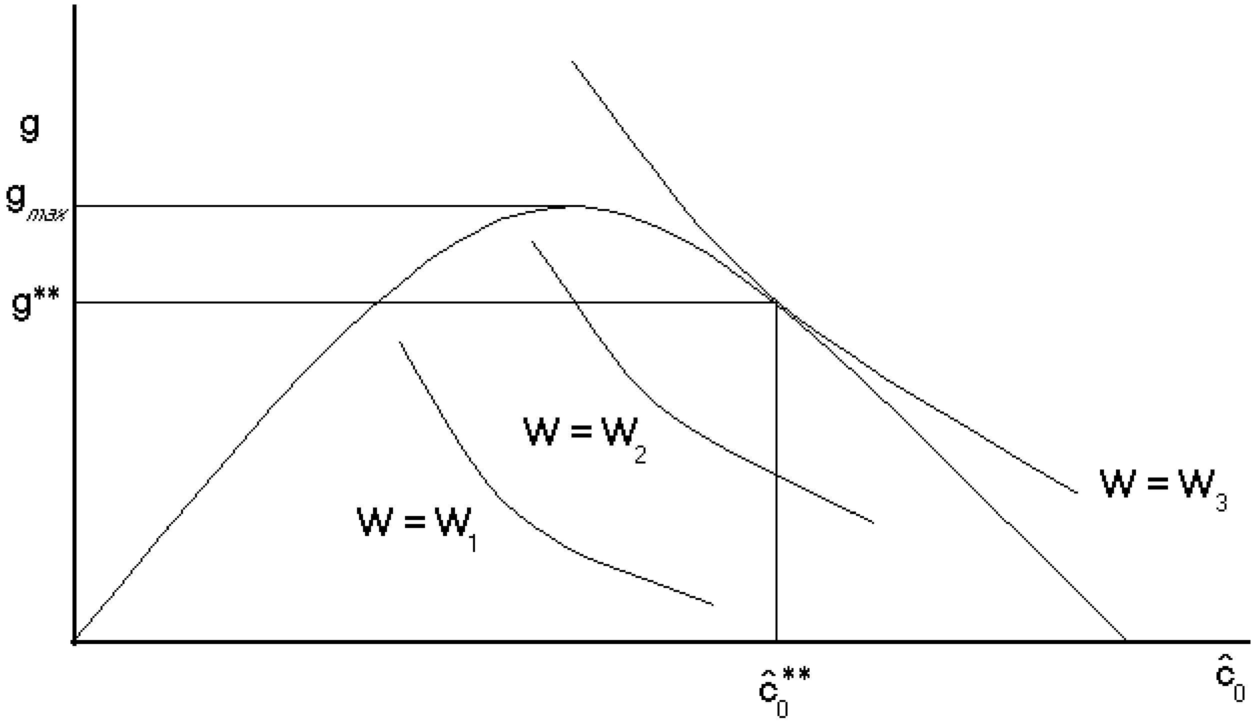

Figure 3

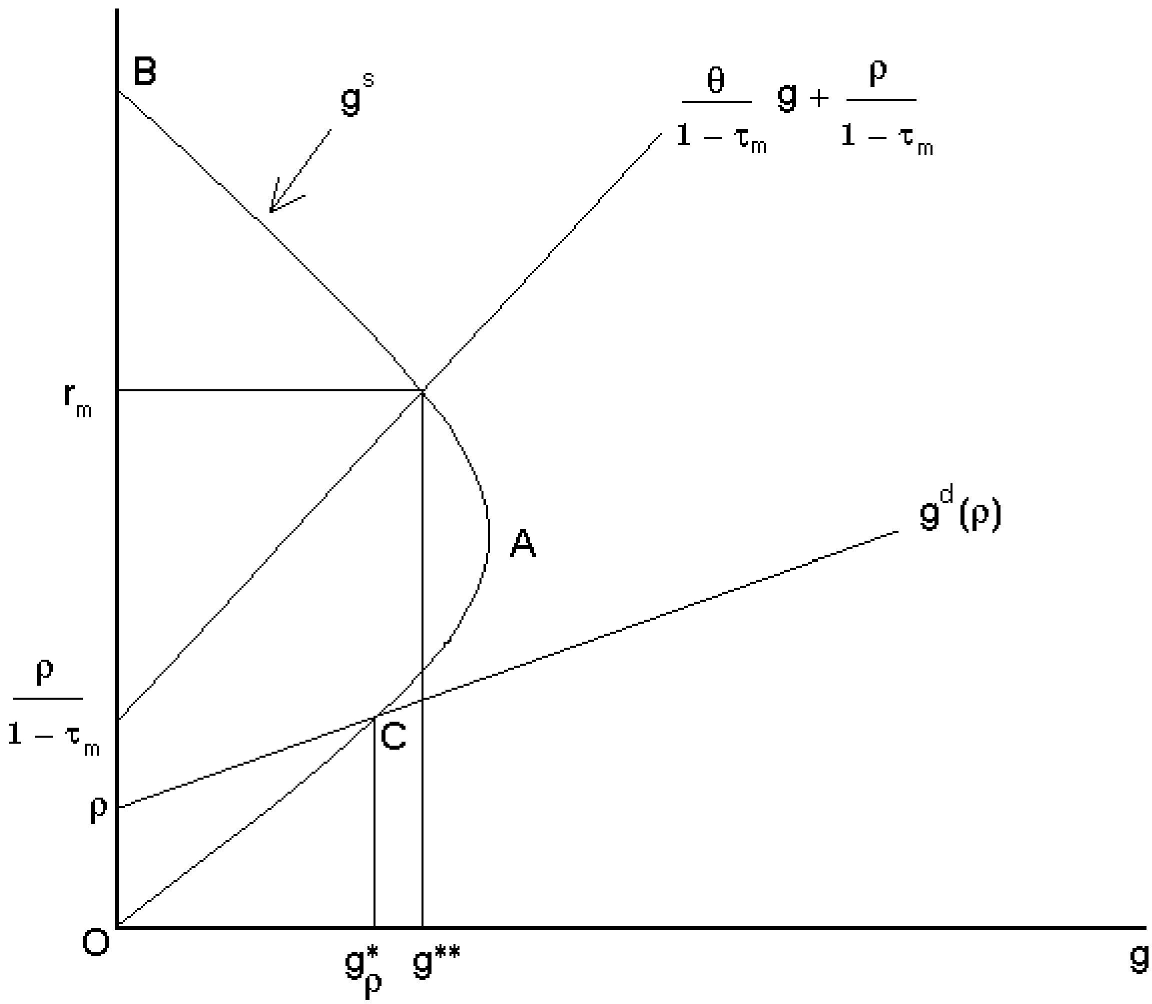
B
g^s
$\frac{\theta}{1-\tau_m} g + \frac{\rho}{1-\tau_m}$
r_m
A
$g^d(\rho)$
$\frac{\rho}{1-\tau_m}$
C
ρ
O
g^*_ρ
g^{**}
g

