Discussion Paper ERU/2010 – 03
July, 2010

On Technology Licensing in a Hotelling Structure

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Abstract: This paper discusses the question of optimal licensing contracts in a Hotelling structure. We show that a royalty equilibrium exists if and only if transport cost lies in a specified interval, but the royalty rate can be higher than the amount of cost saving. While fee licensing only is never profitable, the optimal licensing contract consists of a fee plus royalty. In equilibrium the market is fully covered with monopolistic goods.

Key words: Technology transfer; royalty licensing; fee licensing
JEL Classifications: D43, D45, L13

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1. Introduction

Technology transfer from a low cost firm to a high cost firm is a common phenomenon. By transferring its superior technology the patent holder can achieve a larger profit. There is already a vast literature on technology licensing. One aspect of this literature is to discuss the question of optimal licensing contracts. Generally technology transfer occurs under a fee contract or royalty contract, and sometimes the contract takes a hybrid form consisting both fee and royalty.[1] However, which particular form of contracts is optimal from the viewpoint of the transferor depends on a number of factors such as: the structure of the product market, the nature of market competition, the degree of product differentiation, the extent of cost saving, whether the patentee is an insider or outsider, whether there is asymmetric information, etc.

Given the existing literature, it seems that the question of technology licensing in a Hotelling (1929) structure is not yet fully explored. To our knowledge, only two papers, viz., Poddar and Sinha (2004) and Matsumura and Matsushima (2008) (henceforth, P&S and M&M respectively) have provided some analysis on this issue. While P&S discuss the question of optimal licensing contracts between two firms when the patentee is an insider or outsider, M&M examine how licensing activities affect the location of the firms and their incentives for R&D investment. In this context we restrict to the scenario where the patentee is an insider and firms' locations are exogenous. Initially the firms have asymmetric technologies. The low cost firm then designs a contract to transfer its technology to the other firm.

In such a model, given that the consumers are uniformly distributed over the length of the city, technology transfer under a fixed fee contract is never profitable. So the literature examines the possibility of a royalty contract. P&S and M&M have shown that the optimal royalty will be equal to the amount of cost saving, that is, the difference between the two unit costs of production. The purpose of the present paper is to show

[1] See Kabiraj (2005) and Mukherjee (2010) and the references therein.

that their results may hold only under some self-imposed restrictions. Hence we provide a more general and complete analysis.

One important limitation of the P&S paper is the following. They (implicitly) assume that in the market competition stage the transferor chooses its price so as to maximize its profit from its own product without caring the licensing revenue. P&S do not have this assumption explicitly, but without this their result does not hold. M&M have a correct formulation, but like P&S, M&M put the restriction that the royalty will not exceed the extent of cost saving, because, they think, the licensee will otherwise refuse to accept the contract. But given the profit maximization objective of the firms, our paper shows that such a priori restriction is not only arbitrary but it is inconsistent with the objective of profit maximization. We show that transferee's profit is independent of the royalty rate so long as the market remains fully covered, and transferor's profit is increasing in royalty. Then the optimal royalty rate is determined corresponding to zero net utility of the marginal consumer. And there are situations where the optimal royalty will be higher than the amount of cost saving. In any case, under a royalty contract the transferee gets strictly a positive surplus. Hence non-accepting by the transferee is not subgame perfect.

Then we have extended the discussion of optimal royalty equilibrium if the market is not fully covered. We have introduced an outside good and have allowed the possibility of segmented markets for the monopolistic good so that in the post-transfer situation each firm enjoys a local monopoly. In our model, whether the market will be fully covered in the post-licensing situation is an endogenous decision.

The important results that we have derived are the following. In our paper transport cost plays a significant role in determining and characterizing the licensing equilibrium. A royalty licensing equilibrium exists if and only if transport cost belongs to a given interval. In equilibrium the market is fully covered; thus royalty equilibrium with local monopoly of each firm will never occur. The optimal royalty rate when licensing equilibrium exists can be higher than the amount of cost reduction due to technology transfer. Since the transferor cannot extract all surplus of the transferee by means of royalty only, a fixed fee plus royalty contract is optimal in the Hotelling structure, although only fee contract is never mutually profitable.

