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R&D in a Duopoly under Incomplete Information

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Abstract

Availability of information about rivals may have a significant impact on a firm's decision about R&D investment. This paper investigates how R&D incentive of a firm in a Cournot duopoly may depend on informational structures. We show that asymmetric information about rival's cost reduction may enhance research incentive of each firm compared to complete information case. However, an additional dimension of asymmetry (e.g.the information about whether the rival has invested in R&D or not) will reduce R&D incentive unambiguously compared to one dimensional asymmetry case. We discuss the problem with general distribution function of a firm's type.

Keywords: R&D incentives, Duopoly, Asymmetric information, Type distribution.
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1 Introduction

R&D is regarded essential by both governments and private entities though for separate reasons. Governments hold R&D necessary for the sake of growth and development so that the concerned country can either stay on the technological frontier or push the frontier outward. Private entities, on the other hand, take interest in R&D in order to gain an edge over the rivals and enhance their profits by lowering costs and producing better quality of an existing product or by introducing a totally different or closely related product. Whether a firm will invest in R&D or not depends on a wide range of factors including the availability of the required amount of R&D fund, success probability, market structure, spill-over effects, R&D organisation, patent protection, imitative capability of the rivals and so on.

R&D incentives based on these factors are extensively discussed in the literature. But the role of information in determining R&D investment by a firm has drawn little attention so far. The present paper seeks to discuss how R&D decision of a firm may depend on various informational structures. Quite obviously, R&D investment depends on factors about which a firm does not have perfect or complete information. For instance, a competing firm may not simply know whether its rival is involved in R&D to reduce production cost or innovate a new product. This is certainly the case when the firms are from different countries but possibly they compete in a third market.[1] Even when the competing firms know whether their rivals have invested in R&D or not, the outcomes of R&D may be unknown. Consider cost reducing R&D. Then it is possible that the firm does not know the extent of cost reduction of the rival. This informational asymmetry is well highlighted in the literature of technology transfer and incumbent-entrant interaction.[2]

In this paper we focus on the following scenario. Whether a firm is conducting R&D or not, and if it does then exactly what R&D output it is obtaining, may or may not be observable to its rivals. The extent of observability of a firm's R&D decision and its consequences will lead to different information structures. Then the R&D decision of a firm becomes contingent on the availability of information regarding its rival firms' R&D decisions and R&D outcome. Hence, the purpose of this paper is to study how informational structure can affect the R&D decision of a firm. Does more information provide a larger incentive for investing in R&D? We show in the present paper that the R&D incentive of a firm can be stronger under incomplete information than that under complete information.

R&D incentive in the presence of incomplete information is relatively a less explored area. There are only a few related literature where R&D decision making under incom-

[1]These types of scenarios are observed to prevail in the strategic trade literature, see, for instance, Brander and Spencer(1985).

[2]See, for instance, Milgrom and Roberts (1982), Gallini and Right (1990) and Beggs (1992).

plete information has been studied. The work by Grishagin, Sergeyev and Silipo (2001) considers patent race in a multi-stage game involving duopoly. Here the firms know each other's initial position but cannot observe the rival's R&D progress, and hence do not know their relative positions afterwards. Brocas (2004) analyses regulator's role in providing incentives for cooperative R&D when the firms possess some privately known skills. The contract involves skill-sharing offers and coordination of subsequent efforts where the innovators must get informational rents for disclosing privately known skills. The choice between cooperative and non-cooperative R&D under incomplete information is also studied in Kabiraj and Chattopadhyay (2015), and Chattopadhyay and Kabiraj (2015). Both these papers show that incentive for cooperative R&D increases under incomplete information compared to complete information.[3]

Bacchiega and Garella (2008) discuss the issue of withholding as opposed to disclosing private information about alternative technologies to rivals. Transfer of information to the rivals affects the distribution of knowledge in an industry. The firms make R&D decision as a function of the initial distribution of knowledge in the industry and hence sometimes they conduct R&D just to acquire information and not to use it. Under certain circumstances information is transferred to the rivals.

Vives (2008) analyses the effects of competitive pressure on process innovation and introduction of new product for both Bertrand and Cournot duopoly. In this context an agency model has been studied involving an owner and a manager for every firm. The manager's efforts determine the amount of cost reduction, but efforts are unobservable. The owner designs an incentive scheme for the manager with a cost reduction target and a compensation scheme. Thus the problem considered in this paper focuses on designing R&D incentives for managers rather than R&D incentives of firms.

In the present paper we discuss R&D incentives of a firm under incomplete information in a Cournot duopoly when the firms interact non-cooperatively in R&D and product market. We examine how the level of information can affect the R&D incentive of a firm. We assume that R&D investment by a firm will lower its marginal cost of production, but the extent of cost reduction is not known to the rivals. Since R&D activity involves a cost, therefore each firm will decide strategically whether to invest in R&D or not. We investigate how this decision depends on the availability of information. In particular, we study the relation between R&D incentive and the level of information.

We consider three distinct scenarios regarding the level of information available to a firm:

- Each firm knows whether its rival conducts R&D or not, and in case its rival actually conducts R&D, how much reduction in its marginal cost will occur.

[3] Kabiraj and Chattopadhyay (2015) consider a Cournot duopoly framework with discrete probability of success, while Chattopadhyay and Kabiraj (2015) consider a Bertrand competition with continuous probability distribution over R&D outcomes.

- Each firm knows whether its rival conducts R&D or not, but how much cost reduction occurs due to R&D for each firm is known only to the concerned firm and not to its rival.

- A firm does not know whether its rival at all conducts R&D, nor it knows the actual cost reduction of the rival.

Note that the first case is a game of complete information, while the last two cases are games of incomplete information. When only the extent of cost reduction is private information, but whether the rival has conducted R&D or not is perfectly observable, let us call this *Level I* incomplete information problem. When the extent of cost reduction of a firm, and whether it has conducted R&D or not, are both private information, we call this *Level II* incomplete information problem.[4]

It may be emphasized that under the incomplete information scenario we are portraying, no firm knows at the stage of R&D decision making whether its rival will be doing R&D, although the firm knows that at the beginning of production stage it will have that information. However, whether it is *Level I* or *Level II* incomplete information, a firm cannot observe the actual reduction in its rival's marginal cost. Our paper clearly differs from Grishagin, Sergeyev and Silipo (2001) and Brocas (2004) in that we consider neither patent race nor R&D cooperation. In our paper the regulator's policy choice is simply restricted to disclosure of information. In Bacchiega and Garella (2008), the level of information for each agent depends on incentives of the firms to share information, and there is no role of a regulator. In contrast, our paper considers the situation where the distribution of types in the industry is largely dependent on the regulator's policy choice. Finally, in Vives (2008) incomplete information arises because of non-observability of the manager's efforts which determine the R&D outcome, whereas in our paper R&D output of a firm is private information. Moreover, whether a firm is conducting R&D can be unobservable.

