Cournot And Bertrand Prices
In A Model Of Differentiated Duopoly With R&D

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Abstract

We construct a model of differentiated duopoly with process R&D when goods are substitutes. In the first stage firms decide their technologies (i.e., the average costs of production) and in the second stage they compete in quantities or prices. We have shown that not only the Cournot firms invest a larger amount on R&D than the Bertrand firms, but, contrary to the result in the literature, Cournot price can be less than Bertrand price. This occurs when the R&D technology is relatively inefficient.

Key words: Process R&D, Bertrand price, Cournot price.

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1. Introduction

In a static oligopoly model firms have generally two alternative strategies, quantity and price. These give Cournot model and Bertrand model, respectively. Since the Bertrand model for homogeneous good appears to have less predictive power, economists reasonably work with differentiated products, and specifically with imperfectly substitute goods. Then a logical extension of this research is to provide an analysis based on comparing the results of these two models. A classic contribution in this context is the work by Singh and Vives (1984). Cheng (1985) provides a geometrical representation of the results. Vives (1985) has extended the analysis to the case of general demand and cost structures.

The important results of Singh and Vives are the following. First, equilibrium prices are lower and outputs are higher under Bertrand competition compared to Cournot competition.[1] Second, Bertrand equilibrium is more efficient than Cournot equilibrium in the sense that both consumer surplus and total surplus are greater under price competition. Finally, Cournot profits are larger than Bertrand profits (given that the goods are substitutes).

Since then these results have been facing steep challenges. In particular, economists have examined the question of efficiency in a dynamic setup by introducing R&D interaction of firms prior to the production stage. The idea is that the competing firms can, through a product or process R&D, alter the demand and cost structures, and therefore the outcomes of the production stage is also affected. Since Cournot profits are higher than Bertrand profits, Cournot firms will have a stronger cost reducing R&D incentive.[2] Hence there can be situations when Cournot equilibrium is more efficient than Bertrand equilibrium in the sense that social welfare under quantity competition will be higher compared to price competition. Let us briefly outline the literature below.

Delbono and Denicolo (1990) have a perfectly substitute good duopoly model, with R&D in the form of a patent race. The model shows that the R&D incentive are larger under price competition than that under quantity competition, but the general welfare comparison is ambiguous. Motta (1993) has a vertically differentiated duopoly with quality improving R&D. It shows that to relax fierce market competition firms differentiate more under Bertrand than under Cournot, and that price competition leads to higher consumer as well as social surplus.

On the other hand, Qiu (1997) provides an extended model of Singh and Vives (1984) with introducing a process R&D competition. It also allows for R&D spillovers. It is shown that Cournot firms invest more on R&D than the Bertrand firms and that Cournot equilibrium becomes more efficient if R&D productivity is high, spillovers are large and

[1] In fact, this result holds also for the case of complementary goods.

[2] Arrow (1962) has a pioneering work to the question of R&D incentives. He shows that a competitive firm has a larger R&D incentive than a monopoly firm. Bester and Petrakis (1993) have studied cost reducing incentive in a differentiated duopoly and have shown that Cournot competition provides a stronger incentive to innovate than Bertrand competition if the degree of substitution is low.

products are substitutes. The traditional efficiency results follow when spillovers are small, the degree of substitution is low and the R&D cost is large.[3]

Surprisingly enough, in all the works contributed so far in the literature, Cournot prices are unambiguously higher than Bertrand prices. Thus, the first result of Singh and Vives remains absolutely unchallenged. Even in Hackner (2000) that shows the possibility of a higher Bertrand price vis-à-vis the Cournot price in a framework of more than two firms, but that possibility arises only when the quality differences are large and goods are complementary; and for substitutes the traditional result holds. Symeonidis (2001) has larger consumer surplus, but this is due to quality improvement; Cournot price remains unambiguously higher than Bertrand price. This is our first modest attempt to show that Bertrand price can be higher than Cournot price. This possibility arises when the R&D technology is relatively inefficient or less productive. Our model is close to the Qiu (1997) structure with zero spillovers of R&D.[4] However, in one respect there is an important difference. In Qiu (1997), to start with firms have identical technologies (as given by the same marginal costs of production). Then in the first stage firms do R&D to reduce cost of production. Thus the R&D investment in Qiu is dependent on the initial marginal cost --- the higher the initial marginal cost, the lower is the incentive for R&D. In contrary, we assume that in the beginning firms decide their R&D investment to come up with an appropriate innovation, and then they compete in the product market either in prices or in quantities.[5] This structure appears to us more logical.

