R&D Cooperation in a Three-Firm Cournot Industry*

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Abstract: In a three firm structure, given that research outcome is uncertain, we discuss the choice of R&D organization. Cooperative research takes the form of RJV or knowledge sharing (KS) only, and this is optimally chosen at the research stage. Then the successful firms play Cournot at the production stage. We characterize subgame perfect equilibria for all possible values of the probability of success. We show that KS alone can never be a part of subgame perfect equilibrium. In equilibrium, RJV strictly dominates non-cooperative research for all low and high probabilities of success, and for intermediate values non-cooperative R&D occurs.

Keywords: R&D organization; Cooperative and non-cooperative research; Research joint venture; knowledge sharing.

JEL classifications: D43; L13; O32.

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1. Introduction

The literature on the choice of R&D organization discusses incentives of the interacting firms for cooperative and non-cooperative research. A subset of this literature focuses on uncertainty about success in research. However, the existing works assume that the market structure is duopoly, and cooperative research takes the form of either research joint venture (RJV) or knowledge sharing (KS) agreement. Under RJV the firms do research in a single lab and share both R&D costs and research results, and under KS agreement firms do research independently in their respective labs but agree to share the results of any research lab. The present paper seeks to extend this literature to the case where there are more than two firms interacting in R&D and production, and the firms under research cooperation have the choice between doing research in a single lab in the form of RJV or in their own labs under a KS contract. Hence in our paper the firms first decide whether they will do research cooperatively or non-cooperatively, and then if it is cooperative research, they decide whether to do in a single lab or in all labs. Finally, they compete in quantities in the product market a la Cournot. We examine how uncertainty in the realization of R&D outcomes affects the choice of R&D organization under various scenarios.

In the pioneering work Marjit (1991) analyzes in a duopoly how uncertainty influences the choice between cooperative and non-cooperative R&D, but cooperative R&D always takes the form of RJV. The paper has shown that cooperative R&D will occur if the probability of success in R&D is either small or large; in the intermediate range non-cooperative R&D will dominate. Combs (1992) has considered a special type of R&D process. Under cooperative R&D firms do research in their own research labs independently under a KS agreement. The paper has shown that cooperative R&D will occur if and only if the probability of success is above a critical level. The R&D process (in the form of multiple research projects) is so designed that the success probability is twice of that under non-cooperative research.

In both the papers discussed above, there is no possibility of imitating the rival’s technology, hence no patent protection is required. In the presence of imitation, Kabiraj (2007) has introduced patent protection and has shown that if the R&D cost is low, Combs (1992) results will be reversed, otherwise Marjit (1991) results will follow. Then Mukherjee and Marjit (2011) have introduced the possibility of technology transfer in Combs (1992) structure to
study its effect on the choice of R&D organization. There are some other works (for instance, see d’Aspremont and Jacquemin (1988), Suzumura (1992) and Kamien et al. (1992)) which also discuss the choice between cooperative and non-cooperative research, but these are not in the ‘uncertainty’ framework; this literature focuses on spillovers of R&D.\footnote{There are some other works which discuss the problem under incomplete information. See for instance, Chattopadhyay and Kabiraj (2015), and Kabiraj and Chattopadhyay (2015).} However, none of the above papers consider the choice of the optimal form of R&D cooperation; those papers simply assume a particular form of cooperative research. But in our paper the choice of the form of cooperative research follows from the optimal decision of the firms.

In the present paper we have shown that there are situations when the KS agreement generates a larger payoff than the RJV does, but in the subgame perfect equilibrium KS only will not figure. The reason is that as the number of competing firm increases, the payoff from KS alone will fall, and as there will be no cost saving in research, the increased probability of success under cooperation will be weak compared to the loss of payoff. So in case of more than two firms Combs (1992) type results will not occur. In the subgame perfect equilibrium, however, we have shown that Marjit (1991) type results will reappear, that is, cooperative research will dominate non-cooperative research, but research in a single lab will be the outcome of the of optimal decision of the firms.

The organization of the paper is the following. Section 2 provides the model and results and section 3 summarizes the results and concludes.

2. Model and Results

Consider three firms which are going to invest in research for innovating a product. The R&D cost per research lab is $R > 0$, but research outcome is uncertain. If $R$ is invested in a lab, then success in innovation occurs with probability $\rho$ and failure occurs with probability $(1 - \rho)$; $0 < \rho < 1$. The firms who come up with the innovation at the end of research, play a Cournot game, and hence compete in quantities. While the product market competition is always non-cooperative, however, at the research stage they do research cooperatively ($C$) or non-cooperatively ($NC$), hence at the first stage they have the choice of R&D organization. Under cooperative research we consider two scenarios: they can do cooperative research in a single lab sharing both R&D costs and output ($C1$) (i.e., RJV), or they can do research
independently in their own labs under a KS contract (C2) (i.e., to share research results of any research lab). The research stage problem is depicted in Figure 1.

