Is Effect of Risk on Stock Returns Different in Up and Down Markets? A Multi-Country Study

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Abstract

It has been investigated in the finance literature whether or not the risk associated with any stock market responds differently in two different states of stock market, especially in bull and bear markets. This paper studies this problem in the modelling framework where (*i*) the conditional mean specification considers threshold autoregressive model for two market situations characterized as the up and down markets, (*ii*) the conditional variance (as a measure of time-varying risk) specification is asymmetric in the sense of capturing leverage effect, and (*iii*) the conditional variance directly affects the conditional mean through risk premium term in the risk-return relationship. Using daily returns on stock indices of eight countries comprising four advanced countries -the USA, the UK, Hong Kong, Japan - and four important emerging economies, called the BRIC countries *viz.*, Brazil, Russia, India and China, we have found that the risk aversion parameter is significant in most of the countries, and that it is positive in down market and negative in up market, which supports the Fabozzi and Francis (1977) and Kim and Zumwalt (1979) hypothesis that investors require a premium for taking downside risk and pay a premium for upside variation.

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1 Introduction

Modelling stock market volatility has been the subject of vast theoretical and empirical investigations during the last three decades by academics and practitioners alike. Initially, volatility, as measured by the standard deviation or variance of returns, was often used as a crude measure of the total risk of financial assets. In fact, the well known capital asset pricing model (CAPM) in finance, due to Markowitz (1959), assumed risk to be constant. However, this assumption of constant risk was found to be very restrictive, particularly in the context of financial time series, and researchers started looking for models capturing time varying volatility. This search along with modelling consideration to another important feature of many time series on financial returns, called the 'volatility clustering', led to the seminal work by Engle (1982) where he proposed a model for asset volatility, now well-known as the autoregressive conditional heteroscedastic (ARCH) model. Subsequently, its generalization was suggested by Bollerslev (1986), which is called the generalized ARCH (GARCH) model.

One of the primary restrictions of ARCH/GARCH class of models is that it imposes the symmetric response of volatility to positive and negative shocks. However, it has been argued that a negative shock to financial time series is likely to cause volatility to rise by more than a positive shock of the same magnitude. In the case of equity returns, such asymmetries are typically attributed to leverage effect. As regards models capturing such asymmetry in volatility is concerned, there are two popular such models - one due to Nelson (1991), called the exponential GARCH (EGARCH) model, and the other due to Glosten, Jogannathan and Runkle (1993), known as the threshold GARCH (TGARCH) model.

While there are many models capturing asymmetry in conditional variance, conditional mean models from similar consideration are rather limited. In the context of finance literature, in general, it has long been investigated whether or not the asymmetric risk or beta of the capital asset pricing model(CAPM) responds asymmetrically to good and bad news as measured by positive and negative returns, respectively. More generally, many studies (see, for instance Kim and Zumwalt (1979), Bharadwaj and Brooks (1993), Pettengil, Sundaram and Mathur (1995), Howton and Peterson (1998), Crombez and Vandetr Vennet (2000), and Faff (2001)) have examined the validity of the asset pricing models, especially the CAPM, taking into account the market movements, defined as 'up' and 'down' markets. To classify 'up' and 'down' markets, various definitions have been used. For instance, Kim and Zumwalt (1979) and Chen (1982) have used three threshold levels *viz.*, average monthly market return, average risk free rate and zero. When realized market return is above (below) the threshold level, the market is said to be in the up (down) market state. When the threshold value is taken to be zero in market returns, the up and down markets are often called the bull and bear markets. The overwhelming empirical evidence in empirical studies with different states of market condition is that the CAPM assuming constant risk cannot participate in different market conditions. Levy (1971), and Fabuzzi and Francis (1977) suggested that there is a need to separate betas between bull and bear markets. By defining the bull and bear markets using a threshold model, Kim and Zumwalt (1979) found no evidence to support the beta instability, but concluded that investors should like to receive a positive premium for accepting downside risk, while a negative premium was associated with the up-market beta. Using daily returns data, Granger and Silvafulle (2002) investigated the asymmetric response of beta to different market conditions by modelling the mean and the volatility of CAPM as nonlinear threshold models with three regimes. The results are in accordance with the widely held view that the portfolio beta increases (decreases) when the market is bearish (bullish). Galagedera and Faff (2005) have also investigated whether the risk return relation varies, depending on changing market volatility and up-down market conditions.

To capture such 'asymmetries' or differential effects understood by different market conditions, threshold autoregressive (TAR) models, originally due to Tong (1978), have been used. Such models are linear locally but nonlinear overall. This class of models is characterized by a regime switching mechanism. Several models where such consideration to conditional mean along with volatility, especially of symmetric GARCH kind, have been proposed. For instance, Li and Li (1996) introduced the double threshold GARCH (DTGARCH) model. However, returns models with asymmetry characterized by market conditions like the bull and bear markets along with asymmetry in conditional variance characterized by leverage effect is very limited. In case of the latter, mention may be made of Hagerud (1996) who proposed the smooth transition ARCH model. Gonzalez Rivera (1998) used the smooth transition GARCH (ST-GARCH) model. Lundberg and Teresvirta (1998) used a STAR model for conditional mean and the smooth transition GARCH (STGARCH) which is a modification of the TGARCH model of Glosten *et al.*(1993) for the conditional variance. Brannas and DeGoojer (2004) combined an asymmetric moving average model for the asymmetric mean equation with an asymmetric quadratic GARCH model for the conditional variance equation.

In the literature on 'risk-return' relationship, all the models proposed till the publication of the paper by Engel *et al.* in 1987 were of the kind where risk had no direct effect on returns. They proposed the ARCH-in-mean (ARCH-M) model which incorporated risk premium by introducing volatility directly into the conditional mean equation of returns so that risk, *inter alia*, could affect returns directly. Although such a risk-return relationship is expected to be positive since an increase in risk represented through conditional variance is likely to lead to a rise in expected return, the empirical evidence is somewhat mixed. In respect of correlation behavior of stock returns and subsequent volatility, French, Schwert and Stambaugh (1987) and Champbell and Hentschel (1992) have found the relative risk aversion parameter, the parameter linking volatility to returns in the mean equation, to be positive, while Turner *et al.* (1989) have found it to be negative. As noted by Bekaert and Wu (2000), sometimes this coefficient has been found to be statistically insignificant as well. If the relation between market conditional volatility and the market return is not found to be positive, then the validity of time-varying risk premium is in doubt. However, it might as well be due to the fact that in this theory the relative risk aversion parameter is taken to be constant, which indeed may not be true in some situations, leading to possible error in the estimate of the parameter concerned and consequently of inferences thereof. Therefore, consideration to different market movements[1] in conditional mean as also to asymmetry in conditional variance in terms of behaviour of return shocks in the GARCH-in-mean (GARCH-M) modelling framework, should yield a better understanding of the 'risk-return' relationship. It is important to note that in this framework it is possible to find if the associated risk responds differently to different states of stock markets. In fact, this enables us to test the conclusion of Kim and Zumwalt (1979) that investors would like to receive a positive (negative) premium for accepting down (up) market risk. To the best of our knowledge, there is no such study which considers two different relative risk aversion parameters in this framework.