Before we go to the next section let us briefly explain the intuition of the result. First, Cournot firms have higher incentives to invest on process R&D because of the prospect of a higher profit. Hence Cournot firms will come up with a lower marginal cost compared to Bertrand firms. Secondly, higher is the productivity of the R&D technology, higher will be the innovation incentive for a larger innovation. So, the technological difference between Cournot and Bertrand firms will be less if the R&D productivity is large. Then for an inefficient R&D technology marginal cost differences will be wider. In fact as R&D inefficiency grows, Bertrand firms' R&D incentives fall at a greater rate compared to Cournot firms. Finally, the Bertrand price is more sensitive to the change of the marginal cost. Therefore, as marginal cost increases, the Bertrand price increases more than the Cournot price. All these factors together imply that the Cournot price can be lower than the Bertrand price when the R&D technology is less productive.

The plan of the paper is the following. We provide the model and results of the paper in Section 2. It contains a number of subsections. The first subsection presents Singh and Vives results. The second subsection is the Singh and Vives model with a prior process R&D stage. Then two sub-subsections discuss the technology choices under Cournot game and Bertrand game respectively. Then in the third subsection we compare marginal costs of the two games. Finally, in the fourth subsection Cournot and Bertrand prices are

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[3] Symeonidis (2001) in an otherwise Qiu (1997) model allows R&D both in process innovation and quality improvement. The distinctive feature of this model is that although prices under Cournot competition are always higher, consumer surplus can be higher too due to quality improvement.

[4] Even with zero spillovers, Cournot price is higher than Bertrand price in Qiu (1997).

[5] For technology choice in a conjectural variation duopoly one may look at Mai and Hwang (1999).

compared. The last section is a conclusion. Algebraic details are relegated in the Appendix.

2. Model

2.1 Singh and Vives (1984) Results

We consider an otherwise Singh and Vives model with an introduction of R & D interaction by the firms in the pre-production stage. In the production stage firms compete either in quantities or in prices, and in the R & D stage they decide their optimal production technologies represented by the constant unit costs of production.

Let the inverse (and linear) market demand of the *i-th* firm be

$$p_i = \alpha - \beta q_i - \gamma q_j, \qquad i, j = 1,2, \quad i \neq j, \tag{1}$$

where q_i is the amount of the goods produced by firm i and p_i is the price of the *i-th* firm's product. We assume that the goods are substitutes. The ratio $\frac{\gamma}{\beta}$ captures the degree of substitution --- $\gamma = 0$ means goods are independent and $\gamma = \beta$ means goods are perfect substitutes. The natural restrictions on the demand parameters are $\alpha > 0$ and $0 < \gamma < \beta$.

Denoting $a = \frac{\alpha(\beta - \gamma)}{\beta^2 - \gamma^2}$, $b = \frac{\beta}{\beta^2 - \gamma^2}$ and $c = \frac{\gamma}{\beta^2 - \gamma^2}$, the direct demand functions can be written as

$$q_i = a - bp_i + cp_j, \qquad i, j = 1,2, \quad i \neq j. \tag{2}$$

Now, assuming constant returns to scale technology the symmetric cost function is given by

$$C_i = mq_i \qquad i = 1,2 \tag{3}$$

where m is the constant marginal (and average) cost of production; $m < \alpha$. Then given (1), (2) and (3), we can immediately solve for symmetric Cournot and Bertrand prices as given by:

$$p^C = \frac{\alpha\beta}{2\beta + \gamma} + \frac{m(\beta + \gamma)}{2\beta + \gamma} \qquad \text{and} \qquad p^B = \frac{\alpha(\beta - \gamma)}{2\beta - \gamma} + \frac{\beta m}{2\beta - \gamma} \tag{4}$$

Correspondingly, profits are respectively,

$$\pi^C = \frac{\beta(\alpha - m)^2}{(2\beta + \gamma)^2} \quad \text{and} \quad \pi^B = \frac{(\alpha - m)^2}{(2\beta - \gamma)^2} \frac{\beta(\beta - \gamma)}{\beta + \gamma} \tag{5}$$

Then Singh and Vives (1984) results easily follow because

$$p^C(m) > p^B(m) \quad \text{and} \quad \pi^C(m) > \pi^B(m) \tag{6}$$

that is, for the given marginal cost Cournot price and Cournot profits are larger compared to Bertrand case. We can further see that

$$\left| \frac{d\pi^C}{dm} \right| > \left| \frac{d\pi^B}{dm} \right| \tag{7}$$

This states that Cournot firms have larger incentives to reduce the marginal cost of production than Bertrand firms.

The implication of (6) and (7) is the following. Since Cournot firms has larger incentives to reduce cost compared to Bertrand firms, Cournot firms will invest more on R & D and will come up with a lower marginal cost than the Bertrand firms. When the cost difference will be sufficiently large, Cournot price can be lower than Bertrand price. The following result clearly shows the differential impact of the change of the marginal cost on the respective Cournot and Bertrand price.