Figure 4

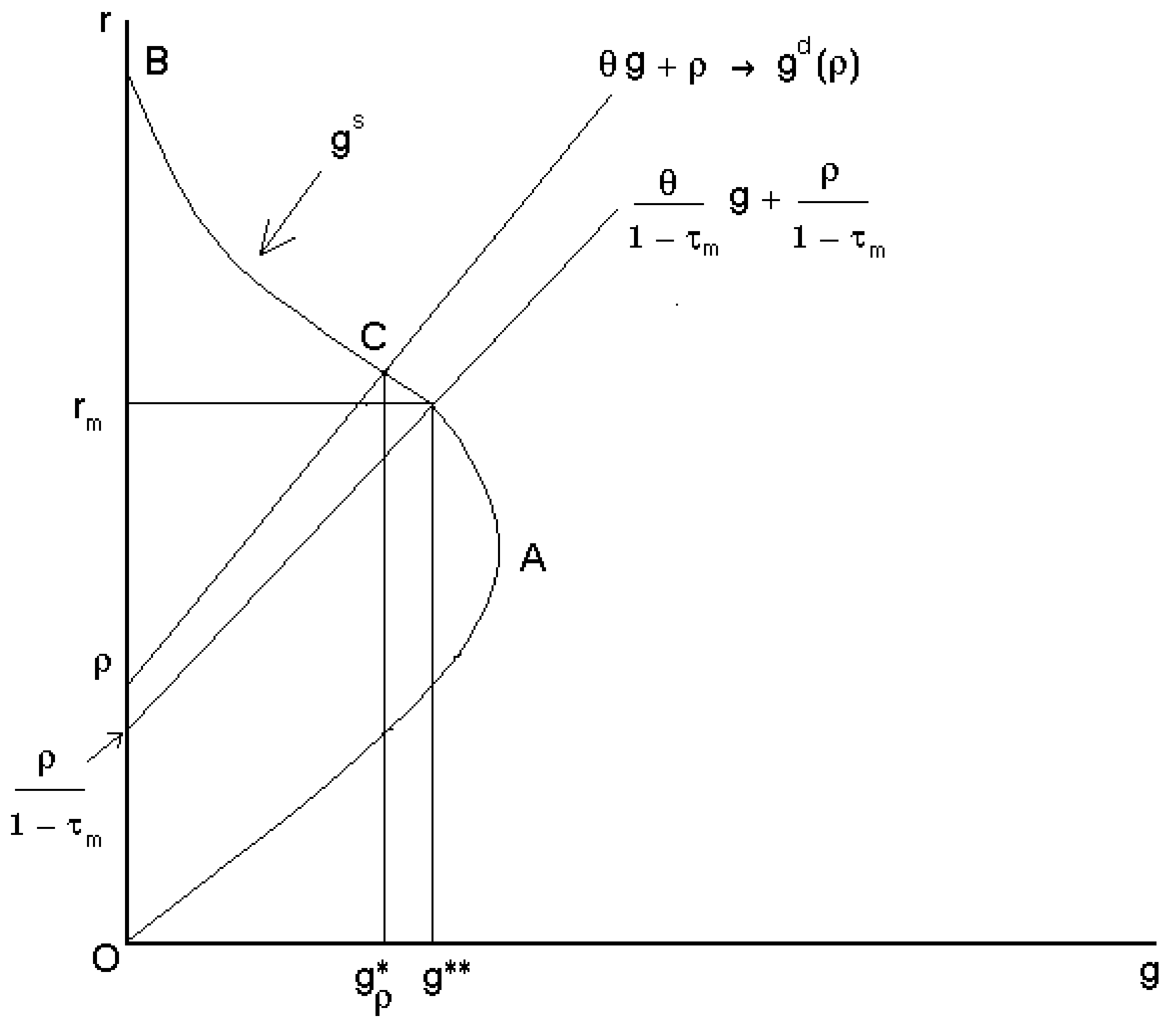
r
B
$\theta g + \rho \rightarrow g^d(\rho)$
g^s
$\frac{\theta}{1-\tau_m} g + \frac{\rho}{1-\tau_m}$
C
r_m
A
ρ
$\frac{\rho}{1-\tau_m}$
O
g^*_ρ
g^{**}
g

Figure 5