This paper proposes two such models each with different risk aversion parameters for two different regimes based on average past returns - to be henceforth called as - up and down markets, which are also called bull and bear markets by some definitions (see, for details, Chen (1982), and Chen S. S. (2009)). The conditional variance model is taken to be the EGARCH model. We have also considered a similar model with (symmetric) GARCH specification for conditional variance. One of the proposed models considers smooth transition mechanism in the conditional mean process, which was proposed by Chen and Tong (1986) and Teresvirta (1993). The proposed models along with two other existing models *viz.*, the AR-GARCH-M and AR-EGARCH-M, which are taken as benchmark models, have been fitted to individual time series of stock returns of eight countries comprising four advanced economies - the USA, the UK, Hong Kong, and Japan - and four important emerging economies - called the BRIC group - Brazil, Russia, India, and China. The benchmark models have been considered in order to find to what extent the performances of the proposed models improve with the introduction of (*i*) up and down movements of the stock market, and (*ii*) two different risk aversion parameters for these two states.

The organization of the paper is as follows. In Section 2, the proposed models are introduced. The empirical results are presented and discussed in Section 3. The paper ends with some concluding remarks in Section 4.

[1]There are some studies where asymmetry in mean in terms of return shocks , which is also called the asymmetric reverting behaviour in return dynamics, have been considered while specifying the model for conditional mean (see, for details, Nam *et al.* (2001), and Nam (2003), Kulp-Tåg (2007)).

2 The Proposed Models

As discussed in the preceding section, all the three proposed models for return, r_t, have the following conditional mean specification[2]:

$$r_t = \begin{cases} \mu_1 + \phi_1 r_{t-1} + \delta_1\sqrt{h_t} + \epsilon_t, & \text{if } \bar{r}_t{}^k \le 0 \\ \mu_2 + \phi_2 r_{t-1} + \delta_2\sqrt{h_t} + \epsilon_t, & \text{if } \bar{r}_t{}^k > 0 \end{cases} \tag{1}$$

where $\varepsilon_t = \nu_t\sqrt{h_t}$ with $\varepsilon_t|\psi_{t-1} \sim N(0, h_t)$, ν_t is independently and identically distributed $N(0,1)$, ψ_{t-1} is the information or conditioning set up to time $t-1$, h_t is the conditional variance at time t and $\bar{r}_t{}^k$ is as defined below.

In the literature on threshold autoregressive (TAR) model, the choice of threshold variable is often taken to be a past value of its own. In our case, the threshold variable, as stated in Section 1, is taken to be $\bar{r}_t{}^k$, where $\bar{r}_t{}^k$ is the average of the past k returns i.e., $\bar{r}_t{}^k = \frac{\sum_{i=1}^{k} r_{t-i}}{k}$. Obviously, appropriate choice of k is a relevant issue. What we have done is to make several choices of k and then choose the one for which the underlying likelihood value for our model is maximum.

Combining the two conditional mean specifications stated in equation(1), we can write the model as

$$r_t = (\mu_1 + \phi_1 r_{t-1} + \delta_1\sqrt{h_t})D(\bar{r}_t{}^k \le 0) + (\mu_2 + \phi_2 r_{t-1} + \delta_2\sqrt{h_t})(1 - D(\bar{r}_t{}^k > 0)) + \varepsilon_t \tag{2}$$

where $D(.)$ is an indicator function taking value 1 if $\bar{r}_t{}^k \le 0$ and 0 otherwise.

Little is known about the rigorous derivations of the conditions for stationarity of nonlinear time series models. In general Chang and Tong (1985) have shown that a sufficient condition for stationarity of TAR model is $\max(|\phi_1|, |\phi_2|) < 1$. Chen *et al.* (1985) has derived less restrictive sufficient conditions. The conditions on the intercepts μ_1 and μ_2 are such that the time series has a tendency to revert to the stationary regime, and the time series is globally stationary. As noted by Franses and van Dijk (2004), a 'rough and ready' check for nonstationarity of nonlinear time series model, in general, is to find if the skeleton is stable. This intuitively means that if the time series tends to explode for certain initial values of the parameters, then the series is nonstationary, and this can be checked by simulation.

The first proposed model, called the TAR-GARCH-M, assumes h_t to have the GARCH(1,1) specification[3] i.e.,

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1} \tag{3}$$

where $\omega > 0$, $\alpha \ge 0$, $\beta \ge 0$, and $\alpha + \beta \le 1$. In case of the second model, designated as the TAR-EGARCH-M, h_t is taken to be the EGARCH(1,1) model[3] which is given by

$$\ln(h_t) = \omega + \alpha_1\left(\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}\right) + \gamma_1\left[\frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}} - E\left(\frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}}\right)\right] + \beta_1\ln(h_{t-1}). \tag{4}$$

Unlike (symmetric) GARCH model, no restrictions on parameters of EGARCH model are needed to be imposed to ensure that h_t is positive. Note that for EGARCH$(1,1)$, if $0 < \gamma_1 < 0$, then the process generates volatility clustering while this condition along with $\alpha_1 < 0$ delivers a leverage effect since under the restrictions, negative shock has a leverage effect on the conditional variance than positive shock of the same size.

This model assumes that the border between the two regimes signifying change in mean is given by a specific value of the threshold variable, which is 0 for our model. A more gradual transition between different regimes can be obtained by replacing the indicator function $D(\bar{r}_t{}^k \le 0)$ in equation (2) by a continuous function $G(\bar{r}_t{}^k, \gamma)$ which changes smoothly from 0 to 1 as $\bar{r}_t{}^k$ increases. The resulting model based on this transition mechanism was first introduced by Teresvirta (1994) in the context of capturing regime switching behaviour of financial variables, and it is called the smooth

[2]The model has been specified with one lag only since this was found to be adequate for all the return series. Further, in specifying the risk premium term, $\sqrt{h_t}$ has been considered although other functional forms like h_t and $\ln h_t$ can as well be taken.

[3]The order $(1,1)$ for the GARCH/EGARCH model was found to be adequate for all the return series, and hence the model has been specified for this order only.

transition autoregressive (STAR) model. While this model has been extensively used in capturing regime switching behaviour, it is recently being also applied to incorporate asymmetry in conditional variance through regime-shifting consideration (see Nam *et al.* (2001, 2003), for instance).