To summarize the contribution of the paper, note that the purpose of the paper is to study the relation between the level of information and the corresponding incentive to do R&D. In a deterministic world where the amount of R&D investment determines the extent of cost reduction, the revealed investment acts as a commitment to produce a higher quantity in the subsequent stage under Cournot competition. This would then imply that the incentive of each firm to engage in R&D under complete information should always be stronger than that under incomplete information about the amount of

[4]To illustrate, consider purchase of a license to use a cost reducing technology. When everything is publicly observable, i.e. who possesses the license and how much cost reduction it experiences is known to everyone, we have the game of complete information. When who holds the license is known to all, but how much cost reduction will be experienced by the license holding firm is private information, we effectively have *Level I* incomplete information scenario. However, if who holds the license is also unknown, we have *Level II* incomplete information scenario.

cost reduction through R&D. In the present paper we have portrayed situations when the above intuition does not always hold. The reason is that each firm at the stage of production decision under (*Level I*) incomplete information perceives an average cost reduction of the rival. Hence a firm which would not invest in R&D under complete information if its cost reduction is small relative to R&D cost is likely to go for investment under incomplete information. However, under *Level II* incomplete information when the concerned firm even does not know whether its rival has invested in R&D, this added uncertainty reduces R&D incentives compared to the case of *Level I* incomplete information. These two results together give the possibility of a non-monotone relation between the availability of information and R&D incentive.

The organisation of the paper is as follows. Section 2 describes the mode, section 3 presents the benchmark case involving complete information, section 4 and 5 discuss R&D incentive under *Level I* and *Level II* incomplete information, respectively, and section 6 compares the results under different scenarios. Finally, section 7 concludes the paper. All major proofs of the paper are relegated to appendix.

2 Model Setup

Consider two firms, A and B. They compete in a Cournot fashion in the common product market. The inverse market demand function is given by $P = \max\{0, a - Q\}$ where Q is the quantity demanded at price P, and $a > 0$ is the demand shift parameter. Let initially the marginal costs of both firms be c where $c > 0$ is a constant.

However, before they compete in the product market, they can invest in a cost reducing research before starting production. The cost of research, denoted by $M > 0$, is same for both firms, although outcome of research may well be different.

We assume that if firm i ($i = A$, B) undertakes the R&D activity, then its marginal cost will be $c - \theta_i$, where θ_i is the extent of marginal cost reduction by firm i. We assume that θ_i is distributed over an interval $\left[\underline{\theta}, \overline{\theta}\right]$ with distribution function $F(\cdot)$ and continuous density function $f(\cdot)$ with full support. Therefore θ_i also denotes the type of firm i, and θ_i is private information to firm i in case of incomplete information. It is assumed that firm i knows its own type before deciding about R&D activity. We assume $c > \overline{\theta}$, $\underline{\theta} \geq 0$, and for technical requirement we further assume $a > c + 3\overline{\theta}$. [5]

To simplify the algebra, we denote:

$$K := a - c$$

$$q(x) := \frac{K + x}{3}$$

[5] This condition states that demand will be sufficiently high.

and

$$\Pi(x) := (q(x))^2$$

where $q(\cdot)$ is one firm's output under Cournot competition and $\Pi(\cdot)$ is profit of a firm. Our subsequent analysis crucially depends on the expected value of the rival's type, given that the type of the rival is greater than the type of the concerned firm. The expected type of a firm conditional on the rival's type being greater than x is denoted by

$$\Theta(x) := \int_x^{\overline{\theta}} y \frac{dF(y)}{1-F(x)}$$

It is routine to check that $q'(\cdot) > 0$, $\Pi'(\cdot) = \frac{2}{3}q(\cdot) > 0$, $\lim_{x\to\overline{\theta}} \Theta(x) = \overline{\theta}$.

Denote 'doing research' by R and 'no research' by N. So when firm A chooses to invest in research and firm B does not, we denote profit (expected profit) of firm A by $\Pi_A^{[RN]}$ ($E\Pi_A^{[RN]}$) and that of firm B by $\Pi_B^{[RN]}$ ($E\Pi_B^{[RN]}$). Similar notation is used for other cases.

We consider the following two stage game:

Stage I The firms decide simultaneously whether to do R&D or not, knowing the prevailing informational structure.

Stage II Given the informational structure, the firms then decide production and compete in the product market a la Cournot.

In the following sections, we investigate R&D incentive of a firm given different levels of informational asymmetry.

3 Complete Information: Benchmark Case

In this section we assume that everything is common knowledge, including the types of the firms. Hence in equilibrium either both, or only one firm, or none will invest in R&D.

The lemma below summarizes the payoffs of the firms under different equilibrium situations.

Lemma 1. *Given two firms i and j ($i, j = A, B$; $i \neq j$):*

a. If none does research, then they get a payoff $\Pi_i^{[NN]} = \Pi_j^{[NN]} = \Pi(0)$.

b. If both invest in research, their payoffs will be $\Pi_i^{[RR]} = \Pi(2\theta_i - \theta_j) - M$.

c. If only firm i invests and firm j does not, then, their payoffs are $\Pi_i^{[RN]} = \Pi(2\theta_i) - M$ *and* $\Pi_j^{[RN]} = \Pi(-\theta_i)$

Define

$$V(x) := \frac{\sqrt{(K+x)^2 + 9M} - (K+x)}{2}$$

We have $V(\cdot) > 0$ and $V'(\cdot) < 0$. Note first that if the rival is not doing research, then it is always optimal for firm i to do research if and only if $\Pi(2\theta_i) \geq \Pi(0) + M$, i.e., iff $\theta_i \geq V(0)$. Second, if the rival firm is doing research, then firm i will do research if and only if $\Pi(2\theta_i - \theta_j) \geq \Pi(-\theta_j) + M$, that is, iff $\theta_i \geq V(-\theta_j)$. Therefore, given $V(-\theta_j) > V(0)$, the sufficient condition that firm i will do research is $\theta_i \geq V(-\theta_j)$. [6]

As we have stated above, different alternative equilibria are possible. The theorem below formally states the conditions under which these equilibria occur.

Theorem 1. *The following results hold:*

a. If $\theta_i \leq V(0)$, then firm i will never invest in research.

b. If $\theta_i \geq V(-\theta_j)$,then firm i will always invest in research.

c. If $V(0) < \theta_i < V(-\theta_j)$, then firm i will invest in research only if firm j has not invested in research.

The results are stated in Figure 1 for all possible (realised) values of θ_A and θ_B.

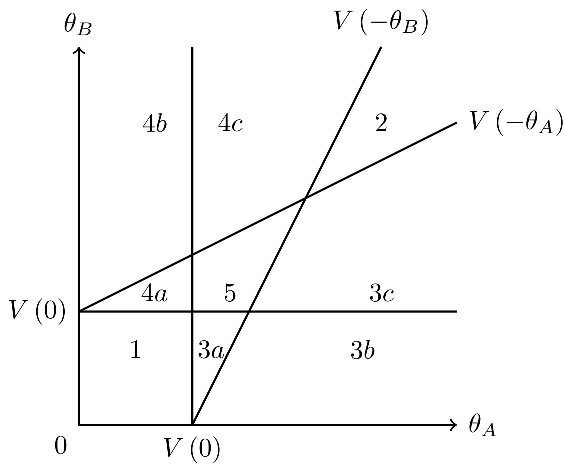

Figure 1: Research Possibilities under Complete Information

Given the possible domain of (θ_A, θ_B), in region 1, none of A and B will invest in R&D, while in region 2, both firms will invest. In region 3 (i.e. $3a$, $3b$ and $3c$), only firm A (and not firm B), and in region 4 (i.e. $4a$, $4b$ and $4c$), only firm B (and not firm A),

[6]It may be noted that unlike the cases of incomplete information (as we see later), here the threshold value of the marginal cost reduction, below which the firm decides not to invest in R&D, is a function of the rival's type, because here the firms do not have to take expectation like the cases of incomplete information. Also the threshold value is a strictly increasing function of the rival's type because if the rival's type is higher, then unless the research activity leads to a significant decline in marginal cost, the firm will not get enough profit to cover the cost of research.

will invest in research. Finally, in region 5, either A or B (but not both) will invest. Thus Figure 1 describes which firm will invest in R&D given the realisation of firms' types.