If only one firm comes up with the innovation at the end of the research stage, it acts as a monopolist in the product market and reaps a profit of \( \Pi(1) \); if two firms possess the innovation, they act as Cournot duopolists and each makes a profit of \( \Pi(2) \); and if all three firms have the innovation, they compete as triopolists, and each gets a profit of \( \Pi(3) \). Then under very general conditions we must have,

\[
\Pi(1) > 2\Pi(2) > 3\Pi(3)
\]  

(1)

that is, industry profit is falling as the number of firms in the industry goes up. We first derive the expected payoff of a firm engaged in research and production under various scenarios.

**Non-cooperative R&D (NC)**

Suppose the firms do R&D independently and non-cooperatively. Then the expected payoff of a firm is:

\[
E\Pi(\text{NC}) = \rho^3 \Pi(3) + 2\rho^2(1-\rho)\Pi(2) + \rho(1-\rho)^2 \Pi(1) - R
\]  

(2)

The above is derived as follows. All three firms are successful in research with probability \( \rho^3 \); the concerned firm along with one firm is successful with probability \( 2\rho^2(1-\rho) \); finally, the concerned firm alone is successful with probability \( \rho(1-\rho)^2 \).

**Research joint venture (C1)**

Under this scenario the firms do research jointly in a single lab and share the research cost and results with probability \( \rho \). Hence the expected payoff of a firm under RJV is:

\[
E\Pi(C1) = \rho \Pi(3) - \frac{R}{3}
\]  

(3)

**Cooperative research with KS contract (C2)**
Under this scenario the firms write a contract on information sharing but they do research independently in their own labs. Hence the expected payoff of a firm is:

\[
E\Pi(C2) = [1 - (1 - \rho)^3]\Pi(3) - R
\]  

(4)

Here a firm comes up with the innovation if at least one firm has success. This occurs with probability \([1 - (1 - \rho)^3]\).

Optimal form of cooperative research

Comparing \(E\Pi(C1)\) and \(E\Pi(C2)\), we have

\[
E\Pi(C1) > E\Pi(C2) \iff \frac{2}{3}R > g(\rho)
\]

(5)

where

\[
g(\rho) = \rho(1 - \rho)(2 - \rho)\Pi(3)
\]

It is easy to check that \(g(\rho)\) is strictly concave over \((0,1)\) with \(g(0) = 0 = g(1)\). Hence we can write the following result.

**Lemma 1**: If \(\frac{2}{3}R < \max_\rho g(\rho)\), \(\exists \bar{\rho} \& \hat{\rho},\ 0 < \bar{\rho} < \hat{\rho} < 1\), such that \(C1 > C2\) \(\forall \rho \in (0, \rho)U(\rho, 1)\) and \(C2 > C1 \forall \rho \in (\rho, \hat{\rho})\).

This lemma says that for low and high probabilities of success, RJV is preferred to KS agreement.

Cooperative vs. Non-cooperative R&D

**Scenario 1**:

Consider the choice between RJV and non-cooperative R&D, hence we compare \(E\Pi(C1)\) and \(E\Pi(NC)\). We have,

\[
E\Pi(C1) > E\Pi(NC) \iff \frac{2}{3}R > h(\rho)
\]

(6)

where

\[h(\rho) = \rho(1 - \rho)(2 - \rho)\Pi(3)\]

\[2\]Here `>` denotes `strictly preferred to`. 

\[ h(\rho) = \rho(1 - \rho)\{\Pi(1) - \Pi(3)\} - \rho[\Pi(1) - 2\Pi(2) + \Pi(3)] \]

We have \( h(0) = 0 = h(1) \) with \( h'(\rho)|_{\rho=0} > 0 \) and \( h'(\rho)|_{\rho=1} < 0 \). Further \( h(\rho) \) is positive and bell-shaped over \((0,1)\).\(^3\) Hence we have the following result.

**Lemma 2:** If \( \frac{2}{3} R < \max_{\rho} h(\rho) \), \( \exists \rho \in (0,\rho) \), \( \rho < \rho < 1 \), such that \( \forall \rho \in (0,\rho) U(\rho, 1) \), \( C1 > NC \), and \( \forall \rho \in (\rho, \rho), NC > C1 \).