We propose our last model by considering the STAR model for the conditional mean i.e., by replacing the indicator function in equation (2) with a popular smooth transition function *viz.*, the logistic function, for the conditional mean and assuming, as before, the EGARCH(1,1) specification, for the conditional variance. This model, called the smooth transition autoregressive model with EGARCH-in-mean (STAR-EGARCH-M), is represented as

$$r_t = (\mu_1 + \phi_1 r_{t-1} + \delta_1 \sqrt{h_t})(1 - G(\bar{r_t}^k;\gamma)) + (\mu_2 + \phi_2 r_{t-1} + \delta_2 \sqrt{h_t})G(\bar{r_t}^k;\gamma) + \varepsilon_t \tag{5}$$

and the specification of h_t is as given in equation (4) where $G(\bar{r_t}^k;\gamma) = \frac{1}{(1+exp^{-\gamma\bar{r_t}^k})}$. This transition function takes a value between 0 and 1 depending on the value of $\bar{r_t}^k$ i.e., $0 < G(\bar{r_t}^k;\gamma) < 0.5$ for $\bar{r_t}^k < 0$, $G(\bar{r_t}^k;\gamma) = 0.5$ for $\bar{r_t}^k = 0$ and $0.5 < G(\bar{r_t}^k;\gamma) < 1$ for $\bar{r_t}^k > 0$. The parameter γ determines the smoothness of the change in the value of the logistic function, and thus the transition from one regime to the other. When the parameter γ approaches 0, yielding $G(\bar{r_t}^k;\gamma) \simeq 0.5$, the STAR-EGARCH-M model reduces to a simple AR-EGARCH-M model. When, on the other hand, γ approaches infinity, the transition function becomes $G(\bar{r_t}^k;\gamma) = 1$ for $\bar{r_t}^k > 0$ and $G(\bar{r_t}^k;\gamma) = 0$ for $\bar{r_t}^k \leq 0$ so that it reduces to an indicator function.

3 Empirical Results

In this section we report as well as discuss the results of estimation of all the models considered in this paper. We begin with describing the data sets used and also the results of some standard statistical tests which were carried out to find the characteristics of these time series.

3.1 Data and summary statistics

Stock index data at daily frequency for eight countries - four from advanced countries and four from important emerging economics - have been considered for this study. Specifically, the countries in these two groups are the USA, the UK, Hong Kong and Japan for the advanced economics, and Brazil, Russia, India and China for the important emerging economies. Most of the empirical studies on such models have been carried out with returns on stock indices of advanced economies. This study extends the horizon by including four important emerging economies as well, which are also known as the BRIC group of countries. The distinctive advantage of such a multi-country study is that these are based on the same models, methodologies, time periods and data frequency. The choice of these two groups of countries has been made from consideration of the fact that stock markets of developed economies are well developed, structured, have time-tested, well-established trading rules and also strong regulatory authorities. For the emerging economics, there are some differences and deficiencies in respect of some of these. Hence, the investors' sentiments and reactions may be somewhat different in terms of risk-return relationship, especially in case of different market conditions. We have made this choice with the purpose of examining empirically if the findings on the nature of this relationship turns out to be similar or different for these two groups of countries.

In all these countries, more than one index on stock prices are available. We have, however, taken only one index for each country. Accordingly, the stock indices considered are: BOVESPA (for Brazil), MICEX (for Russia), SENSEX (for India), SSE COMPOSITE (for China), S&P 500 (for the US), FTSE ALL (for the UK), HANG SENG (for Hong Kong) and NEKKIE 225 (for Japan). All the time series have been downloaded from the official website of Yahoo Finance (http://finance.yahoo.com/). The time period for all the time series is from 01 January 2000 to 31 December 2012. The total number of observations are not the same for all the eight series because of varying number of holidays in different countries when stock markets remain closed. Thus, SSE COMPOSITE of China has the highest number of observations 3329, while the lowest 3191 is for the NEKKIE 225 of Japan. All stock

Table 1: Summary statistics of daily stock returns on the eight stock markets

	Brazil	Russia	India	China	The USA	The UK	Hong Kong	Japan
Mean	0.0399	0.0661	0.0396	0.0152	-0.0006	-0.0004	0.0082	-0.0189
Median	0.0937	0.1526	0.1117	0.0000	0.0488	0.0402	0.0286	0.0051
Maximum	13.68	25.23	15.99	9.40	10.96	8.81	13.40	13.23
Minimum	-12.10	20.66	-11.81	-9.26	-9.47	-8.70	-13.58	-12.11
Std. dev.	1.9048	2.3290	1.6517	1.5882	1.3508	1.2385	1.6158	1.5667
Skewness	-0.0953	-0.1960	-0.1777	-0.0831	-0.1584	-0.1767	-0.0657	-0.3933
Kurtosis	6.6994	15.4304	9.3485	7.5141	10.3268	8.7030	10.5496	9.6856
J-B	1837.04	2086.92	5466.37	2829.46	7323.49	4479.70	7703.89	6023.19
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ADF	-55.90	-54.57	-53.12	-57.45	-44.89	-29.40	-57.93	-58.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$Q(5)$	13.8192	12.1271	21.5550	10.2975	42.0542	48.4091	7.5071	7.0895
	(0.02)	(0.03)	(0.00)	(0.07)	(0.00)	(0.00)	(0.19)	(0.21)
$Q(10)$	23.1694	15.7780	38.5307	18.9498	49.1160	64.2120	17.7793	14.1138
	(0.01)	(0.11)	(0.00)	(0.04)	(0.00)	(0.00)	(0.06)	(0.17)
$Q^2(5)$	943.6102	560.6082	522.6123	250.7641	1340.2018	1302.9052	1351.6053	1617.4151
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$Q^2(10)$	1903.1032	835.0211	845.1342	454.6938	2609.1054	2122.8019	2026.8029	2589.5043
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
D_{max}	6.82	7.62	14.37	11.46	14.66	14.67	8.56	5.31
WD_{max}	9.89	12.34	18.18	14.49	14.66	18.57	10.17	9.44
No. of obs.	3214	3242	3246	3329	3269	3294	3244	3191

Figures in parentheses indicate $p-$ values. J-B stands for the Jarque-Bera normality test. Unit root test is based on augmented Dickey-Fuller test with linear trend and intercept terms.

price indices are taken in logarithmic values and then computed in percentages i.e., $r_t = \ln(\frac{p_t}{p_{t-1}}) \times 100$, where p_t is the stock price index of a country on the t^{th} day.

The summary statistics on the returns of the eight time series are presented in Table 1. The mean returns for all the BRIC countries are positive while all but returns on the HANG SENG of Hong Kong are negative. The skewness coefficients for all the return series have negative values, although small in magnitude, indicating that all the returns distributions are skewed to the left with Japan having the maximum asymmetry in distribution. All the kurtosis values are higher than 3 with the maximum being 15.4304 for MICEX of Russia. Consequently, the J-B test statistic values reject the assumption of normality strongly for all the series.