Lemma 1 *If marginal cost goes up (falls), the Bertrand price will increase (decrease) more than Cournot price i.e.,* $\frac{dp^C}{dm} < \frac{dp^B}{dm}$.

2.2 Singh and Vives Model with R & D

We consider a two-stage (non-cooperative) game. In the first stage firms independently decide their R & D investment (R) that determines production technology (that is, marginal costs of production, *m*). Then in the second stage firms simultaneously and non-cooperatively decide their quantities or prices depending on whether there is Cournot competition or Bertrand competition. We solve the game by the backward induction method, that is, we first solve the second stage problem and then the first stage problem noting the behavior of the second stage. Quite naturally, in the context of the 2nd stage, R & D cost is sunk, and the marginal costs determined in the first stage are treated as constant. Throughout the paper we assume symmetric equilibrium.

Let the R & D technology of a firm be

$$R(m_i) = \frac{\theta}{m_i} \tag{8}$$

where $\theta > 0$ is a parameter --- higher the value of θ, higher is the R & D cost or lower efficiency. The function clearly reveals diminishing returns technology.[6]

2.2.1 Technology Choice under Cournot Game

Profit function of the *i-th* firm under Cournot competition is defined as:

$$\pi_i^C = [\alpha - \beta\, q_i - \gamma\, q_i - m_i^C]\ q_i - \frac{\theta}{m_i^C}, \qquad i, j = 1,2, \quad \mathrm{i} \neq \mathrm{j} \tag{9}$$

where m_i^C is the marginal cost of the *i-th* firm.

Then the 2nd stage problem is

$$\max_{q_i}\ \pi_{\mathrm{i}}^{\mathrm{C}}, \qquad i = 1,2.$$

The first order conditions are given by

$$\frac{\partial \pi_i^C}{\partial q_i} = 0,\ \ i.e.,\ \ \alpha - m_i^C - 2\beta\, q_i - \gamma\, q_j = 0, \qquad i, j = 1,2, \quad i \neq j \tag{10}$$

By solving we get

$$q_i^C = \frac{2\alpha_i \beta - \alpha_j\, \gamma}{\Delta} \ \ and \ \ \ p_i^C = m_i^C + \beta\, q_i^C \tag{11}$$

where

$$\alpha_i = \alpha - m_i^C \quad \text{and} \quad \Delta = 4\beta^2 - \gamma^2 .$$

The second order and stability conditions are also satisfied, that is[7]

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[6] If we take a more general function, $R(m_i) = \dfrac{\theta m_i^{-e}}{e}$, $e \geq 1$, our results will remain unchanged.

[7] $\dfrac{\partial^2 \pi_i^C}{\partial q_i^2} = -2\beta$ and $\dfrac{\partial^2 \pi_i^C}{\partial q_i\, \partial q_j} = -\gamma$.

$$\frac{\partial^2 \pi_i^C}{\partial q_i^2} < 0 \text{ and } \Delta = \frac{\partial^2 \pi_1^C}{\partial q_1^2} . \frac{\partial^2 \pi_2^C}{\partial q_2^2} - \frac{\partial^2 \pi_1^C}{\partial q_1 \, \partial q_2} . \frac{\partial^2 \pi_2^C}{\partial q_2 \, \partial q_1} > 0 \tag{12}$$

Now, given the 2nd stage solution, the first stage problem is:

$$\max_{m_i^C} \pi_i^C \qquad \text{subject to} \qquad \frac{\partial \pi_i^C}{\partial q_i} = 0, \quad i = 1,2.$$

The first order conditions, $\frac{\partial \pi_i^C}{\partial m_i^C} = 0$ ($i = 1,2$) can be reduced to

$$\frac{4\beta^2 q_i}{\Delta} = \frac{\theta}{(m_i^C)^2} \tag{13}$$

We assume that second order and stability conditions are satisfied. Let $m^C = m_1^C = m_2^C$ be the symmetric equilibrium of this (first) stage. This will correspond symmetric equilibrium output and price solutions $q^C = q_i^C$ and $p^C = p_i^C$ for $i = 1,2$ in the 2nd stage. In *Appendix A* we have presented the steps to solve the Cournot game. The conditions describing equilibrium of this game are:

$$\frac{4\beta^2}{\Delta} q^C = \frac{\theta}{(m^C)^2} \qquad 14\text{(i)}$$

$$q^C = \frac{(2\beta - \gamma)(\alpha - m^C)}{\Delta} \qquad 14\text{(ii)}$$

$$p^C = m^C + \beta q^C \qquad 14\text{(iii)}$$

2.2.2 Technology Choice under Bertrand Game

Under Bertrand competition the profit function of a firm is:

$$\begin{aligned} \pi_i^B &= (p_i - m_i^B)\,(a - bp_i + cp_j) - \frac{\theta}{m_i^B} \qquad \text{i, j} = 1,2; \quad \text{i} \neq \text{j} \\ &= (a_i - bp_i + cp_j)\, p_i - m_i^B\,(a + cp_j) - \frac{\theta}{m_i^B} \end{aligned} \tag{15}$$

where $a_i = a + m_i^B b$, and m_i^B is the marginal cost of the *i-th* Bertrand firm. Therefore, the 2nd stage problem is:

$$\max_{p_i} \pi_{\mathrm{i}}^{\mathrm{B}}, \quad \mathrm{i} = 1{,}2 .$$

The first order conditions are

$$\frac{\partial \pi_i^B}{\partial p_i} = 0 \text{, i.e., } a_i - 2bp_i + cp_j = 0 , \tag{16}$$

By solving for p_i and p_j we have[3]

$$p_i^B = \frac{2a_i b + a_j c}{D} \quad \text{and} \quad q_i^B = b(p_i^B - m_i^B) \tag{17}$$

where

$$a_i = a + m_i b \quad \text{and} \quad D = 4b^2 - c^2 .$$

The first stage problem is

$$\max_{m_i^B} \pi_{\mathrm{i}}^{\mathrm{B}} \quad \text{subject to} \quad \frac{\partial \pi_i^B}{\partial p_i} = 0, \quad i = 1{,}2.$$

From the first order conditions, $\frac{\partial \pi_i^B}{\partial m_i^B} = 0$, we shall get

$$\frac{2b(2b^2 - c^2)}{D}(p_i^B - m_i^B) = \frac{\theta}{(m_i^B)^2} \tag{18}$$

Second order and stability conditions are assumed to be satisfied. Assuming symmetric equilibrium we have $m_1^B = m_2^B = m^B$. This leads to $p_1^B = p_2^B = p^B$ in (17).

Appendix B has provided steps to solve the Bertrand game. The equilibrium solutions of the Bertrand game are given by the following equations,

$$\frac{2b}{D}(2b^2 - c^2)(p^B - m^B) = \frac{\theta}{(m^B)^2} \tag{19(i)}$$

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[3] We can easily check that second order and stability conditions are satisfied, that is, $\frac{\partial^2 \pi_i^B}{\partial p_i^2} = -2b < 0$ and $D = \frac{\partial^2 \pi_1^B}{\partial p_1^2}\frac{\partial^2 \pi_2^B}{\partial p_2^2} - \frac{\partial^2 \pi_1^B}{\partial p_1\, \partial \mathrm{p}_2}\frac{\partial^2 \pi_2^B}{\partial p_2\, \partial \mathrm{p}_1} = (\text{-2b})^2 - c^2 = 4b^2 - c^2 > 0$.

$$p^B = \frac{(2b+c)(a+m^B b)}{D} \qquad 19(ii)$$

$$q^B = b(p^B - m^B) \qquad 19(iii)$$

2.3 Comparison between m^C and m^B

Under Cournot game the optimal technology, m^C is solved from (14), the optimal technology under Bertrand game, m^B is solved from (19). Comparing these technology levels we can write the following proposition.

Proposition 1 *In equilibrium Cournot firms have a lower marginal cost than the Bertrand firms (i.e., $m^C < m^B$).*

The proof is given in *Appendix C*. Intuition of the result is simple, Cournot firms have larger incentives to invest in R & D. Hence R & D investment under Cournot competition will be larger, implying that it will have a lower marginal cost of production.

2.4 Comparison between p^C and p^B

From 14(ii) and 14(iii),

$$p^C = m^C + \frac{\beta(2\beta-\gamma)(\alpha-m^C)}{\Delta} \qquad (20)$$

and from 19(ii) and 19(iii),

$$p^B = \frac{(2b+c)(a+m^B b)}{D} = \frac{(2\beta+\gamma)(\alpha(\beta-\gamma)+\beta m^B)}{\Delta} \qquad (21)$$

Therefore,

$$p^C - p^B = \frac{(\alpha-m^C)\gamma^2}{\Delta} - \frac{\beta(2\beta+\gamma)(m^B-m^C)}{\Delta} \qquad (22)$$

Quite obviously, if $m^B = m^C = m$, we have $p^C > p^B$, which is the Singh and Vives result (see (6)). In our case since $m^B > m^C$ (see Proposition 1), we have the possibility that $p^C < p^B$ if the difference between m^B and m^C is large.