This states that when RJV is the form of cooperative research, the firms will prefer cooperative research to non-cooperative research for all low and high values of probability of success. For the intermediate range, non-cooperative R&D is to be preferred. So this is similar to Marjit (1991) result.

**Scenario 2:**

Consider the choice between cooperative research with KS agreement and non-cooperative research. Comparing \( E\Pi(C2) \) and \( E\Pi(NC) \), we have

\[ E\Pi(NC) > E\Pi(C2) \forall \rho \]  \hspace{1cm} (7)

This means cooperative research with KS agreement is strictly dominated by non-cooperative research. Hence cooperative research with research taking place in all labs will never occur in equilibrium.

**Proposition 1:** When three firms interact in research and production, RJV is preferred to non-cooperative R&D for small and large values of success probability, but non-cooperative R&D will strictly dominate cooperative research with KS agreement.

Now we can combine all the above analysis to see the optimal choice of R&D organization when the firms also decide optimally whether they will do research in a single lab (RJV) or in all their labs independently under KS agreement.

Comparing \( g(\rho) \) and \( h(\rho) \), we have:

\[ g(\rho) < h(\rho) \forall \rho \in (0,1) \]  \hspace{1cm} (8)

\(^3\) Note that \( h''(\rho) < 0 \) iff \( \rho < \frac{2(\Pi(1) - \Pi(2))}{3(\Pi(1) - 2\Pi(2) + \Pi(3))} \equiv \rho^* \). It can then be shown that if either the demand function is linear or the industry profit as a function of the number of firms in the industry is concave, \( h(\rho) \) is strictly concave over \((0,1)\); otherwise, it is bell-shaped with initially concave, then convex.
Then we shall have the following possible scenarios depending on the values of the probability of success in R&D.

When \( \frac{2}{3} R < \max_{\rho} g(\rho) \), we must have \( 0 < \rho < \bar{\rho} < \hat{\rho} < \overline{\rho} < 1 \) (see Figure 2). Now given Lemma 1, Lemma 2 and inequality (5) and (7), the following inequalities must be true.

\[
C1 > NC > C2 \ \forall \rho \in (0, \rho)U(\bar{\rho}, 1) \tag{9a}
\]

\[
NC > C1 > C2 \ \forall \rho \in (\hat{\rho}, \bar{\rho})U(\hat{\rho}, \overline{\rho}) \tag{9b}
\]

\[
NC > C2 > C1 \ \forall \rho \in (\hat{\rho}, \bar{\rho}) \tag{9c}
\]

![Figure 2](image)

When \( \max_{\rho} h(\rho) > \frac{2}{3} R > \max_{\rho} g(\rho) \), we must have,

\[
C1 > NC > C2 \ \forall \rho \in (0, \rho)U(\bar{\rho}, 1) \tag{10a}
\]

\[
NC > C1 > C2 \ \forall \rho \in (\hat{\rho}, \overline{\rho}) \tag{10b}
\]

Given the above inequalities underlying (9) and (10), we can now write the final proposition of the paper.

**Proposition 2:** In a three firm structure with the form of cooperative research chosen optimally, cooperative R&D with research in a single lab (i.e., RJV) will occur for all low and high values of the probability of success, but for intermediate values, non-cooperative research will occur.

The above result supports the findings of Marjit (1991) even in case of three firms. However, in our paper RJV is the outcome of an optimal choice whereas in Marjit (1991) RJV is just
assumed. This rules out the possibility of Combs’ (1992) result as the number of firms increases.

4. Conclusion

In this paper we have considered a model of three firms. The firms interact both at the research stage and production stage. Research outcome is uncertain, and the successful firms play Cournot at the production stage. We discuss the choice between cooperative research and non-cooperative research, hence the choice of R&D organization.

We have considered two forms of cooperative research, viz., RJV and KS. Under RJV the firms do cooperative research in a single lab and share both R&D costs and output. Under KS firms do research independently in their own labs but agree to share the result of any lab. The research cooperation optimally decides the form of cooperative research. We have characterized the subgame perfect equilibria for all possible values of the probability of success. We have shown that in equilibrium the KS agreement will never occur. However, when the probability of success is either high or low, cooperative research in the form of RJV will strictly dominate non-cooperative research. When the probability of success is in the intermediate range, non-cooperative R&D will dominate cooperative research. Thus we rediscover Marjit (1991) results even in the context of more than two firms. However, the main difference between Marjit (1991) and our paper is that in Marjit (1991) RJV as a form of cooperative research is taken as an assumption, but in our paper it follows as a part of the optimal decision of the firms.

References