All the series were found to have unit roots at their level i.e., stock price, values by the augmented Dicky Fuller (ADF) test. However, the ADF test on returns confirmed that all the return series were stationary. It is evident from the ADF test statistics values given for return series in Table 1 that all the return series are stationary. In the ADF estimating equation, the linear deterministic trend term was statistically insignificant for all the series. Also, all the return series were found to have (linear) autocorrelations as well as squared autocorrelation, as exhibited by the values of $Q(5)$, $Q(10)$ and $Q^2(5)$ and $Q^2(10)$ test statistics in the table.

Finally, the structural stability of each of the series during the sample period was examined by carrying out the Bai-Perron test (1998, 2003) for multiple structural breaks. This test envisages a two step procedure. In the first step, Bai and Perron proposed test statistics, called the D_{max} and WD_{max}, for testing the null hypothesis of 'no structural break' against the alternative of 'the number of breaks/changes is arbitrary/unknown, but upto some specified maximum'. In case these tests reject the null hypothesis of 'no structural break', then in the second stage a sequential test procedure designated as $SupF(2|1)$, $SupF(3|2)$, *etc.* is to be applied. It is evident from the D_{max} test statistic values presented in Table 1 that at 1% level of significance the computed values are less than the critical value of 5.4 for all the eight return series, leading to the conclusion that the null hypothesis of 'no structural break' cannot be rejected, and hence all the series are structurally stable. The conclusion by the WD_{max} test remains the same for all but SENSEX (India) and FTSE ALL (the UK) indices. For this two series, the test statistic values were found to have just exceeded the critical value of 17.01 at 1% level of significance. Combing these findings, it can be concluded that all the eight series are structurally stable.

3.2 Findings on the models

In this study, we have considered, in all, five models and obtained the ML estimates of the parameters involved. Besides the three proposed models - TAR-GARCH-M, TAR-EGARCH-M and STAR-EGARCH-M - the two others are AR-GARCH-M and AR-EGARCH-M. The two latter models refer to two simple models in 'volatility-in-mean' framework, which may as well be considers as benchmark models, where the conditional mean model has no consideration to stock market movements and the conditional volatility model is taken to be GARCH and EGARCH for the two models, respectively. These two models, therefore, are special cases of the models proposed in this paper. These models are considered to find to what extent consideration to up and down market movements in conditional mean with and/or asymmetry in conditional variance captured through EGARCH model leads to improvement in explaining return dynamics, and the consequent impact of the same on risk aversion parameter and hence on risk premium.

Assuming the distributional assumption about ε_t to be normal i.e., $\varepsilon_t|\psi_{t-1} \sim N(0, h_t)$, the likelihood function is written which is obviously highly nonlinear. Imposing stationarity conditions, the programmes for obtaining the ML estimates of the parameters of these models, have been written in GAUSS[4]. The computations involved turned out to be complicated and substantial. In all cases, global convergence of the underlying nonlinear objective function has been achieved. The parametric restrictions imposed from consideration of stationarity and finite unconditional variance of GARCH i.e., $-1 < \phi_1, \phi_2 < 1$, $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ and $\alpha + \beta < 1$ were taken into the algorithms. The ML estimates thus obtained for all the models are reported and discussed below.

We first report the estimates of the AR-GARCH-M[5] model for all the eight return series in Table 2. In the context of our study, this is the simplest model since it considers neither different market movements like up and down for conditional mean nor leverage effect in conditional variance.

Table 2: Estimates of the parameters of AR(1)-GARCH(1,1)-M model

Para	Brazil	Russia	India	China	the US	the UK	Hong Kong	Japan
μ	-0.1506	0.1290	0.0941	-0.1119	-0.0078	-0.0011	0.0129	0.0536
	(0.23)	(0.20)	(0.17)	(0.16)	(0.87)	(0.97)	(0.83)	(0.47)
ϕ	0.0112	0.0238	0.0797	0.0036	-0.0586	-0.0450	0.01498	-0.0044
	(0.54)	(0.20)	(0.00)	(0.84)	(0.00)	(0.01)	(0.41)	(0.81)
δ	0.1421	0.0018	0.0048	0.1030	0.0597	0.0568	0.0345	-0.0106
	(0.06)	(0.97)	(0.93)	(0.09)	(0.22)	(0.21)	(0.50)	(0.86)
ω	0.0655	0.0902	0.0551	0.0288	0.0151	0.0132	0.0142	0.0417
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α	0.0720	0.1056	0.1291	0.0625	0.0864	0.1141	0.0671	0.1052
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	0.9086	0.8767	0.8527	0.9267	0.9043	0.8799	0.9271	0.8794
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
MLLV	-6312.9611	-6706.2816	-5684.6883	-5895.7102	-4862.3414	-4679.8743	-5529.8469	-5527.3343

In Table 2, we specify the following models for r_t and h_t:

$$r_t = \mu + \phi r_{t-1} + \delta\sqrt{h_t} + \epsilon_t \qquad , \epsilon_t|\psi_{t-1} \sim N(0, h_t)$$

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}$$

The entries in the parentheses are the $p-$ values; MLLV is the maximized log-likelihood value.

It is noted that the intercept μ is statistically insignificant for all the series. The first-order autocorrelation coefficient ϕ is significant only for three returns series *viz.*, SENSEX for India, S&P 500 for the USA and FTSE ALL for the UK. The coefficients of GARCH model are significant at 1%

[4]Codes available from the website of P. Perron, http://people.bu.edu/perron/, have been used for carrying out tests for multiple structural breaks

[5]Since the order of the AR model for the conditional mean has been found to be 1 and the orders of GARCH/EGARCH model for the conditional variance have been obtained as (1,1) for all the eight return series, as discussed in this section, nowhere in the running text these orders are mentioned.

level for all the return series indicating presence of strong volatility in all the series. From consideration of risk-return relationship, the risk aversion parameter, δ, is the most important one. And the findings on δ are somewhat unexpected since for all the return series, δ is statistically insignificant meaning thereby that time-varying risk does not directly influence returns irrespective of whether the stock markets refer to advanced or BRIC group of emerging economies. This has been observed by others as well. For instance, Bekaert and Wu (2000) have stated that it is often found that the coefficient linking volatility to return is statistically insignificant. Of course, if the relation between market conditional volatility and market expected return is not positive, then the validity of the time-varying risk premium set-up is in doubt. Such a finding for returns on all the eight stock markets raises the question of asymmetry in volatility not being duly considered in the analysis, especially because leverage effect is so common and prevalent in stock markets.