4 R&D under *Level I* Incomplete Information

In this section we consider the problem under *Level I* incomplete information, hence we assume that θ_i is private information to firm i. However, each firm knows before its production decision whether its rival has invested in R&D or not. We first discuss R&D incentive of a firm under this situation and in subsection 4.1 we compare the results with those under complete information.

Since R&D decision is taken at the first stage, therefore, at the beginning of the production stage, each firm can observe whether its rival has performed R&D or not. It is easy to see that if M is "sufficiently" high, then no firm will do R&D. On the other hand, if M is very small, then both the firms will invest in research. In between, suppose θ^1 is the threshold value, given M, such that a firm will invest in research if and only if its type is greater than or equal to θ^1. For the time being let us assume that for each M there exists a unique θ^1. We will show in subsequent analyses that θ^1 is a strictly increasing function of M in the range stated above. So, our primary objective in this section is to find out θ^1 as a function of M.

Like the case of complete information we start our analysis by estimating the (expected) payoffs of firms under different situations. The following lemma derives the (expected) profits. Note that we denote expected profit by $E\Pi(\cdot)$.

Lemma 2. *The following results hold:*

a. If no firm invests in research, then their payoffs are $\Pi_A^{[NN]} = \Pi_B^{[NN]} = \Pi(0)$.

b. If both the firms invest in research then firm i *gets* $E\Pi_i^{[RR]} = \Pi\left(\frac{3\theta_i - \Theta(\theta^1)}{2}\right) - M$.

c. Suppose firm A *conducts the research and firm* B *does not. The expected profits of firm* A *and firm* B *are given by* $E\Pi_A^{[RN]} = \Pi\left(\frac{3\theta_A + \Theta(\theta^1)}{2}\right) - M$ *and* $E\Pi_B^{[RN]} = \Pi\left(-\Theta\left(\theta^1\right)\right)$.

Proof. See Appendix I and Appendix II for proof. □

The firms face the following payoff matrix:

Table 1: Payoff Matrix under *Level I* Incomplete Information

		B: R	B: N
A	R	$\Pi\left(\frac{3\theta_A-\Theta(\theta^1)}{2}\right)-M, \Pi\left(\frac{3\theta_B-\Theta(\theta^1)}{2}\right)-M$	$\Pi\left(\frac{3\theta_A+\Theta(\theta^1)}{2}\right)-M, \Pi\left(-\Theta\left(\theta^1\right)\right)$
	N	$\Pi\left(-\Theta\left(\theta^1\right)\right), \Pi\left(\frac{3\theta_B+\Theta(\theta^1)}{2}\right)-M$	$\Pi(0), \Pi(0)$

Since $\Pi(\cdot)$ is strictly increasing, we have the following inequalities:

$$\Pi_i\left(\frac{3\theta_i+\Theta\left(\theta^1\right)}{2}\right) > \Pi_i\left(\frac{3\theta_i-\Theta\left(\theta^1\right)}{2}\right) > \Pi\left(-\Theta\left(\theta^1\right)\right) \tag{1}$$

$$\Pi_i\left(\frac{3\theta_i+\Theta\left(\theta^1\right)}{2}\right) > \Pi(0) > \Pi\left(-\Theta\left(\theta^1\right)\right) \tag{2}$$

$$\Pi_i\left(\frac{3\theta_i-\Theta\left(\theta^1\right)}{2}\right) \gtreqless \Pi(0) \tag{3}$$

If we consider only the "gross" profit[7] of a firm, then the first two sets of inequalities as obtained from (1) and (2) indicate that a firm will always be better off by investing in research irrespective of what its rival is doing. This is easy to understand, because if research cost is not considered, then doing research makes a firm more efficient, and therefore its profit will increase.

The inequality (3) says that even if both the firms invest in research, their gross expected profit may not be higher than that compared to the case where neither firm invests in research.

The threshold parameter θ^1 will be solved by equating the expected profit of a firm when it invests in research and the expected profit when it does not. Now, the expected profit of firm i when it invests in research is

$$F\left(\theta^1\right)\Pi\left(\frac{3\theta_i+\Theta\left(\theta^1\right)}{2}\right) + \left(1-F\left(\theta^1\right)\right)\Pi\left(\frac{3\theta_i-\Theta\left(\theta^1\right)}{2}\right) - M$$

and when it does not, the expected profit is

$$F\left(\theta^1\right)\Pi\left(0\right) + \left(1-F\left(\theta^1\right)\right)\Pi(-\Theta\left(\theta^1\right))$$

Let $T(x;\theta^1)$ denote the "gross" opportunity "gain" from doing research when the type

[7] If a firm is not investing in research, then its gross profit is equal to its net profit. On the other hand, if a firm is investing in research, then its net profit is the gross profit minus the cost of research.

of the firm is x. Then $T(x;\theta^1)$ is defined as

$$T(x;\theta^1) := (1-F(\theta^1))\left[\Pi\left(\frac{3x-\Theta(\theta^1)}{2}\right)-\Pi\left(-\Theta(\theta^1)\right)\right]+F(\theta^1)\left[\Pi\left(\frac{3x+\Theta(\theta^1)}{2}\right)-\Pi(0)\right]$$

Clearly, $T(x;\theta^1)$ is an increasing function of x. When θ^1 exists, it is solved from

$$T(\theta^1;\theta^1) = M$$

We have,

$$T\left(\underline{\theta};\underline{\theta}\right) = \Pi\left(\frac{3\underline{\theta}-\Theta\left(\underline{\theta}\right)}{2}\right)-\Pi(-\Theta\left(\underline{\theta}\right))$$

and with slight abuse of notation let

$$T(\overline{\theta};\overline{\theta}) = \lim_{x\to\overline{\theta}} T\left(x;x\right) = \Pi\left(2\overline{\theta}\right)-\Pi(0)$$

As stated above, our objective is to find out θ^1 as a function of M. However, till now there is nothing that tells us that for a particular M there will be a unique θ^1. The following lemma ensures uniqueness.

Lemma 3. *$T(x;x)$ is strictly increasing in $x \in \left(\underline{\theta},\overline{\theta}\right)$.*

Proof. See Appendix III for proof. □

Like the case of complete information, the theorem below provides the conditions for pooling and separating equilibria.

Theorem 2. *The following results hold:*

a. If $M \leq T\left(\underline{\theta};\underline{\theta}\right)$, then all the firms will invest in research.

b. If $M \geq T\left(\overline{\theta};\overline{\theta}\right)$, then no firm will invest in research.

c. Finally, when $T\left(\underline{\theta};\underline{\theta}\right) < M < T\left(\overline{\theta};\overline{\theta}\right)$, there exists a unique θ^1 such that a firm will invest in research if and only if its type is greater than or equal to θ^1 when θ^1 is solved from the equality $T\left(\theta^1;\theta^1\right) = M$.[8]

The uniqueness of θ^1 for any given M in (c) above follows from Lemma 3.