Before we describe the nature of the relations $p^C(\theta)$ and $p^B(\theta)$, let us first consider the effect of the change of θ on m^C and m^B. The comparative static results are given in the following Lemma.

Lemma 2 : $\dfrac{dm^C}{d\theta} > 0$, $\dfrac{dm^B}{d\theta} > 0$ and $\dfrac{dm^B}{d\theta} > \dfrac{dm^C}{d\theta}$.

Proof is given in *Appendix D*. Lemma 2 says that *as the R & D technology becomes more and more inefficient, marginal cost under each of Cournot and Bertrand game increases, but it increases more under Bertrand competition.*

Now we are to compare $p^C(m^C)$ and $p^B(m^B)$. From (20) and (21) we have, respectively,

$$\frac{dp^C}{dm^C} = \frac{\beta(2\beta+\gamma)-\gamma^2}{\Delta} \qquad \text{and} \qquad \frac{dp^B}{dm^B} = \frac{\beta(2\beta+\gamma)}{\Delta}.$$

Therefore,

$$\frac{dp^C}{dm^C} > 0, \ \frac{dp^B}{dm^B} > 0 \qquad \text{and} \qquad \frac{dp^B}{dm^B} > \frac{dp^C}{dm^C} \tag{23}$$

This result tells that as marginal cost changes, Bertrand price is affected more compared to Cournot price.

Finally,

$$\frac{d[p^C - p^B]}{d\theta} = \frac{dp^C}{d\theta} - \frac{dp^B}{d\theta} = \frac{dp^C}{dm^C}.\frac{dm^C}{d\theta} - \frac{dp^B}{dm^B}.\frac{dm^B}{d\theta} \tag{24}$$

Using (23) through (24) we must have

$$\frac{dp^C}{d\theta} > 0, \ \frac{dp^B}{d\theta} > 0 \qquad \text{and} \qquad \frac{dp^C}{d\theta} < \frac{dp^B}{d\theta} \tag{25}$$

Therefore, both $p^C(\theta)$ and $p^B(\theta)$ are upward sloping with $p^B(\theta)$ having a greater slope than $p^C(\theta)$. This means,

$$\exists \ \theta = \theta^* \mid \mathrm{p}^C \underset{<}{>} p^B \ \Leftrightarrow \ \theta \underset{>}{<} \theta^* \tag{26}$$

Hence we have the following proposition.

Proposition 2 *If $\theta > \theta^*$, that is, if the R & D technology is relatively inefficient, Cournot price will be lower than Bertrand price.*

The result is portrayed in *Figure 1*. The result can be explained as follows. Low θ means that R & D cost of innovating a big sized technology is low. Hence both Cournot and Bertrand firms will come up with quite low marginal costs each, and therefore, the difference between their technologies will be small. For example, in the extreme, think that θ=0. Then both Cournot and Bertrand firms will come up with the most efficient technology, i.e., $m^B(0) = m^C(0) = 0$. In this case Singh and Vives result works, that is, $p^C(0) > p^B(0)$. But as θ goes up, marginal cost under Bertrand competition increases at a greater rate than that under Cournot competition. Therefore, for larger θ, the difference between m^B and m^C will be larger.

Moreover, by (25) as marginal cost increases, Bertrand price increases more than Cournot price. This explains why Bertrand price can be larger for a large θ. This is a new result in the literature. In other works, although Cournot firms have larger R & D investment, but Cournot price never falls below Bertrand price. One implication of this result is that if R & D is relatively costly, Cournot market structure will generate a larger consumers' welfare.

3. Conclusion

There is an extensive literature comparing Cournot and Bertrand results. In a pioneering work Singh and Vives show in terms of a differentiated duopoly that Bertrand price is lower than Cournot price and that the Bertrand equilibrium is more efficient. The literature thereafter has thoroughly examined these findings in a number of alternative set ups and has come to the conclusion that if goods are substitutes there can be situations when Cournot equilibrium generates more social welfare. However, Cournot price continues to be higher than Bertrand price in the literature. In an otherwise Singh and Vives framework with introducing process R&D in the first stage prior to the production stage we have shown in the present paper that Cournot price can be lower than Bertrand price when the R&D technology is relatively inefficient. Low R&D productivity leads to a wide difference in marginal costs in equilibrium under Cournot and Bertrand game, with Bertrand marginal cost being higher than Cournot marginal cost. This gives the possibility of a higher Bertrand price than the Cournot price. While we have proved the result assuming a specific innovation function, our result will go through for a wide class of R&D technology so long as there are diminishing returns to R&D investment. Finally, it may be mentioned that in this paper we have not provided separately any analysis on the overall welfare effect; instead, we have focused only on the price effect. However, it is understood that the welfare result will be similar to the existing literature.