The next section provides the model and results, and the third section is a conclusion.

2. Model and Results

Consider two firms producing homogeneous goods but located at two end points of a Hotelling linear city of length 1. Assume that firm 1 is located at 0 and firm 2 at 1; consumers are uniformly distributed over the length of the city. Each consumer buys at most one unit of the monopolistic good. We say that the market is fully covered if all consumers buy the good. Assume further that there is an outside competitive good, and the consumer's net utility from it is normalized to zero. Therefore a consumer will buy the monopolistic good if and only if her net utility from it is non-negative.

The utility function of a consumer located at $x \in [0,1]$ is given by

$$u = \begin{cases} v - tx - p_1 & \text{if to buy from firm 1} \\ v - t(1-x) - p_2 & \text{if to buy from firm 2} \end{cases}$$

where $v > 0$ denotes the basic utility, same for all consumers, p_i is the unit price charged by firm i, and $t > 0$ is the Hotelling transport cost of travel per unit distance.[2] We restrict to the scenario where each firm has a positive market share; therefore, $p_2 - t < p_1 < p_2 - t$.

Let $\bar{x}$ be the consumer indifferent between buying from firm 1 and firm 2,

$$\bar{x} = \frac{1}{2t}\left[t + p_2 - p_1\right]$$

[2] In their specification of the utility function both P&S and M&M assume $v = 0$. Further, P&S assume $t = 1$.

Then the market will be fully covered if and only if $2v \geq p_1 + p_2 + t$. This gives demand for firm 1 and firm 2's product as $D_1(p_1, p_2) = \bar{x}$ and $D_2(p_1, p_2) = 1 - \bar{x}$, respectively. On the other hand, the market is segmented if $2v < p_1 + p_2 + t$. This is the situation when some consumers (in particular, the one located at $\bar{x}$) fail to buy the monopolistic product. In this case firm i ($i = 1, 2$) faces the demand, $D_i(p_i) = \frac{v - p_i}{t}$.

The profit function of firm i is:

$$\Pi_i(p_1, p_2) = (p_i - c_i)\, D_i(p_1, p_2) \quad i = 1, 2$$

where c_i is firm i's unit production cost. Assume that firm 2 possesses the superior technology; therefore, $c_2 < c_1$. And initially both the firms have positive market shares. Hence under price competition, the equilibrium prices, market shares and profits of the firms are

$$p_1^0 = \frac{1}{3}\left[3t + 2c_1 + c_2\right], \qquad p_2^0 = \frac{1}{3}\left[3t + c_1 + 2c_2\right]$$

$$D_1^0 = x^0 = \frac{1}{6t}\left[3t + c_2 - c_1\right], \qquad D_2^0 = 1 - x^0 = \frac{1}{6t}\left[3t + c_1 - c_2\right]$$

$$\pi_1^0 = \frac{1}{18t}\left[3t + c_2 - c_1\right]^2, \qquad \pi_2^0 = \frac{1}{18t}\left[3t + c_1 - c_2\right]^2$$

Then, the assumption that both firms' market shares are positive implies that

$$t > \frac{c_1 - c_2}{3} \equiv \underline{t} \tag{1}$$

Now consider the possibility of technology transfer from firm 2 to firm 1 under a royalty contract. The game is the following. First, firm 2 proposes a royalty, r, per unit of firm 1's output. Firm 1 either accepts or rejects the contract. It accepts if it is not worse off in the post-transfer situation. In the second stage the firms choose prices simultaneously.