Since in the second stage the firms are informed about the R&D decision of the rival, this information acts as a signal. When a firm is observed to do R&D, it signifies that its realised θ is above a critical level. So, it is important now to check the incentive compatibility. We claim above that a firm will invest in R&D if and only if the type of the firm is greater than or equal to θ^1. Suppose firm A follows this strategy and believes

[8]Note that the value of θ^1 depends on the value of M.

firm B to be also following the same strategy. Firm B knows firms A's strategy and belief.

Remark 1. Suppose firm B's type is less than θ^1 but it decides to invest in R&D. Here, from the second stage onwards firm A believes that the type of firm B is greater than θ^1. Therefore, firm A will produce accordingly.

So the expected profit of firm B is

$$F\left(\theta^1\right)\Pi\left(\frac{3\theta_B+\Theta\left(\theta^1\right)}{2}\right)+\left(1-F\left(\theta^1\right)\right)\Pi\left(\frac{3\theta_B-\Theta\left(\theta^1\right)}{2}\right)-M$$

However, if it had not invested, then its expected profit would have been

$$F\left(\theta^1\right)\Pi\left(0\right)+\left(1-F\left(\theta^1\right)\right)\Pi(-\Theta\left(\theta^1\right))$$

From the definition of θ^1, and since $T(x;\theta^1)$ is strictly increasing in x, we know that for all $\theta_B<\theta^1$ the following holds:

$$\begin{aligned}&\left[F\left(\theta^1\right)\Pi\left(0\right)+\left(1-F\left(\theta^1\right)\right)\Pi(-\Theta\left(\theta^1\right))\right]\\&>\left[F\left(\theta^1\right)\Pi\left(\frac{3\theta_B+\Theta\left(\theta^1\right)}{2}\right)+\left(1-F\left(\theta^1\right)\right)\Pi\left(\frac{3\theta_B-\Theta\left(\theta^1\right)}{2}\right)-M\right]\end{aligned}$$

So, if firm B's type is less than θ^1, then given firm A's strategy and belief, it will never invest in research.

Remark 2. Suppose firm B's type is greater than or equal to θ^1 but it decides not to invest in R&D. Here, from the second stage onwards firm A believes that the type of firm B is less than θ^1. So, firm A will produce accordingly.

So the expected profit of firm B is

$$F\left(\theta^1\right)\Pi\left(0\right)+\left(1-F\left(\theta^1\right)\right)\Pi(-\Theta\left(\theta^1\right))$$

However, if it had invested then its expected profit would have been

$$F\left(\theta^1\right)\Pi\left(\frac{3\theta_B+\Theta\left(\theta^1\right)}{2}\right)+\left(1-F\left(\theta^1\right)\right)\Pi\left(\frac{3\theta_B-\Theta\left(\theta^1\right)}{2}\right)-M$$

Again from the definition of θ^1 and since $T(x;\theta^1)$ is strictly increasing in x, we know that for all $\theta_B\geq\theta^1$ the following holds:

$$\begin{aligned}&\left[F\left(\theta^1\right)\Pi\left(\frac{3\theta_B+\Theta\left(\theta^1\right)}{2}\right)+\left(1-F\left(\theta^1\right)\right)\Pi\left(\frac{3\theta_B-\Theta\left(\theta^1\right)}{2}\right)-M\right]\\&\geq\left[F\left(\theta^1\right)\Pi\left(0\right)+\left(1-F\left(\theta^1\right)\right)\Pi(-\Theta\left(\theta^1\right))\right]\end{aligned}$$

So, if firm B's type is greater than or equal to θ^1, then given firm A's strategy and belief, it will always invest in research.

By optimal strategy under *Level I* incomplete information we mean that the firm will invest in R&D if and only if its type is greater than or equal to θ^1 and it believes that the rival is following the same strategy. The above two remarks show that given that the rival is following the optimal strategy mentioned above, it is also optimal for a firm to follow the same strategy. So, both the firms following this strategy is a perfect Bayesian Nash equilibrium.

Below we illustrate our findings with an example.

Example 1. Assume that: $a = 10$, $c = 2$, $M = 2$, $\theta \in [0, 1]$ is distributed uniformly. Therefore, $F(\theta^1) = \theta^1$, $K = 8$ and $\Theta(\theta^1) = \frac{1+\theta^1}{2}$. We have $\Pi(0) = \frac{64}{9}$, $\Pi(-\Theta(\theta^1)) = \left(\frac{15-\theta^1}{6}\right)^2$, $\Pi\left(\frac{3\theta^1+\Theta(\theta^1)}{2}\right) = \left(\frac{33+7\theta^1}{12}\right)^2$ and $\Pi\left(\frac{3\theta^1-\Theta(\theta^1)}{2}\right) = \left(\frac{31+5\theta^1}{12}\right)^2$.

Then θ^1 is solved from

$$\theta^1\left[\left(\frac{33+7\theta^1}{12}\right)^2 - \frac{64}{9}\right] + \left(1-\theta^1\right)\left[\left(\frac{31+5\theta^1}{12}\right)^2 - \left(\frac{15-\theta^1}{6}\right)^2\right] = 2$$

The above equation solves uniquely $\theta^1 \approx 0.487$. If research cost is too high (i.e. more than 4) then no firm will do the research. On the other hand, if the cost is too low (i.e. less than 61/144) then both of them will invest in research.

4.1 *Level I* Incomplete Information vs. Complete Information

To compare the results under *Level I* incomplete information and complete information, we basically need to compare the threshold values under these two situations. It is important to note that in case of complete information the threshold value depends on the type of the rival firm. But this is not the case under incomplete information since rival's type is not observable. So, to compare, we must fix the rival's type in terms of its own type. The following theorem fixes the rival's type at $\Theta(\theta_i)$ where θ_i is the type of firm i and it shows that there are cases when less information leads to more investment in R&D.

Theorem 3. *Let $\Theta(\theta_i)$ be the type of firm i's rival. Then there exists M such that in case of incomplete information firm i will invest in research, but it will not do so under complete information.*

Proof. See Appendix IV for proof. □

This theorem shows the possibility that R&D incentive of a firm can be larger under incomplete information compared to that under complete information. To give an intuition of the possibility, consider that a firm's realised cost reduction is low such that, given the R&D cost, R&D investment under complete information is not profitable for this firm. Under incomplete information, the firm does not know the rival's actual type. So it forms a conditional expectation about rival's possible cost reduction, which could be lower than the rival's actual cost reduction. Alternatively, given low cost reduction of the concerned firm, its rival under incomplete information, does not know it to be a low cost firm. This means, under incomplete information, a firm may expect to be confronted with a less competing rival, hence its expected payoff would be larger.

To illustrate, consider Example 1 again. We can derive $\theta^1 \approx 0.487$, $V(0) \approx 0.528$ and $V(-1) \approx 0.593$. Now if $\theta_A = 0.5$, then firm A will invest in research under *Level I* incomplete information and not under complete information.

When the type of a firm is θ^1, it is indifferent between doing and not doing R&D. Assume in this case that a firm will invest in R&D. Our next theorem, will show using Lemma 4 that, when the types of both the firms are fixed at the threshold value, although it is optimal for both the firms to invest in research in case of incomplete information, under complete information at most one of them will invest in R&D.

