Stability in Poisson Geometry

Plan of Lectures

IST

August 1, 2008

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In these lectures I will discuss the theory of stability of leaves in various geometric settings. The main focus will be in Poisson geometry. I will give a detailed proof of a fundamental result about the stability of symplectic leaves of Poisson manifolds. I will also discuss an analogous result for Lie algebroids.

Lecture 1: Review of Poisson Geometry

Poisson brackets. Basic examples: symplectic manifolds, duals of Lie algebras, symplectic foliations. The splitting theorem. Cartan calculus. Lie algebroids. Connections and representations. Poisson cohomology. Cotangent paths and parallel transport. Poisson fundamental group. Linear Poisson holonomy.

Lecture 2: Stability of leaves: overview

Stability in dynamical systems: periodic orbits of vector fields. Stability in foliated geometry: stability of leaves of foliations; the theorem of Reeb, Thurston, Langevin & Rosenberg. Stability in equivariant geometry: the theorem of Hirsch and Stowe. The stability problem for symplectic leaves: weak and strong stability.

Lecture 3: Poisson geometry around a leaf

Fixed points (zero dimensional leaves): isotropy Lie algebra; the linearization problem; Conn's linearization theorem. Higher dimensional leaves: transverse Poisson structure; the short exact sequence of a leaf; couplings and geometric data (after Vorobjev). Poisson fibrations and gauge theory. First order approximation around a leaf. Relative Poisson cohomology.

Lecture 4: Stability of symplectic leaves: main theorem

Precise statement of the stability theorem for symplectic leaves. Spectral sequence of the relative Poisson cohomology. Flat sections and minimizing functionals. The proof.

Lecture 5: Stability of leaves of Lie algebroids

Stability of leaves of Lie algebroids. Precise statement of the "Universal Stability Theorem". Relationships with the classical stability theorems. Necessary versus sufficient conditions for stability.

Main References:

[1] M. Crainic and R.L. Fernandes, Rigidity and flexibility in Poisson geometry, *Trav. Math.*, **16** (2005), 53-68.

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[3] R. Langevin and H. Rosenberg, On stability of compact leaves and fibrations, *Topology* **16** (1977), 107-111.

[4] K. Mackenzie, *Lie groupoids and Lie algebroids in differential geometry*, London Mathematical Society Lecture Notes Series, 124. Cambridge University Press, Cambridge, 1987.

[5] D. Stowe, Stable orbits of differentiable group actions, *Trans. Amer. Math. Soc.* **277** (1983), 665-684.

[6] Y. Vorobjev, Coupling tensors and Poisson geometry near a single symplectic leaf, *Banach Center Publ.* **54**, (2001), 249-274.