First consider royalty equilibrium with full market coverage. Given any r, the second stage problems of firm 1 and firm 2 are respectively[3],

$$\max_{p_1} \ (p_1 - c_2 - r)\ D_1(p_1, p_2) \quad \text{and} \quad \max_{p_2} \ (p_2 - c_2)\ D_2(p_1, p_2) + rD_1(p_1, p_2)$$

The second stage outcomes are:

$$\hat{p}_1(r) = \hat{p}_2(r) = t + c_2 + r$$

$$\hat{D}_1(r) = \hat{D}_2(r) = \frac{1}{2}$$

$$\hat{\pi}_1(r) = \frac{t}{2} \quad \text{and} \quad \hat{\pi}_2(r) = \frac{t}{2} + r$$

Recall that the assumption of full market coverage requires that $2v \geq \hat{p}_1(r) + \hat{p}_2(r) + t$. That is,

$$r \leq v - c_2 - \frac{3}{2}t \equiv \hat{r} \tag{2}$$

We further need to restrict $\hat{r} > 0$, that is,

$$t < \frac{2}{3}(v - c_2) \equiv \bar{t} \tag{3}$$

Note that $\hat{D}_1$, $\hat{D}_2$ and $\hat{\pi}_1$ are independent of r, but $\hat{p}_1$, $\hat{p}_2$ and $\hat{\pi}_2$ are linear and increasing in r. Hence firm 2 has an incentive to increase r as much as possible; in response firm 1 will just raise its price linearly without losing its market share and profits as long as the indifferent consumer $\bar{x}$ continues to buy. Therefore, the optimal royalty is determined corresponding to $u(\bar{x}) = 0$. This gives

$$\hat{r} = v - c_2 - \frac{3}{2}t \tag{4}$$

Given that $t > \underline{t}$ (which ensures that both firms' market shares are positive and payoffs are strictly positive), we have $\hat{\pi}_1(r) > \pi_1^0$. Therefore any royalty $r \leq \hat{r}$ is

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[3] As pointed out in the introduction, firm 2's second stage problem in P&S is: $\max_{p_2} \ (p_2 - c_2)\ D_2(p_1, p_2)$.

acceptable to firm 1. Clearly, given (4), there are parameter situations where $\hat{r}-(c_1-c_2)>0$ is possible.[4]

Now firm 2 will offer the royalty $\hat{r}$ if and only if $\hat{\pi}_2(\hat{r})>\pi_2^0$, that is

$$\hat{r}+\frac{t}{2}>\frac{1}{18t}(3t+c_1-c_2)^2$$

$$\Rightarrow \quad LHS(t)\equiv v-c_2-\frac{3}{2}t>\frac{(c_1-c_2)}{3}+\frac{(c_1-c_2)^2}{18t}\equiv RHS(t) \qquad (5)$$

$LHS(t)$ is linear and decreasing in t, and $RHS(t)$ is convex and decreasing in t, with $RHS(t)\to\frac{(c_1-c_2)}{3}$ as $t\to\infty$. We can further check that (at $t=\underline{t}$) $LHS(\underline{t})>RHS(\underline{t})$. Therefore,

$$\exists\hat{t},\underline{\mathrm{t}}<\hat{\mathrm{t}}<\bar{\mathrm{t}},|\,\mathrm{LHS(t)}>\mathrm{RHS(t)}\;\forall\mathrm{t}\in(\underline{\mathrm{t}},\hat{\mathrm{t}}) \qquad (6)$$

This is shown in *Figure 1*. We can now write the first result of our paper.

Proposition 1: Technology transfer under the royalty contract with full market coverage is mutually profitable if and only if $t\in(\underline{t},\hat{\mathrm{t}})$. The optimal royalty rate is $\hat{\mathrm{r}}$.

Clearly, there will be no royalty licensing with full market coverage if $t\geq\hat{\mathrm{t}}$. Now we examine the possibility of royalty equilibrium when the market is not fully covered. That is, we restrict to the scenario where in equilibrium, $2v<p_1+p_2+t$.

Consider local monopoly of firm 1 under technology transfer. If a royalty r is accepted, firm 1's problem is:

$$\max_{p_1}\;(p_1-c_2-r)\,\frac{v-p_1}{t}.$$

The corresponding product price and market share of firm 1 are, respectively,

$$\tilde{p}_1(r)=\frac{1}{2}(v+c_2+r)\quad\text{and}\quad\tilde{D}_1(r)=\frac{1}{2t}(v-c_2-r)$$

[4] If in the utility function, $v=0$, then r should go on increasing unless there is an upper bound. P&S and M&M assume that the maximum r cannot exceed the cost difference.