Lemma 4. $\Pi\left(\frac{3x-\Theta(x)}{2}\right) - \Pi(-\Theta(x)) > \Pi(x) - \Pi(-x)$ *for all* $x \in \left(\underline{\theta}, \overline{\theta}\right)$.

Proof. See Appendix V for proof. □

Theorem 4. *Suppose the type of each firm is* θ^1*, and* M *is such that* θ^1 *is the threshold value in case of incomplete information (in which case, we assume, both the firms invest in research under incomplete information). Then,*

a. *it will never be the case that in case of complete information both the firms will invest in research;*

b. *moreover, if* $\Theta(\theta^1) \geq 3\theta^1$ *holds, then under complete information no firm will invest in research.*[9]

Proof. See Appendix VI for proof. □

This theorem is another case to show that less information may encourage more investment in R&D. Since the above results are derived under specific assumptions regarding the types of the firms, therefore counterexamples where more information leads to higher amount of investment are obviously possible.[10] We claim that whether more information will lead to a greater investment in R&D depends on the underlying parameters including the types of the firms.

[9] The condition holds only when θ^1 is "very close" to $\underline{\theta}$.

[10] Let $\theta \in [0,1]$ be distributed uniformly and $K = 3.001$ and $M = 1.4$ Then $V(0) \approx 0.8236$ and $\theta^1 \approx 0.8307$; clearly $V(0) < \theta^1$. So if the type of the firm lies between $V(0)$ and θ^1, then the firm may invest in research (if the rival is not investing in research) under complete information but clearly it will never invest in research under any incomplete information scenario.

5 R&D under *Level II* Incomplete Information

In this section we consider the problem under *Level II* incomplete information. Here, in addition to the assumption that θ_i is private information to firm i ($i = A, B$), we assume at the stage of production decision that neither firm knows whether its rival has performed research or not. We first discuss R&D incentive under this situation, then compare the results with the case of complete information.

Let θ^2 be the threshold value in this case given M, such that a firm will invest in research if and only if its type is greater than or equal to θ^2. Assume for the time being that for each M within a certain range, θ^2 is unique. We then show that θ^2 is also a strictly increasing function of M in that range.

Lemma 5 below derives the expected payoffs of a firm when it invests in R&D and when it does not.

Lemma 5. *If a firm invests in research, its second stage expected profit is given by*

$$E\Pi_i^{[R]} = \Pi\left(\frac{3\theta_i - (1 - F(\theta^2))\,\Theta(\theta^2)}{2}\right) - M$$

and if a firm does not invest in research, its second stage expected profit is given by

$$E\Pi_i^{[N]} = \Pi\left(-\frac{(1 - F(\theta^2))\,\Theta(\theta^2)}{2}\right)$$

Proof. See Appendix VII for proof. □

The "gross" opportunity "gain" from doing research when the type of the firm is y, is given by

$$W(y;\theta^2) := \Pi\left(\frac{3y - (1 - F(\theta^2))\,\Theta(\theta^2)}{2}\right) - \Pi\left(-\frac{(1 - F(\theta^2))\,\Theta(\theta^2)}{2}\right)$$

.

Clearly, $W(y;\theta^2)$ is increasing in y. Then θ^2 is solved from

$$W(\theta^2;\theta^2) = M$$

.

Note that,

$$W(\underline{\theta};\underline{\theta}) = \Pi\left(\frac{3\underline{\theta} - \Theta(\underline{\theta})}{2}\right) - \Pi\left(-\frac{\Theta(\underline{\theta})}{2}\right)$$

With slight abuse of notation we have,

$$W(\overline{\theta};\overline{\theta}) := \lim_{y\to\overline{\theta}} W(y;y) = \Pi\left(\frac{3\Theta(\overline{\theta})}{2}\right) - \Pi(0)$$

The following lemma then proves uniqueness of θ^2.

Lemma 6. *The function $W(y;y)$ is strictly increasing in $y \in (\underline{\theta}, \overline{\theta})$.*

Proof. See Appendix VIII for proof. □

We can now characterise different equilibria in the case of *Level II* incomplete information. This is stated in the next theorem.

Theorem 5. *We have:*

a. *If $M \leq W(\underline{\theta}; \underline{\theta})$, then both the firms will invest in research.*

b. *If $M \geq W(\overline{\theta}; \overline{\theta})$, then no firm will invest in research.*

c. *Finally, when $W(\underline{\theta}; \underline{\theta}) < M < W(\overline{\theta}; \overline{\theta})$, there exists a unique θ^2 such that a firm will invest in research if and only if its type is greater than or equal to θ^2, where θ^2 is solved from the equality $W(\theta^2; \theta^2) = M$.* [11]

To illustrate, for the parameter values of Example 1, θ^2 is solved from

$$\left(8 + \frac{(\theta^2)^2 + 6\theta^2 - 1}{4}\right)^2 - \left(8 - \frac{1 - (\theta^1)^2}{4}\right)^2 = 18$$

This solves uniquely $\theta^2 \approx 0.713$. If research cost is too high (i.e. more than 35/12) then no firm will do the research. On the other hand, if there is no research cost, only then both will invest in research, irrespective of their types.

5.1 *Level II* Incomplete Information vs. Complete Information

Compared to the case of complete information, under *Level II* incomplete information, not only the rival's type is unknown to a firm, it also does not know whether the rival has invested in R&D or not. In the following theorem, we focus on the fact that this growing uncertainty would provide a lower R&D incentive under incomplete information compared to complete information.

Theorem 6. *If the type of each firm is θ and θ is "sufficiently" close but strictly less than θ^2, and M is such that θ^2 is the threshold value in case of incomplete information, then,*

a. *it will always be the case that in case of complete information at least one firm will invest in research;*

[11] Note that the value of θ^2 depends on the value of M.

b. moreover, if $2\theta^2 < (1-F(\theta^2))\Theta(\theta^2)$ holds, then under complete information all the firms will invest in research.[12]

Proof. See Appendix IX for proof. $\square$

For the case of Example 1, suppose $\theta_A = 0.6$. Then firm A will invest in R&D under complete information, but will never invest under *Level II* incomplete information.[13] Since our results are derived by construct, hence completely opposite results are not ruled out.[14]

6 *Level I* vs. *Level II* Incomplete Information vis-a-vis Complete Information

To compare R&D incentives of the interactive firm under *Level I* and *Level II* incomplete information it will be sufficient to compare the two threshold values of θ (viz. θ^1 and θ^2) given the cost of research, whereas the critical levels are the extent of cost reductions under *Level I* and *Level II* incomplete information for which the R&D firms would be indifferent between doing and not doing R&D; a firm invests in R&D when its actual cost reduction is higher than the critical level.