Table 3: Estimates of the parameters of AR(1)-EGARCH(1,1)-M model

Para	Brazil	Russia	India	China	the US	the UK	Hong Kong	Japan
μ	-0.0947	0.0152	0.0867	-0.0365	0.0409	0.0077	0.0733	0.0866
	(0.42)	(0.09)	(0.18)	(0.63)	(0.30)	(0.83)	(0.21)	(0.23)
ϕ	0.0263	0.0161	0.0907	0.0047	-0.0541	-0.0292	0.0235	-0.0085
	(0.15)	(0.38)	(0.00)	(0.79)	(0.00)	(0.10)	(0.19)	(0.68)
δ	0.0771	-0.0419	-0.0318	0.0562	-0.0434	-0.0001	-0.0432	0.0770
	(0.28)	(0.43)	(0.54)	(0.34)	(0.34)	(0.99)	(0.39)	(0.19)
ω	0.0312	0.0438	0.0306	0.0180	0.0046	-0.0005	0.0107	0.0218
	(0.00)	(0.00)	(0.00)	(0.00)	(0.07)	(0.84)	(0.00)	(0.00)
α	-0.0819	-0.0457	-0.0997	0.0192	-0.1274	-0.1247	0.0628	-0.0980
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
λ	0.1284	0.2246	0.2431	0.1207	0.1067	0.1224	0.1301	0.1867
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	0.9724	0.9733	0.9631	0.9873	0.9832	0.9830	1.9869	0.9697
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
MLLV	-6284.1864	-6712.2342	-5660.9226	-5893.7096	-4791.0282	-4606.7713	-5497.6119	-5490.2236

In Table 3 we specify the following models for r_t and h_t:

$$r_t = \mu + \phi r_{t-1} + \delta\sqrt{h_t} + \epsilon_t, \qquad \epsilon_t|\psi_{t-1} \sim N(0, h_t)$$

$$\ln h_t = \omega + \alpha\left(\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}\right) + \lambda\left[|\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}| - \frac{\sqrt{2}}{\pi}\right] + \beta \ln h_{t-1}$$

The entries in the parentheses are the $p-$ values; MLLV is the maximized log-likelihood value.

The estimation results for the AR-EGARCH-M model where instead of GARCH, conditional variance is taken to be the EGARCH model are presented in Table 3. It is noted from this table that the first-order autocorrelation coefficient is significant only for three series i.e., SENSEX, S&P 500 and FTSE ALL, which are, as expectedly, the same as obtained in case of AR-GARCH-M model. From the estimates of the parameters of the EGARCH model, it is found that all the four parameters i.e., ω, α, λ and β are significant for all the eight series. The parameter α has been found to be negative for all but returns on SSE COMPOSITE of China and HANG SENG of Hong Kong. For these two countries $\hat{\alpha}$ has been found to be 0.0192 and 0.0628, respectively. Thus, while asymmetry in volatility in returns of all the eight developed and emerging economies have been empirically found, it may be worthwhile to note that the nature of this asymmetry is somewhat mixed, as also noted by Bekaert and Wu (2000). To be more specific, in six out of eight, the relation between current volatility and past returns is found to be negative, which is indeed often the case (see, for instance, Turner *et al.* (1989), Glosten *et al.* (1993), and Nelson (1991)), for the remaining two, it is positive. While the explanations for observed negative correlations are provided in terms of leverage effect and volatility feedback, the same for the positive correlation is given in terms of time-varying risk premium theory.

Table 4: Estimates of the parameters of TAR(1)-GARCH(1,1)-M model

Para	Brazil	Russia	India	China	US	UK	Hong Kong	Japan
μ_1	-0.4201	-0.1394	0.1304	-0.1087	0.0023	-0.0649	-0.0812	-0.09992
	(0.02)	(0.37)	(0.22)	(0.35)	(0.97)	(0.31)	(0.37)	(0.41)
ϕ_1	-0.0031	0.0066	-0.0152	-0.0643	-0.0659	-0.0311	-0.0155	-0.0177
	(0.90)	(0.81)	(0.76)	(0.10)	(0.01)	(0.24)	(0.56)	(0.52)
δ_1	0.3024	0.1342	-0.0910	0.0643	0.0768	0.1433	0.0928	0.1066
	(0.01)	(0.10)	(0.37)	(0.50)	(0.28)	(0.03)	(0.22)	(0.25)
μ_2	0.1032	0.3369	0.1528	-0.0452	0.0138	0.0566	0.0895	0.1759
	(0.54)	(0.10)	(0.10)	(0.68)	(0.81)	(0.26)	(0.27)	(0.09)
ϕ_2	0.0340	0.0435	0.1712	0.1188	-0.0347	-0.0478	0.0430	0.0248
	(0.23)	(0.01)	(0.00)	(0.00)	(0.22)	(0.09)	(0.14)	(0.40)
δ_2	-0.0267	-0.1317	-0.1059	-0.0332	0.0090	-0.0256	-0.0343	-0.1261
	(0.80)	(0.10)	(0.22)	(0.72)	(0.89)	(0.69)	(0.63)	(0.14)
ω	0.0670	0.0917	0.0570	0.0280	0.0154	0.0136	0.0148	0.0448
	(0.00	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α	0.0720	0.1058	0.1314	0.0609	0.0869	0.1154	0.0670	0.1072
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	0.9079	0.8757	0.8496	0.9281	0.9034	0.8784	0.9266	0.8759
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
MLLV	-6310.52	-6702.94	-5680.57	-5884.76	-4861.36	-4679.05	-55299.00	-5527.05

In Table 4, we specify the following models for r_t and h_t:

$$r_t = \begin{cases} \mu_1 + \phi_1 r_{t-1} + \delta_1\sqrt{h_t} + \epsilon_{1t}, & \text{for down market} \\ \mu_2 + \phi_2 r_{t-1} + \delta_2\sqrt{h_t} + \epsilon_{2t}, & \text{for up market} \end{cases} \qquad \epsilon_t, |\psi_{t-1} \sim N(0, h_t)$$

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}$$

The entries in parentheses are the $p-$ values; MLLV is the maximized log-likelihood value.

Insofar as the risk aversion parameter δ is concerned, the findings are exactly the same as for the AR-GARCH-M model *viz.*, this parameter is insignificant for all the eight return series. Thus, the conclusion, based on these two benchmark models, is that there is no direct effect of risk on the expected returns for all the eight series. Since GARCH and EGARCH models are not nested, it cannot be formally tested if asymmetry in volatility represents a statistically better risk-return relationship. However, looking at the maximized log-likelihood values, it is noted that except for returns on Russia's stock market index MICEX, there is substantial improvement in the likelihood values for all other returns series, and, to that extent the AR-EGARCH-M model is a better model for returns that the AR-GARCH-M model.