Appendix

Appendix A : An Outline of Cournot Game Solution

Payoff functions under Cournot game :

$$\pi_i^C = [\alpha_i - \beta\, q_i - \gamma\, q_j]\; q_i - \frac{\theta}{m_i^C}, \quad \alpha_\mathrm{i} = \alpha - \mathrm{m_i^C}, \quad i = 1,2;\ \mathrm{i} \neq \mathrm{j}$$

For any given m_1^C and m_2^C, the second stage solution is given by (11) (see section 2.2.1), that is,

$$q_i^C = \frac{2a_i\beta - \alpha_j\gamma}{\Delta} \qquad \text{and} \qquad p_i^C = m_i^C + \beta\, \mathrm{q}_i^C \tag{A.1}$$

Then for the first stage problem the first order conditions are

$$\frac{\partial \pi_i^C}{\partial m_i^C} = 0 \qquad\qquad \mathrm{i} = 1,2\,,$$

$$\text{i.e.} \quad -(1+\gamma\,\frac{\partial q_j}{\partial m_i^C})\ \mathrm{q_i} + (\alpha_\mathrm{i} - 2\beta \mathrm{q_i} - \gamma q_j)\,\frac{\partial \mathrm{q_i}}{\partial \mathrm{m_i^C}} + \frac{\theta}{(\mathrm{m_i^C})^2} = 0 \tag{A.2}$$

Now, the first order conditions of the 2nd stage game are (see (10))

$$\frac{\partial q_i}{\partial m_i^C} = -\frac{2\beta}{\Delta} \qquad \text{and} \qquad \frac{\partial q_j}{\partial m_i^C} = \frac{\gamma}{\Delta} \qquad \text{where} \qquad \Delta = 4\beta^2 - \gamma^2 > 0\,.$$

Inserting these values in (A.2) we shall get

$$\frac{\partial \pi_i^C}{\partial m_i^C} = -\frac{4\beta^2}{\Delta}\,\mathrm{q_i} + \frac{\theta}{(\mathrm{m_i^C})^2} = 0 \tag{A.3}$$

We assume that second order and stability conditions for the first stage game are also satisfied. Then symmetric equilibrium implies $m_1^C = m_2^C = m^C$. This will lead to symmetric quantity equilibrium $q_1^C = q_2^C = q^C$. Hence using (A.1) and (A.3) we characterize the equilibrium solutions of the Cournot game by the equations 14(i) through 14(iii) in section 2.2.1.

Appendix B : An Outline of Bertrand Game Solution

The profit function of i-*th* firm is :

$$\pi_i^B = (a_i - bp_i + cp_j)\,\mathrm{p_i} - m_i^B\,(a + \mathrm{cp_j}) - \frac{\theta}{m_i^B}, \quad a_\mathrm{i} = a + m_i^B b, \quad \mathrm{i, j} = 1{,}2;\ \mathrm{i} \neq \mathrm{j}$$

For any given m_1^B and m_2^B, the second stage solution is given by (17) (see section 2.2.2), that is,

$$p_i^B = \frac{2a_i b + a_j c}{D} \qquad \text{and} \qquad q_i^B = \mathrm{b}\,(p_i^B - \mathrm{m}_i^B) \tag{B.1}$$

Then the first order conditions of the first stage problem are

$$\frac{\partial \pi_i^B}{\partial m_i^B} = 0 \qquad \mathrm{i} = 1{,}2\,,$$

i.e., $$(\mathrm{b} + c\frac{\partial p_j}{\partial m_i^B})\ \mathrm{p_i} + (a_\mathrm{i} - 2b\mathrm{p_i} + cp_j)\frac{\partial \mathrm{p_i}}{\partial \mathrm{m_i^B}} - (a + \mathrm{cp_j}) - \mathrm{cm_i^B}\ \frac{\partial \mathrm{p_j}}{\partial \mathrm{m_i^B}} + \frac{\theta}{(\mathrm{m_i^B})^2} = 0 \tag{B.2}$$

The first order conditions of the 2nd stage game (see (16)) are

$$a_i - 2bp_i + cp_j = 0$$

From this, the comparative static results are:

$$\frac{\partial p_i}{\partial m_i^B} = \frac{2b^2}{D} \qquad \text{and} \qquad \frac{\partial p_j}{\partial m_i^B} = \frac{bc}{D} \qquad \text{where} \quad D = 4b^2 - c^2 > 0$$

Using the above, (B.2) becomes

$$\frac{\partial \pi_i^B}{\partial m_i^B} = -\frac{2b}{D}\ (2\mathrm{b}^2 - \mathrm{c}^2)\,(\mathrm{p_i} - \mathrm{m_i^B})\ + \frac{\theta}{(\mathrm{m_i^B})^2} = 0 \tag{B.3}$$

Again we assume that second order and stability conditions of this stage game are satisfied. Then the assumption of symmetric equilibrium leads to $m_1^B = m_2^B = m^B$ and $p_1^B = p_2^B = p^B$. Hence from (B.1) and (B.3), the equilibrium solutions of the Bertrand game are characterized by the equations 19(i) through 19(iii) in section 2.2.2.