Let

$$D(x) := \Pi\left(\frac{3x-\Theta(x)}{2}\right) - \Pi\left(-\Theta(x)\right)$$

Then we have,

$$\begin{aligned} T\left(\underline{\theta};\underline{\theta}\right) = D\left(\underline{\theta}\right) &= \left[\Pi\left(\tfrac{3\underline{\theta}-\Theta(\underline{\theta})}{2}\right) - \Pi\left(-\Theta(\underline{\theta})\right)\right] \\ &> W\left(\underline{\theta};\underline{\theta}\right) = \left[\Pi\left(\tfrac{3\underline{\theta}-\Theta(\underline{\theta})}{2}\right) - \Pi\left(-\tfrac{\Theta(\underline{\theta})}{2}\right)\right] \end{aligned}$$

and

$$T\left(\overline{\theta};\overline{\theta}\right) = \left[\Pi\left(2\overline{\theta}\right) - \Pi\left(0\right)\right] > D\left(\overline{\theta}\right) = \left[\Pi\left(\overline{\theta}\right) - \Pi\left(-\overline{\theta}\right)\right] > W\left(\overline{\theta};\overline{\theta}\right) = \left[\Pi\left(\frac{3\overline{\theta}}{2}\right) - \Pi\left(0\right)\right]$$

Note also that $D\left(\overline{\theta}\right) = \frac{K\overline{\theta}}{3} + \frac{K\overline{\theta}}{9}$, $W\left(\overline{\theta};\overline{\theta}\right) = \frac{K\overline{\theta}}{3} + \frac{\overline{\theta}^2}{4}$ and $K > 3\overline{\theta}$ account for the last inequality.

The following lemma shows that given the types of the firms, the expected opportunity gain of investing in research in case of *Level I* incomplete information is more than that in case of *Level II* incomplete information.

[12] The condition holds only when θ^2 is "very close" to $\underline{\theta}$.

[13] For $\theta_A = 0.6$, however, firm A would invest in R&D under *Level I* incomplete information.

[14] Suppose $K = 3.001$ and $M = 0.07$ and θ is uniformly distributed over $[0,1]$. Then, $V(-1) \approx 0.0758$ and $\theta^2 \approx 0.0748$, therefore $V(-1) > \theta^2$. So if the type of the firm lies between $V(-1)$ and θ^2, then the firm may not invest in research (if the rival is investing in research) under complete information but clearly it does invest in research under incomplete information.

Lemma 7. *For all* $x \in [\underline{\theta}, \overline{\theta}]$

$$D(x) > W(x; x)$$

Proof. See Appendix X for proof. □

Since the opportunity gain of investing in research in case of *Level I* incomplete information is greater than that in case of *Level II* incomplete information, and the threshold values are unique given M, it must be the case that the threshold value for *Level I* incomplete information is strictly less than that under *Level II* incomplete information, ceterus paribus. The theorem below formally states this.

Theorem 7. *Given the research cost the threshold value of marginal cost reduction above which a firm decides to invest in R&D, is lower for the case of Level I incomplete information compared to the case of Level II incomplete information i.e.* $\theta^1 < \theta^2$.

Proof. See Appendix XI for proof. □

Note that the above result is proved under very general condition. The result clearly proves that when the firms operate under different information structures, the less the information they possess, the less will be the incentive for R&D because the firms will tend to take less risk of research. We have already seen for Example 1 that $\theta^1 \approx 0.487$ and $\theta^2 \approx 0.713$, supporting our findings. If the realized cost reduction (θ) of a firm be less than θ^1, the firm will not invest whatever be the level of incomplete information. On the other hand if it is above θ^2, the firm will invest even under *Level II* incomplete information. If the realized θ lies in between θ^1 and θ^2, then the firm will invest only under *Level I* incomplete information but not under *Level II* incomplete information.[15]

We are now in a position to comapre all the results underlying complete and incomplete information and study fully the relation between the avaiability of information and R&D incentive. Interestingly, we find the possibility of non-monotone relation between the two. In particular, there are situations when as available information goes up, the R&D incentive first increases then falls. When a firm has too little information about its rival in respect of R&D, its incentive for doing research is least, but compared to complete information case, if the firm has some information about the R&D capability of the rival, the concerned firm, under some situations, expects to have a competitive advantage, hence its R&D incentives become higher. R&D incentives under *Level I* and *Level II* incomplete information are captured by θ^1 and θ^2. Possibility of non-monotone

[15] To illustrate a policy implication of our result, suppose two firms are considering to get a license of a cost reducing innovation from the government but no firm knows the type of its rival. Will the government disclose the names of the firm who are going to buy the license? Since the threshold value of type decisive R&D investment is lower when only the amount of cost reduction is private information compared to when even the R&D decision of a firm is not observable by its rival (i.e. $\theta^1 < \theta^2$), disclosing the information by the government (i.e. reducing information asymmetry) will lead to a greater probability of R&D investment, hence a higher expected social welfare.

result follows from the results of the previous two sections. Further, we have noted that whether a firm will do R&D or not under complete information depends on comparing θ with $V(0)$ and $V(-1)$. Although we have in general, $\theta^1 < \theta^2$ and $V(0) < V(-1)$, but since θ^1 and θ^2 depend on underlying parameters and type distribution, therefore depending on the parameter values and type distribution function, the overall ranking of these four critical values (θ^1, θ^2, $V(0)$ and $V(-1)$) is ambiguous. Hence whether mitigating all information asymmetry will generate a larger or lower incentive is not conclusive in general. For Example 1, however, we have the ranking $\theta^1 \leq V(0) < V(-1) \leq \theta^2$. This clearly illustrates the non-monotone relation between the amount of available information and the incentive to perform R&D. Thus under certain conditions the incentive to invest in R&D is highest under *Level I* incomplete information, lowest under *Level II* incomplete information, but moderate under complete information.

7 Conclusion

This paper studies R&D incentives of firms in a duopoly market. The firms interact non-cooperatively in both R&D and final production stage in a framework of incomplete information. The amount of cost reduction through R&D is private information and constitutes the types of the firms. Nature reveals the types to the respective firms before they decide whether to undertake R&D investment. We have considered two levels of incomplete information. Under *Level I* incomplete information, each firm privately knows how much cost reduction it can achieve by its R&D investment. This presumes that at the stage of production each firm knows with certainty whether its rival has invested in R&D or not. When in addition to a firm's type, whether or not it has invested in R&D is also private information, we are in the scenario of *Level II* incomplete information. In the benchmark case, however, all are common knowledge.

First, we have shown that, under both complete and incomplete information structure, if both the firms' cost reduction through the R&D activity is above a critical level, given the R&D cost, both firms will perform R&D. This happens because when the rival does not invest in R&D, a firm will invest in R&D provided it covers the R&D cost, and when the rival invests in R&D, the concerned firm will then invest in R&D if and only if it covers the R&D cost plus the loss of profit due to the increase in efficiency of the rival firm

One analytical difference between complete and incomplete information is that in case of the former the threshold value crucially depends on the exact type of the rival, whereas in the latter case it does not depend on the rival's type. The general result is that the relationship between the threshold value for complete information and that for incomplete information is ambiguous. This is not unexpected because of the extra information a firm has about its rival in case of complete information. We have shown the possibility that R&D incentive under incomplete information can be higher compared to the case of complete information. However, things become interesting when we compare the threshold values under different incomplete information structures. We have shown unambiguously that the threshold value under *Level I* incomplete information structure is strictly less than that under *Level II* incomplete information implying also that the probability of doing R&D is higher under *Level I* incomplete information. The less the information, the more risky is it is to invest in R&D.

R&D decision of firms in an oligopoly framework under incomplete information may be modeled in other alternative ways. A slight departure from the present model may involve the assumption of discrete probability distribution of success, where this probability is private information, that is, the probability of success of a particular firm is known to its rival only probabilistically. Another interesting study would be to introduce spillovers, where the extent of spillovers is private information. We leave these issues here as our further plan of research.

