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Patent Licensing in Spatial Competition: Non-existence of Royalty Contracts

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Abstract: *This note seeks to show that optimal royalty licensing contracts in a Hotelling framework do not exist when patentee is an insider and the innovation is non-drastic. This proves that the corresponding results of Poddar and Sinha (2004, ER) are not correct.*

Key words: Insider patentee; non-drastic innovation; royalty licensing.
JEL Classifications: D43, D45, L13

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Introduction

In a recent paper Poddar and Sinha (2004) have studied the question of optimal licensing contracts in a Hotelling linear city model which is characterized by spatial competition. They have shown in particular that when patentee is an insider (i.e., a competitor in the same product market) and the innovation is non-drastic, there is a unique royalty licensing equilibrium superior to no licensing (see their Proposition 6). Since fixed fee licensing is not profitable under spatial competition in this scenario, therefore in equilibrium royalty licensing will occur. In their exercise the unique royalty so determined is equal to the amount of cost saving under the transferred technology. In their analysis Podar and Sinha have assumed the utility function as typically assumed in such a framework (for instance, see Shy (1996), and Shy and Thisse (1999)).

Unfortunately, the formulation of the problem in the price subgame in Poddar and Sinha (2004) is wrong, and hence the results they have derived are incorrect. If the problem is correctly formulated, then, as we show, there is no royalty licensing equilibrium in the given scenario, although there exist royalties for which royalty licensing is better than no licensing for the patentee.

We first present the Poddar and Sinha (2004) results for the case of insider patentee with non-drastic innovation. Then we point out the problem in their paper. Finally, we correct the problem and prove our claim.

Poddar and Sinha (2004) Results

In a linear city of length 1, firm A is located at 0 and firm B at 1. Consumers, uniformly distributed over the length of the city, buy each exactly one unit of the product either from firm A or from firm B.

The utility function of the consumer located at $x \in [0,1]$ is given by

$$u = \begin{cases} -x - p_A & \text{if } \textit{bought} \text{ from firm } A \\ -(1-x) - p_B & \text{if } \textit{bought} \text{ from firm } B \end{cases}$$

where p_A and p_B are the prices charged by firm A and firm B, respectively. For convenience they have assumed the transport cost to be unity per unit of length.

If $\bar{x}$ be the indifferent consumer between firm A and firm B, then

$$\bar{x} = \frac{1}{2}[1 + p_B - p_A]$$

Hence the demands as faced by firm A and firm B are respectively, $D_A(p_A, p_B) = \bar{x}$ and $D_B(p_A, p_B) = 1 - \bar{x}$.

With this in the pre-transfer situation the equilibrium prices, sales and profits of the firms are (see page 210, Poddar and Sinha (2004))

$$p_A^0 = \frac{1}{3}[3 + 2c_A + c_B] = \frac{1}{3}[3 + 3c - 2\varepsilon] \quad \text{and} \quad p_B^0 = \frac{1}{3}[3 + c_A + 2c_B] = \frac{1}{3}[3 + 3c - \varepsilon]$$

$$D_A^0 = \frac{1}{6}[3 + c_B - c_A] = \frac{1}{6}[3 + \varepsilon] \quad \text{and} \quad D_B^0 = \frac{1}{6}[3 + c_A - c_B] = \frac{1}{6}[3 - \varepsilon]$$

$$\Pi_A^0 = \frac{1}{18}[3 + c_B - c_A]^2 = \frac{1}{18}[3 + \varepsilon]^2 \quad \text{and} \quad \Pi_B^0 = \frac{1}{18}[3 + c_A - c_B]^2 = \frac{1}{18}[3 - \varepsilon]^2$$

where the unit costs of the firms are respectively $c_A = c - \varepsilon$ and $c_B = c$; $\varepsilon > 0$. The assumption of non-drastic innovation (and so duopoly in the pre-transfer situation) implies that $\varepsilon < 3$.