We now consider the first proposed model *viz.*, the TAR-GARCH-M model where two regimes based on positive and non positive past average returns - called up and down market movements - are considered for the conditional mean model. As mentioned in Section 2 the two regimes - up and down- have been chosen based on the value of $\bar{r_t}^k$, the average of k past returns, being > 0 and ≤ 0, respectively. As regards the choice of k, we have considered several values, especially because the data are at daily frequency. Thus , starting with $k = 5$, the values were considered with small gaps *viz.*, $k = 7$, 10, 15, 20, 25, 30 and finally with big jumps till up to $k = 150$ i.e., 50, 75, 100 and 150. For each of the choices, this model was estimated and that value of k was finally chosen for which the log-likelihood value was found to be the maximum. It may be noted that the maximized likelihood value was found to be almost the same for $k > 20$ for all the series. The values of k for the eight return series were thus obtained as 20 for returns on BOVESPA, SENSEX, SSECOMPOSITE,S&P 500, 15 for returns on MICEX, 10 for returns on FTSE ALL and 5 for returns on HANG SENG and NEKKIE 225.

From the estimates of this model, which are presented in Table 4, it is noted that with the introduction of two regimes from consideration to market conditions and specification of the model for conditional mean accordingly, the findings are now found to be somewhat different. The autocorrelation coefficient for the down state, ϕ_1, is found to be significant only for S&P 500 returns, while for the other i.e., the up state, ϕ_2 is significant for Russia, India, China and the UK. The parameters of the

GARCH model are found to be statistically significant for all the returns, as expectedly. As regards the two risk aversion parameters, δ_1 and δ_2, for the two mean regimes, δ_1 is found to be significant for the stock returns on Brazil and the UK while δ_2 is significant for Russia only, although at 10% level of significance for the latter. Thus, it is found that with introduction of regimes characterized by up and down movements of stock market in conditional mean and the time-varying risk being represented by GARCH, the expected return is found to be significantly influenced by (symmetric) volatility directly, unlike in the first two models. There is also some improvement as compared to the AR-GARCH-M model in terms of maximized log-likelihood value for most of the stock indices, thus suggesting that consideration of market conditions in mean model is useful. However, similar improvement does not occur if comparison is made with the AR-EGARCH-M model. This indicates that while volatility captured by the GARCH model is statistically valid, it is not the appropriate model for the volatility for all the series, and hence even with the introduction of market condition, there is no improvement in terms of maximized log-likelihood values.

Therefore, we now consider the other two proposed models - TAR-EGARCH-M and STAR-EGARCH-M – the conditional mean specification of which are specified in equations (2) and (5), respectively, and the conditional variance is given by the EGARCH model (equation (4)) for both. Empirical results of these two models are presented in Tables 5 and 6, respectively. It is noted first that all the three parameters of the EGARCH model *viz.*, α, λ and β are significant as in case of AR-EGARCH-M model. But the estimates of α is now negative for all the eight return series establishing thereby the presence of leverage effect in the returns of all eight stock markets considered in this study. The most significant observation, however, is that either of the two risk aversion parameters referring to the two market movements considered, δ_1 and δ_2, is significant for all but one return series i.e., those on BOVESPA, SENSEX, SSE COMPOSITE, S&P 500, FTSE ALL, HANG SENG and NEKKIE 225 stock indices. It is only for the returns on MICEEX index of Russia that both δ_1 and δ_2 have been found to be significant. It is thus clearly established that the proposed TAR-EGARCH-M model is a better model in representing risk-return relationship since unlike the others models, at least one of the two risk aversion parameters is significant in all the stock returns. Further, $\hat{\delta_1}$ has been found to be positive for all the five - Brazil, Russia, India, China and the USA - stock returns which are significant while $\hat{\delta_2}$ is negative for all the four return series *viz.*, MICEX, SSE COMPOSITE, HANG SENG and NEKKIE 225, where δ_2 has been found to be significant. In terms of the two regimes characterized by two different market conditions – up and down - these findings suggest that response of risk (as measured by conditional variance) to these two market conditions is asymmetric since we have found that in the down market, which is the unfavourable market, expected return is higher in direct response to higher volatility while for up market, higher risk has been found to lead to lower expected return. Thus, not only that the risk aversion parameter are now found to be significant in one or both the market states, but also its signs for these two market conditions have been found to be different, as expected. The empirical findings also show that there is no substantial difference in the nature of the risk-return behaviour of investors belonging to advanced and BRIC countries from consideration of up and down markets.

Table 5: Estimates of the parameters of TAR(1)-EGARCH(1,1)-M model

Para	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
μ_1	-0.6719	-0.3300	0.0712	-0.3025	-0.3081	-0.1548	-0.1012	-0.1062
	(0.00)	(0.02)	(0.49)	(0.00)	(0.00)	(0.02)	(0.28)	(0.38)
ϕ_1	0.0041	-0.0035	-0.0107	-0.0096	-0.0821	-0.0248	-0.0254	-0.0328
	(0.88)	(0.90)	(0.84)	(0.70)	(0.00)	(0.36)	(0.35)	(0.29)
δ_1	0.3242	0.1694	-0.0916	0.1810	0.1940	0.1134	0.0551	0.0558
	(0.00)	(0.01)	(0.36)	(0.02)	(0.00)	(0.07)	(0.45)	(0.53)
μ_2	0.0338	0.3551	0.2209	0.1087	0.0793	0.0420	0.1898	0.2001
	(0.83)	(0.00)	(0.01)	(0.31)	(0.10)	(0.35)	(0.01)	(0.04)
ϕ_2	0.0423	0.0233	0.1926	-0.0023	-0.0270	-0.0448	0.0514	0.0292
	(0.08)	(0.32)	(0.00)	(0.93)	(0.27)	(0.07)	(0.06)	(0.30)
δ_2	0.0232	-0.1586	-0.2142	-0.0155	-0.0711	-0.0120	-0.1372	-0.1976
	(0.82)	(0.02)	(0.01)	(0.85)	(0.24)	(0.84)	(0.06)	(0.02)
ω	0.0336	0.0450	0.0300	0.0188	0.0032	-0.0012	0.0099	0.0223
	(0.00)	(0.00)	(0.00)	(0.00)	(0.25)	(0.66)	(0.00)	(0.00)
α	-0.1020	-0.0502	-0.1045	-0.0268	-0.1446	-0.1365	-0.0672	-0.1023
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
λ	0.1126	0.2209	0.2422	0.1262	0.0789	0.1145	0.1251	0.1830
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	0.9698	0.9719	0.9629	0.9863	0.9817	0.9814	0.9870	0.9690
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
MLLV	-6274.58	-6704.53	-5655.56	-5877.21	-4776.91	-4603.53	-5493.31	-5488.43

In Table 5, we specify the following models for r_t and h_t:

$$r_t = \begin{cases} \mu_1 + \phi_1 r_{t-1} + \delta_1\sqrt{h_t} + \epsilon_{1t}, & \text{for down market} \\ \mu_2 + \phi_2 r_{t-1} + \delta_2\sqrt{h_t} + \epsilon_{2t}, & \text{for up market} \end{cases}, \qquad \epsilon_t|\psi_{t-1} \sim N(0, h_t)$$

$$\ln h_t = \omega + \alpha\left(\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}\right) + \lambda\left[|\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}| - \frac{\sqrt{2}}{\pi}\right] + \beta \ln h_{t-1}.$$

The entries in the parentheses are the $p-$values; MLLV is the maximized log-likelihood value.