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Appendices

I Proof of Lemma 2.b

Proof. Expected profit of firm i is given by

$$E\Pi_i^{[RR]} = \int_{\theta^1}^{\overline{\theta}} \left[(K + \theta_i) q_i - q_i^2 - q_i q_j \right] \frac{dF(\theta_j)}{1 - F(\theta^1)} - M$$

Therefore, the reaction function of firm i is

$$2q_i + \int_{\theta^1}^{\overline{\theta}} q_j \frac{dF(\theta_j)}{1 - F(\theta^1)} = K + \theta_i$$

Solving the two reaction functions we get

$$q_i = \frac{K}{3} + \frac{3\theta_i - \Theta(\theta^1)}{6}$$

The rest of the proof is trivial. □

II Proof of Lemma 2.c

Proof. Expected profit of firm A is

$$E\Pi_A^{[RN]} = (K + \theta_i) q_A - q_A^2 - q_A q_B - M$$

And expected profit of firm B is

$$E\Pi_B^{[RN]} = \int_{\theta^1}^{\overline{\theta}} \left[K q_B - q_B^2 - q_A q_B \right] \frac{dF(\theta_A)}{1 - F(\theta^1)}$$

Reaction function of firm A is

$$2q_A + q_B = K + \theta_A$$

And reaction function of firm B is

$$2q_B + \int_{\theta^1}^{\overline{\theta}} q_A \frac{dF(\theta_A)}{1 - F(\theta^1)} = K$$

Solving the two reaction functions we get

$$q_A = \frac{K}{3} + \frac{3\theta_A + \Theta(\theta^1)}{6}$$

and

$$q_B = \frac{K}{3} - \frac{\Theta\left(\theta^1\right)}{3}$$

The rest of the proof is trivial. □

III Proof of Lemma 3

Proof. Note that $\lim_{x \to \bar{\theta}} T(x;x) > T\left(\underline{\theta};\underline{\theta}\right)$, since $\Pi(\cdot)$ is increasing and strictly convex. Also $\frac{d\Pi(x)}{dx} = \frac{2}{3}q(x)q'(x)$. Therefore,

$$\begin{aligned} \frac{dT(x;x)}{dx} &= \left[\Pi\left(\frac{3x+\Theta(x)}{2}\right) - \Pi(0) - \Pi\left(\frac{3x-\Theta(x)}{2}\right) + \Pi\left(-\Theta(x)\right)\right] f(x) \\ &+ \frac{2}{3}(1-F(x))\left[q\left(\frac{3x-\Theta(x)}{2}\right)\frac{3-\Theta'(x)}{2} + q\left(-\Theta(x)\right)\Theta'(x)\right] \\ &+ \frac{2}{3}F(x)q\left(\frac{3x+\Theta(x)}{2}\right)\frac{3+\Theta'(x)}{2} \end{aligned}$$

We have

1. $\left[\Pi\left(\frac{3x+\Theta(x)}{2}\right) - \Pi(0) - \Pi\left(\frac{3x-\Theta(x)}{2}\right) + \Pi\left(-\Theta(x)\right)\right] > 0$ as $\Pi(\cdot)$ is increasing and strictly convex
2. $\left[q\left(\frac{3x-\Theta(x)}{2}\right)\frac{3-\Theta'(x)}{2} + q\left(-\Theta(x)\right)\Theta'(x)\right] > 0$
3. $q\left(\frac{3x+\Theta(x)}{2}\right)\frac{3+\Theta'(x)}{2} > 0$

Note that, the second inequality holds even if $\Theta'(x) > 3$, as then $\Theta'(x) > \frac{3-\Theta'(x)}{2}$ and therefore,

$$\begin{aligned} & q\left(\frac{3x-\Theta(x)}{2}\right)\frac{3-\Theta'(x)}{2} + q\left(-\Theta(x)\right)\Theta'(x) \\ &= \frac{K+\frac{3x-\Theta(x)}{2}}{3}\frac{3-\Theta'(x)}{2} + \frac{K-\Theta(x)}{3}\Theta'(x) \\ &= \frac{K}{3}\frac{3+\Theta'(x)}{2} + \frac{1}{3}\left[\frac{3x-\Theta(x)}{2}\frac{3-\Theta'(x)}{2} - \Theta(x)\Theta'(x)\right] \\ &= \frac{K}{3}\frac{3+\Theta'(x)}{2} - \frac{\Theta(x)\left(1+\Theta'(x)\right) + x\left(\Theta'(x)-3\right)}{4} > 0 \end{aligned}$$

As $K\left(1+\frac{\Theta'(x)}{3}\right) > \Theta(x)\left(1+\Theta'(x)\right)$ and $K\left(1+\frac{\Theta'(x)}{3}\right) > x\left(\Theta'(x)-3\right)$ [16]. □

[16] Since $K > 3\bar{\theta}$

IV Proof of Theorem 3

Proof. Suppose $\theta_j = \Theta(\theta_i)$ in case of complete information . Note that in this case firm i will surely do the research if $\Pi(2\theta_i - \Theta(\theta_i)) - \Pi(-\Theta(\theta_i)) > M$ [17]. Also in case of incomplete information firm i will surely do the research if $\Pi\left(\frac{3\theta_i - \Theta(\theta_i)}{2}\right) - \Pi(-\Theta(\theta_i)) > M$.

Since, Π is strictly increasing, we have

$$\Pi\left(\frac{3\theta_i - \Theta(\theta_i)}{2}\right) - \Pi(-\Theta(\theta_i)) > \Pi(2\theta_i - \Theta(\theta_i)) - \Pi(-\Theta(\theta_i))$$

Finally, assume that in case of incomplete information firm i is doing research. If the condition below holds then in case of complete information firm i will not invest in research, i.e.

$$\Pi\left(\frac{3\theta_i - \Theta(\theta_i)}{2}\right) - \Pi(-\Theta(\theta_i)) > M > \Pi(2\theta_i - \Theta(\theta_i)) - \Pi(-\Theta(\theta_i))$$

□

V Proof of Lemma 4

Proof. Note that

$$9\left[\Pi(x) - \Pi(y)\right] = (2K + x + y)(x - y)$$

So

$$9\left[\Pi\left(\frac{3x - \Theta(x)}{2}\right) - \Pi(-\Theta(x))\right] = 4Kx + 3\left[\frac{\Theta(x) - x}{2}\right]\left[K - \frac{3x + \Theta(x)}{2}\right]$$

Similarly,

$$9\left[\Pi(x) - \Pi(-x)\right] = 4Kx$$

Finally, we know that $K > 3\overline{\theta}$ and $\overline{\theta} > \Theta(x) > x$. So $K - \frac{3x+\Theta(x)}{2} > 0$ and $\frac{\Theta(x)-x}{2} > 0$. □

VI Proof of Theorem 4

Proof. The first claim follows straight from the above lemma.