Appendix C : Proof of $m^B > m^C$

Given the symmetric equilibrium for any arbitrary marginal cost *m*, we have from (B.3)

$$\frac{\partial \pi_i^B}{\partial m} = -\frac{2b}{D}\,(2b^2 - c^2)\,(\mathrm{p^B} - m) + \frac{\theta}{m^2}$$

$$= -\frac{2\mathrm{b}\,(2\mathrm{b}^2 - c^2)(a-(b-c)m)}{D(2b-c)} + \frac{\theta}{m^2} \qquad \text{[using 19(ii))]} \qquad \text{(C.1)}$$

$$= -\ \frac{2\mathrm{b}\,(2\mathrm{b}^2 - c^2)\,(b-c)\,(\alpha - \mathrm{m})}{D(2b-c)} + \frac{\theta}{m^2}$$

We shall now evaluate $\frac{\partial \pi_i^B}{\partial m}$ at $m = m^C$. We may remember that m^C is solved from (14). Using 14(i) and 14(ii),

$$\frac{4\beta^2(\alpha - m^C)}{\Delta(2\beta+\gamma)} = \frac{\theta}{(m^C)^2} \qquad \text{(C.2)}$$

Therefore,

$$\frac{\partial \pi_i^B}{\partial m}\Big|_{m=m^C} = (\alpha - \mathrm{m^C})\ [-\frac{2b\,(2b^2 - c^2)\,(\mathrm{b - c})}{D(2b-c)} + \frac{4\beta^2}{\Delta(2\beta+\gamma)}]$$

$$= (\alpha - \mathrm{m^C})\ [-\frac{2\mathrm{b}\,(2\mathrm{b}^2 - c^2)(b-c)}{D(2b-c)} + \frac{4b^2(b^2-c^2)}{D(2b+c)}]$$

$$= \frac{2\mathrm{b}\,(\mathrm{b}-c)\ (\alpha - \mathrm{m^C})}{D}[-\ \frac{2b^2-c^2}{2b-c} + \frac{2b(b+c)}{2b+c}] \qquad \text{(C.3)}$$

$$= \frac{2b(b-c)\,(\alpha - \mathrm{m^C})}{D}\ \frac{\mathrm{c}^3}{\mathrm{D}} > 0$$

<u>Appendix D</u> : <u>Proof of Lemma 2</u>

From (14) we have

$$K^C(\alpha - m^C)(\mathrm{m}^C)^2 = \theta \tag{D.1}$$

where

$$K^C = \frac{4\beta^2}{\Delta}\frac{(2\beta - \gamma)}{\Delta}$$

and from (19)

$$K^B(\alpha - m^B)(\mathrm{m}^B)^2 = \theta \tag{D.2}$$

where

$$K^B = \frac{2b(2b^2 - c^2)(\mathrm{b} - \mathrm{c})}{D(2b - c)}$$

Therefore from (D.1)

$$\frac{dm^C}{d\theta} = \frac{1}{K^C m^C(2\alpha - 3m^C)} \tag{D.3}$$

and from (D.2)

$$\frac{dm^B}{d\theta} = \frac{1}{K^B m^B(2\alpha - 3m^B)} \tag{D.4}$$

We can now check that the second order conditions of the first stage (Cournot or Bertrand) game imply $2\alpha > 3m^s$ where $s = C, B$. Consider first the Cournot game. We already know from (A.3) that

$$\frac{\partial \pi_i^C}{\partial m_i^C} = -\frac{4\beta^2 q_i}{\Delta} + \frac{\theta}{(\mathrm{m}_\mathrm{i}^\mathrm{C})^2}$$

The second order conditions require

$$\frac{\partial^2 \pi_i^C}{\partial (m_i^C)^2} = -\frac{4\beta^2}{\Delta}\frac{\partial \mathrm{q}_\mathrm{i}}{\partial \mathrm{m}_\mathrm{i}^\mathrm{C}} - \frac{2\theta}{(\mathrm{m}_\mathrm{i}^\mathrm{C})^3} < 0$$