Since now two markets are segmented, the licensing revenue maximizing royalty rate will be $\tilde{r} = \arg\max_{r} r\tilde{D}_1(r)$, where

$$\tilde{r} = \frac{v - c_2}{2} \tag{7}$$

The corresponding equilibrium price, market share and profit of firm 1 are:

$$\tilde{p}_1(\tilde{r}) = \frac{1}{4}(3v + c_2),\ \tilde{D}_1(\tilde{r}) = \frac{1}{4t}(v - c_2) \text{ and } \tilde{\pi}_1(\tilde{r}) = \frac{1}{16t}(v - c_2)^2$$

and the royalty income of firm 2 is

$$\tilde{r}\tilde{D}_1(\tilde{r}) = \frac{1}{8t}(v - c_2)^2$$

Firm 2's profit from its own products can be separately solved from:

$$\max_{p_2} \ (p_2 - c_2)\, \frac{v - p_2}{t}$$

This gives:

$$\tilde{p}_2 = \frac{1}{2}(v + c_2),\ \tilde{D}_1 = \frac{1}{2t}(v - c_2) \text{ and } \tilde{\pi}_2 = \frac{1}{4t}(v - c_2)^2$$

Hence, firm 2's total payoff under royalty licensing is:

$$\tilde{\Pi}_2 = \tilde{\pi}_2 + \tilde{r}\tilde{D}_1(\tilde{r}) = \frac{3}{8t}(v - c_2)^2 \tag{8}$$

Finally, the licensing contract $\tilde{r}$ with local monopoly will be an equilibrium contract if and only if the following *three* conditions hold simultaneously. First, the assumption that each firm has local monopoly requires $2v < \tilde{p}_1 + \tilde{p}_2 + t$, that is,

$$t > \frac{3}{4}(v - c_2) \equiv t^0 \tag{9}$$

Second, the contract on $\tilde{r}$ is acceptable to firm 1 if and only if $\tilde{\pi}_1(\tilde{r}) \geq \pi_1^0$, that is,

$$t \leq \frac{v - c_2}{2\sqrt{2}} + \frac{c_1 - c_2}{3} \equiv t_1 \tag{10}$$

Third, offering the licensing contract $\tilde{r}$ is profitable to firm 2 if and only if $\tilde{\Pi}_2 > \pi_2^0$, that is,

$$t \leq \frac{\sqrt{3}}{2}(v - c_2) - \frac{c_1 - c_2}{3} \equiv t_2 \tag{11}$$

These three conditions (7) through (9) will be satisfied simultaneously if and only if

$$\min\{t_1, t_2\} > t^o \tag{12}$$

But the condition (12) is *never* satisfied (see Appendix A). Hence we have the following proposition.

Proposition 2: In the royalty model discussed above, there exists no equilibrium in which the market is not fully covered.

Further note that $\hat{t} < t^0$. Therefore, in the Hotelling structure if royalty licensing is ever mutually profitable to the firms, it is always optimal to cover the market fully, and such an equilibrium will exist if and only if $t \in (\underline{t}, \hat{t})$.

Proposition 3: In a Hotelling structure with uniform distribution of consumers, the optimal royalty contract is $\hat{r}$, and in equilibrium, the market is fully covered.