For the second part note that

[17] Since $\Theta(\theta_i) > \theta_i$ for $\theta_i \in \left(\underline{\theta}, \overline{\theta}\right)$, if firm i is doing the research, then her rival firm (who is more efficient) must do the research.

$$\begin{aligned}
&\Pi\left(\frac{3\theta^1-\Theta\left(\theta^1\right)}{2}\right)-\Pi\left(-\Theta\left(\theta^1\right)\right)\geq\Pi\left(2\theta^1\right)-\Pi(0)\\
\Leftrightarrow&\Pi(0)-\Pi\left(-\Theta\left(\theta^1\right)\right)\geq\Pi\left(2\theta^1\right)-\Pi\left(\frac{3\theta^1-\Theta\left(\theta^1\right)}{2}\right)\\
\Leftrightarrow&\left(2K-\Theta\left(\theta^1\right)\right)\Theta\left(\theta^1\right)\geq\left(2K+\frac{7\theta^1-\Theta\left(\theta^1\right)}{2}\right)\left(\frac{\theta^1+\Theta\left(\theta^1\right)}{2}\right)\\
\Leftrightarrow&K\left[\Theta\left(\theta^1\right)-\theta^1\right]-\left[\Theta\left(\theta^1\right)\right]^2\geq\left(\frac{7\theta^1-\Theta\left(\theta^1\right)}{2}\right)\left(\frac{\theta^1+\Theta\left(\theta^1\right)}{2}\right)
\end{aligned}$$

We have

$$K\left[\Theta\left(\theta^1\right)-\theta^1\right]-\left[\Theta\left(\theta^1\right)\right]^2\geq 2\left[\Theta\left(\theta^1\right)\right]^2-3\theta^1\Theta\left(\theta^1\right)$$

and

$$\left(\frac{7\theta^1-\Theta\left(\theta^1\right)}{2}\right)\left(\frac{\theta^1+\Theta\left(\theta^1\right)}{2}\right)\leq 3\theta^1\Theta\left(\theta^1\right)$$

The rest of the proof is trivial. □

VII Proof of Lemma 5

Proof. Suppose firm i does research in the first stage. So in the second stage her expected profit function is

$$E\Pi_i^{[R]}=\left(K+\theta_i\right)q_i^{[R]}-\left(q_i^{[R]}\right)^2-q_i^{[R]}\left[F\left(\theta^2\right)q_j^{[N]}+\left(1-F\left(\theta^2\right)\right)\int_{\theta^2}^{\Theta\left(\theta^2\right)}q_j^{[R]}\frac{dF\left(\theta_j\right)}{1-F\left(\theta^2\right)}\right]$$

On the other hand if firm i does not do the research in the first stage, then its expected profit function is

$$E\Pi_i^{[N]}=Kq_i^{[N]}-\left(q_i^{[N]}\right)^2-q_i^{[N]}\left[F\left(\theta^2\right)q_j^{[N]}+\left(1-F\left(\theta^2\right)\right)\int_{\theta^2}^{\Theta\left(\theta^2\right)}q_j^{[R]}\frac{dF\left(\theta_j\right)}{1-F\left(\theta^2\right)}\right]$$

The corresponding reaction functions are

$$2q_i^{[R]}+\left[F\left(\theta^2\right)q_j^{[N]}+\left(1-F\left(\theta^2\right)\right)\int_{\theta^2}^{\Theta\left(\theta^2\right)}q_j^{[R]}\frac{dF\left(\theta_j\right)}{1-F\left(\theta^2\right)}\right]=K+\theta_i$$

and

$$2q_i^{[N]}+\left[F\left(\theta^2\right)q_j^{[N]}+\left(1-F\left(\theta^2\right)\right)\int_{\theta^2}^{\Theta\left(\theta^2\right)}q_j^{[R]}\frac{dF\left(\theta_j\right)}{1-F\left(\theta^2\right)}\right]=K$$

respectively.

So we have four reaction functions with four unknowns viz. $q_A^{[N]}$, $q_B^{[N]}$, $q_A^{[R]}$, $q_B^{[R]}$.

Solving them we get

$$q_i^{[N]} = \frac{K}{3} - \frac{1 - F\left(\theta^2\right)}{6}\Theta\left(\theta^2\right)$$

and

$$q_i^{[R]} = \frac{K}{3} - \frac{1 - F\left(\theta^2\right)}{6}\Theta\left(\theta^2\right) + \frac{\theta_i}{2}$$

Plugging these values we get the payoffs. □

VIII Proof of Lemma 6

Proof. Note that $W(\overline{\theta};\overline{\theta}) > W(\underline{\theta};\underline{\theta})$. Let $A := \frac{3y-(1-F(y))\Theta(y)}{2}$ and $B := -\frac{(1-F(y))\Theta(y)}{2}$. We have, $\frac{dB}{dy} > 0$ and therefore $\frac{dA}{dy} > 0$.

Also $\frac{dA}{dy} > \frac{dB}{dy}$ (in particular, $\frac{dA}{dy} = \frac{3}{2} + \frac{dB}{dy}$).

We have

$$\frac{d}{dy}\left[\Pi\left(A\right) - \Pi\left(B\right)\right] = \frac{2}{3}\left[q(A)\frac{dA}{dy} - q(B)\frac{dB}{dy}\right] > 0$$

□

IX Proof of Theorem 6

Proof. Since $\Pi(\cdot)$ is increasing and strictly convex we have for all $y \in (\underline{\theta}, \overline{\theta})$

$$\Pi(2y) - \Pi(0) > \Pi\left(\frac{3y - (1 - F\left(y\right))\Theta(y)}{2}\right) - \Pi\left(-\frac{(1 - F\left(y\right))\Theta(y)}{2}\right)$$

The above inequality proves the first claim of the theorem.

Suppose, $2\theta^2 < (1 - F(\theta^2))\Theta(\theta^2)$ holds, then $\theta^2 > \frac{3\theta^2 - (1-F(\theta^2))\Theta(\theta^2)}{2}$. Again since $\Pi(\cdot)$ is increasing and strictly convex we have

$$\Pi(\theta^2) - \Pi(-\theta^2) > \Pi\left(\frac{3\theta^2 - (1 - F\left(\theta^2\right))\Theta(\theta^2)}{2}\right) - \Pi\left(-\frac{(1 - F\left(\theta^2\right))\Theta(\theta^2)}{2}\right)$$

which proves the second claim. □

X Proof of Lemma 7

Proof. We have already proved that $D\left(\overline{\theta}\right) > W\left(\overline{\theta};\overline{\theta}\right)$ and $D\left(\underline{\theta}\right) > W\left(\underline{\theta};\underline{\theta}\right)$. So here we will only prove that the inequality holds for all $x \in \left(\underline{\theta}, \overline{\theta}\right)$.

Let $z \in \left(\underline{\theta}, \overline{\theta}\right)$, we will then prove $D(z) > W(z; z)$.

Note that

$$D(z) = \left[\frac{Kz}{3} + \frac{K\Theta(z)}{9} - \frac{z\left(\Theta(z) - z\right)}{4} - \frac{\Theta(z)\left(\Theta(z) - z\right)}{12}\right] > \left[\frac{Kz}{3} + \frac{z\left(3z + \Theta(z)\right)}{12}\right]$$

and

$$W(z;z) = \left[\frac{Kz}{3} + \frac{z\left(3z - 2\left(1 - F(z)\right)\Theta(z)\right)}{12}\right]$$

Clearly $D(z) > W(z;z)$. Since z is arbitrary, we have proved the lemma. $\square$

XI Proof of Theorem 7

Proof. Since for all $z \in \left[\underline{\theta}, \overline{\theta}\right]$, we have $\left[\Pi\left(\frac{3z+\Theta(z)}{2}\right) - \Pi(0)\right] > D(z)$ and $D(z) > W(z;z)$. Hence we have $T(z;z) > W(z;z)$.

Finally, since both $T(\cdot)$ and $W(\cdot)$ are increasing, we have $\theta^1 < \theta^2$. $\square$