Now to consider the problem of royalty licensing equilibrium, Poddar and Sinha have formulated the following problem for the patentee (firm A) (see page 215):

$$\max_{r} \ \pi_A^R = \frac{(3+r)^2}{18} + r\frac{(3-r)}{6} \qquad (1)$$

This has unique royalty solution $r^* = \varepsilon\ (< 3)$. And for r^* , $\pi_A^R > \pi_A^0$ (and $\pi_B^R = \pi_B^0$).

Source of the Problem

In the case of royalty licensing, the firms have the following game to play. In the first stage firm A (patentee) offers a royalty r for transferring its technology $c-\varepsilon \equiv \tilde{c}$ to B. Then firm B decides whether to accept or reject the offer. If it is rejected, in the second stage they will play the pricing game as under no licensing situation. And if the offer is accepted, they will play the following price game in this subgame, that is, firm A's problem is:

$$\max_{p_A} \ (p_A - \tilde{c}) \ D_A(p_A, p_B) + rD_B(p_A, p_B) \tag{2a}$$

and firm B's problem is:

$$\max_{p_B} \ (p_B - \tilde{c} - r) \ D_B(p_A, p_B) \tag{2b}$$

But, as it appears, Poddar and Sinha (2004) have wrongly considered the following problem in this subgame under royalty licensing:

$$\max_{p_A} \ (p_A - \tilde{c}) \ D_A(p_A, p_B) \tag{3a}$$

$$\max_{p_B} \ (p_B - \tilde{c} - r) \ D_B(p_A, p_B) \tag{3b}$$

The solution to the problem (3) yields $D_B = \dfrac{(3-r)}{6}$ and $\pi_A = \dfrac{(3+r)^2}{18}$, and hence they come up with first stage problem as given by (1). But, as we can now understand, this is absolutely wrong.

Royalty Licensing

Given any royalty r, the second stage problem is defined by (2). This generates the following two reaction functions:

$$2p_A - p_B = 1 + \tilde{c} + r$$
$$-p_A + 2p_B = 1 + \tilde{c} + r$$

Hence the equilibrium values for any given r are:

$$\tilde{p}_A = \tilde{p}_B = 1 + \tilde{c} + r$$

$$\tilde{D}_A = \tilde{D}_B = \frac{1}{2}$$

$$\tilde{\pi}_A^R = \frac{1}{2} + r \quad \text{and} \quad \tilde{\pi}_B^R = \frac{1}{2}$$

Note that both $\tilde{D}_A$ and $\tilde{D}_B$ are independent of r, and both $\tilde{p}_A$ and $\tilde{p}_B$ are linear function of r, that is, as r increases, both $\tilde{p}_A$ and $\tilde{p}_B$ increase at the same rate, keeping $\bar{x}$ at the constant level. Further, $\tilde{\pi}_B^R$ is independent of r, but $\tilde{\pi}_A^R$ depends directly (and linearly) on r. Therefore, the patentee can go on increasing the royalty rate indefinitely, but firm B will not be affected by it; it will raise its price linearly without losing its market share. Hence in the present structure we don't have any equilibrium royalty solution. Even we don't need to restrict to $r < \varepsilon$.

Finally note that $\tilde{\pi}_B^R = \frac{1}{2} > \pi_B^0 = \frac{1}{18}(3-\varepsilon)^2$. Then, to check whether royalty licensing is at all feasible we need to check whether the following inequality holds, that is, $\tilde{\pi}_A^R > \pi_A^0 \Leftrightarrow r > \frac{\varepsilon(\varepsilon+6)}{18}$. Clearly there always exists r satisfying this inequality.

To close this model, let us assume that there exists some basic utility level $v > 0$ so that the consumer located at x enjoys a surplus $v - x - p_A$ if he buys from firm A, and $v - (1-x) - p_B$ if he buys from firm B. This means, increase in r only extracts consumer surplus. So r can be increased increasing $\tilde{\pi}_A^R$ as long as the $\bar{x}$ -th consumer participates in consumption. Once r crosses that level, the $\bar{x}$ -th consumer drops out, and then there is a trade off between fall in demand and increase in royalty.

Reference

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