The estimates of the last model considered *viz.*, the STAR-EGARCH-M are presented in Table 6. This model is different from the TAR-EGARCH-M model in that now there is a smooth transition mechanism from one market condition to the other through the assumption of logistic function. The two models are otherwise the same. It is obvious from the entries of this table that the empirical results are almost the same, the values of maximized log-likelihood is also almost the same as that for the TAR-EGARCH-M model for each of the eight stock returns. This is so because the estimate of γ , the parameter of smoothness in the logistic transition function, has been found to be very high for all the stock returns except for returns on SENSEX of India and SSE COMPOSITE of China. As already stated in Section 2, under this condition, the transition is almost instantaneous at $\bar{r}_t^{\,k} = 0$ and hence the STAR model for conditional mean reduces to the TAR model. Accordingly, no further discussions of the findings of this model are made.

Table 6: Estimates of parameters of STAR(1)-EGARCH(1,1)-M model

Para	Brazil	Russia	India	China	The US	The UK	Hong Kong	Japan
μ_1	-0.6769	-0.3281	-0.0402	-0.3761	-0.3079	-0.1747	-0.1152	-0.1090
	(0.00)	(0.02)	(0.84)	(0.00)	(0.00)	(0.01)	(0.18)	(0.36)
ϕ_1	0.0035	-0.0028	-0.0587	-0.0132	-0.0830	-0.0232	-0.0264	-0.0329
	(0.90)	(0.92)	(0.49)	(0.60)	(0.01)	(0.41)	(0.36)	(0.28)
δ_1	0.3241	0.1677	-0.0743	0.2183	0.1910	0.1249	0.0647	0.0580
	(0.00)	(0.01)	(0.58)	(0.01)	(0.00)	(0.05)	(0.35)	(0.51)
μ_2	0.0184	0.3467	0.3913	0.1505	0.0778	0.0513	0.2063	0.1971
	(0.91)	(0.00)	(0.06)	(0.21)	(0.10)	(0.26)	(0.01)	(0.04)
ϕ_2	0.0436	0.0240	0.1409	0.0036	-0.0261	-0.0454	0.0582	0.0279
	(0.09)	(0.30)	(0.07)	(0.88)	(0.30)	(0.07)	(0.05)	(0.33)
δ_2	0.0350	-0.2426	-0.2426	-0.0278	-0.0688	-0.0234	0.1538	-0.1939
	(0.73)	(0.03)	(0.04)	(0.75)	(0.25)	(0.70)	(0.03)	(0.02)
ω	0.0339	0.0450	0.0302	0.0188	0.0032	-0.0011	0.0098	0.0223
	(0.00)	(0.00)	(0.00)	(0.00)	(0.20)	(0.68)	(0.00)	(0.00)
α	-0.1030	-0.0503	-0.1048	-0.0289	-0.1453	-0.1374	-0.0675	-0.1023
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
λ	0.1126	0.2208	0.2427	0.1255	0.0785	0.1130	0.1248	0.1830
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β	0.9694	0.9719	0.9629	0.9862	0.9817	0.9815	09871	0.9689
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
γ	18.85	341.57	1.50	0.71	45.13	26.36	13.70	500.01
	(0.42)	(0.47)	(0.13)	(0.12)	(0.38)	(0.37)	(0.36)	(0.49)
MLLV	-6274.42	-6704.73	-5654.42	-5875.65	-4776.62	-4602.71	-5492.66	-5488.52

In Table 6, we specify the following models for r_t and h_t:

$$r_t = (\mu_1 + \phi_1 r_{t-1} + \delta_1 \sqrt{h_t})(1 - G(\bar{r_t}^k;\gamma,c)) + (\mu_2 + \phi_2 r_{t-1} + \delta_2 \sqrt{h_t})(G(\bar{r_t}^k;\gamma,c)) + \epsilon_t, \qquad \epsilon_t|\psi_{t-1} \sim N(0,h_t)$$

$$G(\bar{r_t}^k;\gamma,c) = \frac{1}{1+\exp(-\gamma[\bar{r_t}^k - c])}$$

$$\ln h_t = \omega + \alpha\left(\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}\right) + \lambda\left[|\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}| - \frac{\sqrt{2}}{\pi}\right] + \beta \ln h_{t-1}$$

Values in parentheses are the $p-$ values; MLLV is the maximized log-likelihood value.

In order to find if the chosen lag values of the models for both the conditional mean and conditional variance were adequate, we carried out the Ljung-Box test $Q(.)$ with both standardized residual and squared standardized residuals for all the five models. The values of this test statistic are provided in Table 7 for the proposed TAR-EGARCH-M model only[6]. It is quite evident from this table that the chosen lag value of unity for the conditional mean is adequate for all the eight return series since the test statistic values suggest that the null hypothesis of 'no autocorrelation in standardized errors' cannot be rejected. As regards the adequacy of the order (1,1) for the conditional variance model EGARCH , it is noted that at1% level of significance the $Q^2(.)$ values indicate that the null hypothesis of 'no autocorrelation squared standardized errors' is rejected for returns on stock indices of Russia, the US and Hong Kong. For the remaining five series, the choice of the order was found to be adequate.

[6]The test statistic values are almost the same for the STAR-EGARCH-M model and hence these are not reported. As for the other models, the conclusions are, by and large, the same.

Table 7: $Q(.)$ and $Q^2(.)$ test statistic values for the residuals of the TAR-EGARCH-M model

Statistic \ Country	Brazil	Russia	India	China	The USA	The UK	Hong-Kong	Japan
$Q(5)$	4.4053	5.1346	8.1960	6.4387	9.1276	6.0071	8.3223	0.8842
	(0.49)	(0.40)	(0.15)	(0.26)	(0.10)	(0.31)	(0.14)	(0.89)
$Q(10)$	8.3952	10.324	16.430	22.715	13.919	8.6823	11.046	7.6421
	(0.59)	(0.41)	(0.09)	(0.01)	(0.18)	(0.56)	(0.35)	(0.66)
$Q^2(5)$	12.508	19.417	4.0754	3.0411	26.005	1.6951	19.449	7.1519
	(0.03)	(0.00)	(0.53)	(0..69)	(0.00)	(0.89)	(0.00)	(0.21)
$Q^2(10)$	19.989	21.270	8.3980	5.0272	42.202	9.4082	29.384	9.9140
	(0.03)	(0.01)	(0.59)	(0.89)	(0.00)	(0.49)	(0.00)	(0.45)

$p-$values are given in parentheses.