Finally note that under spatial competition, with uniform distribution of consumers, there will be no licensing under the fixed fee contract (see Appendix B). But a royalty contract, when it is profitable, leaves a surplus profit for the transferee, because $\hat{\pi}_1(r) > \pi_1^0$. Therefore, the transferor can extract this surplus if it designs a fee plus royalty contract (L, r) for the transferee, where L is the fixed fee. It can then easily be understood that the optimal royalty and fixed fee will be $\hat{r}$ and $\hat{L}$, where

$$\hat{L} = \hat{\pi}_1(r) - \pi_1^0 = \frac{(c_1 - c_2)(6t - c_1 + c_2)}{18t} \tag{13}$$

But now firm 2's profitability condition becomes

$$\hat{\pi}_2(\hat{r}, \hat{L}) = \hat{\pi}_2(\hat{r}) + \hat{L} > \pi_2^0$$

$$\Leftrightarrow LHS(t) \equiv v - c_2 - \frac{3}{2}t > \frac{2(c_1 - c_2)^2}{18t} \equiv \overline{RHS}(t) \tag{14}$$

This is satisfied[5] for all $t \in (\underline{t}, \hat{t}')$ where $\hat{t} < \hat{t}' < \bar{t}$ (see *Figure 1*). This gives the final result of the paper:

Proposition 4: Given $t \in (\underline{t}, \hat{t}')$, the optimal licensing contract under spatial competition is $(\hat{r}, \hat{L})$; there will be no licensing if $t \notin (\underline{t}, \hat{t}')$.

Note that the availability of a fee plus royalty licensing in fact relaxes the constraint of technology transfer, because now the interval of t becomes bigger.

3. Conclusion

We discuss the question of technology licensing under spatial competition. We assume that firms' locations are exogenous and consumers are uniformly distributed over the length of the Hotelling city. In such a model a fixed fee contract is never profitable. On the other hand, we show that a royalty licensing exists if and only if the transport cost lies in a specified interval. But transferor cannot extract all surplus of the transferee by means of a royalty contract only. Hence the optimal licensing contract consists of both fee and royalty. The optimal royalty rate extracts all surplus of the marginal (indifferent) consumer, and the optimal fee takes away transferee's remaining surplus. In equilibrium, however, all consumers buy the monopolistic goods; hence in the post-transfer equilibrium segmented markets will never occur.

[5] Note that $\overline{RHS}(t) \underset{<}{\geq} RHS(t) \Leftrightarrow t \underset{>}{\leq} \frac{(c_1 - c_2)}{6}$.

Appendices

Appendix A:

We have,

$$t^0 = \frac{3}{4}(v-c_2)$$

$$t_1 = \frac{v-c_2}{2\sqrt{2}} + \frac{c_1-c_2}{3} = \frac{3}{4}(v-c_2)\frac{\sqrt{2}}{3} + \frac{c_1-c_2}{3}$$

and

$$t_2 = \frac{\sqrt{3}}{2}(v-c_2) - \frac{c_1-c_2}{3} = \frac{3}{4}(v-c_2)\frac{2\sqrt{3}}{3} - \frac{c_1-c_2}{3}$$

Therefore,

$$t_2 \underset{<}{\overset{>}{-}} t_1 \quad \Leftrightarrow \quad \frac{3}{4}(v-c_2)\frac{(2\sqrt{3}-\sqrt{2})}{2} \;\underset{<}{\overset{>}{-}}\; \frac{c_1-c_2}{3}$$

Finally, one can check that $\min\{t_1, t_2\} < t^0$

Appendix B:

Under the fee contract the market structure will be duopoly with symmetric production technology (with each firm having low unit cost of production). Then the market-operated profits of the firms will be:

$$\pi_1^* = \frac{t}{2} = \pi_2^*$$

Then technology transfer under the fixed fee contract L is mutually profitable if and only if $\pi_2^+ + L > \pi_2^0$ and $\pi_1^* - L \geq \pi_1^0$. Then $\exists L > 0$ if and only if

$$\pi_1^* + \pi_2^* > \pi_1^0 + \pi_2^0$$

But this condition is never satisfied.

References

Hotelling, H. (1929), "Stability in Competition", *Economic Journal 39*, 41-57.

Kabiraj, T. (2005), "Technology transfer in a Stackelberg structure: licensing contracts and welfare", *The Manchester School* 73, 1-28.

Matsumura, T. and N. Matsushima (2008), "On Patent Licensing in Spatial Competition with Endogenous Location Choice". http://www.iss.u-tokyo.ac.jp/~matsumur/LI.pdf.