Since in equilibrium we have $m_i^C = m^C$ and $q_i = q^C$ satisfying (14), that is

$$-\frac{4\beta^2 q^C}{\Delta} + \frac{\theta}{(\mathrm{m}^\mathrm{C})^2} = 0 \quad \text{and} \quad \mathrm{q}^\mathrm{C} = \frac{(2\beta - \gamma)(\alpha - m^C)}{\Delta}$$

we have

$$\frac{\partial^2 \pi_i^C}{\partial (m_i^C)^2} = -\frac{4\beta^2}{\Delta}\left(-\ \frac{(2\beta - \gamma)}{\Delta}\right) - \frac{2}{\mathrm{m}^{\mathrm{C}}}\frac{4\beta^2}{\Delta}\frac{(2\beta - \gamma)(\alpha - \mathrm{m}^{\mathrm{C}})}{\Delta}$$

$$= \frac{4\beta(2\beta - \gamma)}{\Delta^2 m^C})(3\mathrm{m}^{\mathrm{C}} - 2\alpha) = \frac{K_C}{\mathrm{m}^{\mathrm{C}}}\ (3\mathrm{m}^{\mathrm{C}} - 2\alpha)$$

Hence.

$$\frac{\partial^2 \pi_i^C}{\partial (m_i^C)^2} < 0 \Rightarrow 2\alpha > 3\mathrm{m}^{\mathrm{C}}.$$

Similarly, we shall get,

$$\frac{\partial^2 \pi_i^B}{\partial (m_i^B)^2} = \frac{\mathrm{K}_{\mathrm{B}}}{\mathrm{m}^{\mathrm{B}}}\ (3\mathrm{m}^{\mathrm{B}} - 2\alpha) < 0 \Rightarrow 2\alpha > 3\mathrm{m}^{\mathrm{B}}$$

Hence from (D.3) and (D.4)

$$\frac{dm^C}{d\theta} > 0 \quad \text{and} \quad \frac{\mathrm{dm}^{\mathrm{B}}}{d\theta} > 0 \tag{D.5}$$

Again from (D.3) and using (D.1) we have

$$\frac{dm^C}{d\theta} = \frac{1}{K^C m^C (2\alpha - 3m^C)} = \frac{1}{\dfrac{2\theta}{\mathrm{m}^{\mathrm{C}}} - \dfrac{\theta}{\alpha - m^C}}$$

Similarly, from (D.4) and using (D.2) we have

$$\frac{dm^B}{d\theta} = \frac{1}{K^B m^B (2\alpha - 3m^B)} = \frac{1}{\dfrac{2\theta}{\mathrm{m}^{\mathrm{B}}} - \dfrac{\theta}{\alpha - m^B}}$$

Finally, therefore,

$$\frac{\dfrac{dm^B}{d\theta}}{\dfrac{dm^C}{d\theta}} = \frac{\dfrac{2\theta}{m^C} - \dfrac{\theta}{\alpha - m^C}}{\dfrac{2\theta}{m^B} - \dfrac{\theta}{\alpha - m^B}} > 1 \tag{D.6}$$

Thus, (D.5) and (D.6) together give Lemma 2.

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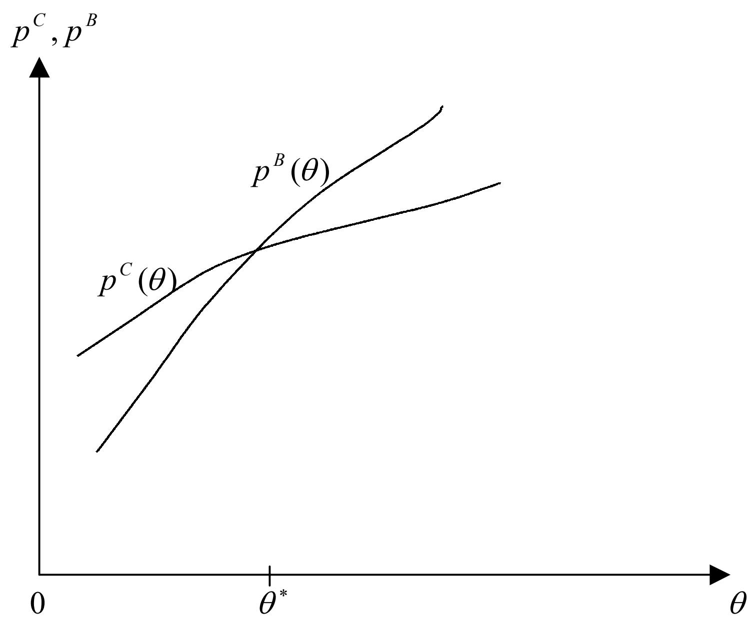

Figure 1: Comparison of Cournot and Bertrand prices