Summing up the empirical findings so far, we can state that consideration to stock market movements like up and down in specifying conditional mean model along with asymmetry in the sense of leverage effect in specifying the conditional variance model are very important in proper modelling of risk-return relationship where time-varying risk is assumed to directly affect conditional mean. As noted in the Table 2 through 6, in terms of maximized log likelihood values substantial gains are made in the model hierarchy considered in this paper *viz.*, starting with AR-GARCH-M where there is no regime in conditional mean as well as no asymmetry in volatility, the modelling performance improves when asymmetry in conditional variance only is considered. The performance of the TAR-EGARCH-M model shows that in all return series, EGARCH, rather than GARCH, is the appropriate model for volatility. The proposed models i.e., TAR-EGARCH-M and STAR-EGARCH perform almost the same suggesting thereby that there is no smooth transition from one market state to the other. Further, these two models perform superior to all other models considered in terms of maximized log-likelihood values. Finally, it can be observed from Table 8 that the proposed model TAR-EGARCH-M (also STAR-EGARCH-M) performs better than the benchmark model AR-EGARCH-M in terms of likelihood ratio (LR) test for all countries other than Japan.

Table 8: Likelihood ratio test statistic values

Country	Brazil	Russia	India	China	USA	UK	Hong-Kong	Japan
H_0 \ H_1	TAR-EGARCH-M							
AR-EGARCH-M	19.21	15.4224	10.7052	33.00384	28.22688	6.45232	8.62372	3.57168
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.09)	(0.03)	(0.31)

$p-$values are given in parentheses.

3.3 Results of the Wald test

The most important hypothesis of interest for the proposed models with EGARCH volatility specification is whether risk responds to returns differently in the up and down market movements. In terms of the parameters, the null and alternative hypotheses are $H_0 : \delta_1 = \delta_2$ and $H_1 : \delta_1 \neq \delta_2$, respectively. This null hypothesis has been tested by using the Wald test and the statistic values are given in Table 9 for the TAR-EGARCH-M model[7]. Before testing this null hypothesis we first tested the null hypothesis $\mu_1 = \mu_2, \phi_1 = \phi_2, \delta_1 = \delta_2$ in order to infer if introduction of the two states of market is statistically tenable. The Wald test statistic values for this null hypothesis are also presented in Table 9. It is evident from this table that this null hypothesis is rejected for all the eight return series, thus empirically supporting the introduction of two market states for the conditional mean model. To be more specific, this finding suggests that the two market situations indeed require two different conditional mean models. Now, to find whether rejection of this null hypothesis is due to differences in autocorrelation coefficient values only, we also tested the null hypothesis given as $\phi_1 = \phi_2$. This hypothesis was found to be 'not rejected' in all but two return series. These two exceptions are the

[7]The test statistic values in case of STAR-EGARCH-M model are almost the same and hence these are not reported seperately.

returns on SENSEX of India and HANG SENG of Hong Kong. It may be worth recalling that ϕ_1 and /or ϕ_2 were found to be statistically significant only in case of few stock returns. Finally, the results of the Wald test for $H_0 : \delta_1 = \delta_2$ show that this null hypothesis is rejected in all but returns on SENSEX and FTSE ALL. Thus it can be inferred that except for these two stock indices, the relative risk aversion parameters in the two market conditions are different for the other six return series *viz.*, BOVESPA, MICEX, SSE COMPOSITE, S&P 500, HANG SENG and NEKKIE 225. This empirically establishes the fact that investors' reactions to returns in response to risk are different in these two states of market characterized by up and down movements, and that this is regardless of the fact whether the stock market is from an advanced economy or form an important emerging economy called the BRIC group.

Table 9: Results of the Wald test for the TAR-EGARCH-M model

H_0 \ Country	Brazil	Russia	India	China	The USA	The UK	Hong-Kong	Japan
$\mu_1 = \mu_2, \phi_1 = \phi_2, \delta_1 = \delta_2$	27.1837	17.3195	12.0445	22.01208	48.9482	12.4868	11.9175	7.34215
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)	(0.06)
$\phi_1 = \phi_2$	0.9026	0.4838	8.9617	0.0432	2.0035	0.2758	3.4002	2.1367
	(0.34)	(0.48)	(0.00)	(0.83)	(0.15)	(0.59)	(0.07)	(0.14)
$\delta_1 = \delta_2$	5.7236	11.6848	0.94870	3.3650	14.3499	2.2395	3.7520	4.1912
	(0.02)	(0.00)	(0.33)	(0.07)	(0.00)	(0.13)	(0.06)	(0.04)

4 Conclusions

This paper has proposed models for studying the risk-return relationship in the framework where (*i*) risk directly affects returns, as in the GARCH-in-mean model, (*ii*) two stock market movements, called the up and down markets - based on past returns, are incorporated in the conditional mean returns model, and (*iii*) the risk aversion parameter is taken to be different so that it can be investigated if risk responds differently in the two market situations. The specification of the conditional variance has been taken to be the EGARCH model which takes into account the leverage effect which defines the asymmetric behaviour of return shocks on conditional variance. The two models capturing this features and designated as the TAR-EGARCH-M and STAR-EGARCH-M models, differ only in respect of the fact that the logistic transition function is considered for smooth transition from one regime to the other in case of the latter model. We have taken daily level data on stock indices for a group of eight countries of which four are developed and four important emerging economies - comprising BRIC countries. The stock indices considered at daily frequency are: S&P 500 (the US), FTSE ALL (the UK), Hang Seng (Hong Kong), NEKKIE 225 (Japan), BOVESPA (Brazil), MICEX (Russia), BSE SENSEX (India) and SSE COMPOSITE (China). Returns from these stock indices have been used to estimate the proposed models along with two benchmark models where the up and down market conditions have not been considered.

The empirical findings are overwhelmingly in favour of the proposed models - TAR-EGARCH-M and STAR-EGARCH-M. It is found that the mean return regimes referring to up and down markets are statistically valid for all the eight return series. Further and more importantly, risk in terms of time-varying conditional variance is found to respond 'asymmetrically' in the two market conditions in the sense that the risk aversion parameter has been found to be positive in the down market and negative in the up market. This empirical finding gives support to the observations made by Fabozzi and Francis (1977) and Kim and Zumwalt (1979) that investors require a premium for taking downside risk and pay a premium for upside variation. Finally, it is also observed that modelling consideration to stock market conditions through the introduction of regimes yields statistically better model since the AR-EGARCH-M model is found to be rejected, by LR test, in favour of the proposed TAR-EGARCH-M model in seven countries.

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