Mukherjee, A. (2010), "Competition and welfare: the implications of licensing", *The Manchester School* 78, 20-40.

Poddar, S. and U. Sinha (2004), "On Patent Licensing in Spatial Competition", *The Economic Record* 80, 208-218.

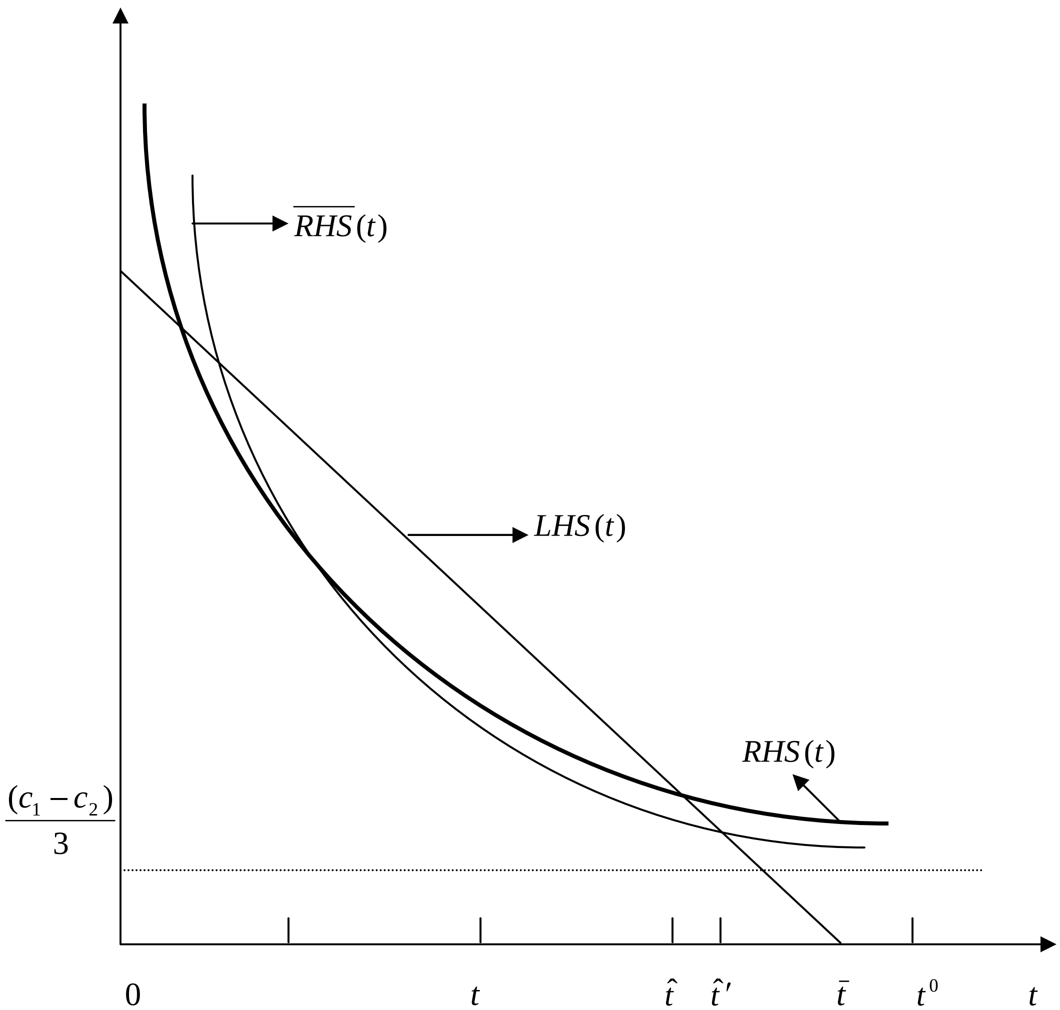
$\overline{RHS}(t)$
$LHS(t)$
$RHS(t)$
$\frac{(c_1 - c_2)}{3}$
0
$\underline{t}$
$\hat{t}$
$\hat{t}'$
$\bar{t}$
t^0
t

Figure 1