Reconsidering Generic Composition: the Tag-then-Encrypt case

Francesco Berti, Olivier Pereira and Thomas Peters

ICTEAM/ELEN/Crypto Group, Université catholique de Louvain, Louvain-la-neuve, Belgium
Objective

Obtain an Authenticated Encryption (AE) scheme composing:

- Encryption scheme
  - IV-based (ivE)
  - nonce-based (nE)
- PRF-MAC

**Problem** Distinguish secure from insecure combinations.
Objective

Obtain an Authenticated Encryption (AE) scheme composing:

- Encryption scheme
  - IV-based (ivE)
  - nonce-based (nE)
- PRF-MAC

Problem Distinguish secure from insecure combinations.
Objective

Obtain an Authenticated Encryption (AE) scheme composing:

- Encryption scheme
  - IV-based (ivE)
  - nonce-based (nE)
- PRF-MAC

Problem Distinguish secure from insecure combinations. NRS did most of the job (Eurocrypt 2014)
(160 [ivE] + 20 [nE] possible, 9 + 3 secure)
3+1 elusive in the Tag-then-Encrypt case
Outline

- Background
- Problem
- Results
- Attack on N4
Authenticated Encryption

**Objective:** Provide privacy and authenticity to a message and associated data (AD) with scheme $\Pi = (K, AEnc, ADec)$
Authenticated Encryption

**Objective:** Provide privacy and authenticity to a message and associated data (AD) with scheme $\Pi=(\mathcal{K}, \text{AEnc}, \text{ADec})$

Real and Ideal should be indistinguishable. (nAE)
Not repeat $n$, if $c \leftarrow \text{AEnc}_k(n, a, m)$ not ask $\text{ADec}_k(n, a, c)$
Authenticated Encryption

**Objective**: Provide privacy and authenticity to a message and associated data (AD) with scheme $\Pi = (\mathcal{K}, \text{AEnc}, \text{ADec})$

**Correctness + Tidiness**

$\text{AEnc}_k^{n,a}(m) = c \iff \text{ADec}_k^{n,a}(c) = m$ (Tidiness $\iff$)
Authenticated Encryption

Objective: Provide privacy and authenticity to a message and associated data (AD) with scheme $\Pi = (\mathcal{K}, \text{AEnc}, \text{ADec})$

Privacy (nAE-E)  
AEnc indistinguishable from $\$$. 
Authenticated Encryption

**Objective**: Provide privacy and authenticity to a message and associated data (AD) with scheme 
\( \Pi = (K, AEnc, ADec) \)

**Authenticity**
Difficult to provide a *valid and fresh* ciphertext \( c \).
Authenticated Encryption

Objective: Provide privacy and authenticity to a message and associated data (AD) with scheme \( \Pi = (K, AEnc, ADec) \)

Privacy + Authenticity = nAE.
Encryption schemes

Scheme $\Pi = (\mathcal{K}_E, \text{Enc}, \text{Dec})$:

- $c$ should be **indistinguishable** from random
  - if $n$ is not repeated (nonce) [$nE$]
  - if $iv$ is picked randomly (iv) [$ivE$]
(PRF)-MAC

Security:

- $\tau$ should be *indistinguishable* from random
  (PRF-security)
(PRF)-MAC

Security:

- $\tau$ should be *indistinguishable* from random
  (PRF-security)

PRF-security $\Rightarrow$ (unforgeability)

hard to find a fresh valid tag $\tau$
Composition

Many possible ways:
**MAC-then-Enc** (other Enc-then-MAC and Enc-and-MAC)

- **Iv-based**
  - $n \rightarrow m \rightarrow a \rightarrow \text{Mac}_{k_M}^V$
  - $m \rightarrow \tau \rightarrow \text{Mac}_{k_M}^{Tag}$
  - $\tau \rightarrow \text{Enc}_{k_E}$
  - $c$

- **Nonce-based**
  - $n \rightarrow a \rightarrow m \rightarrow \text{Mac}_{k_M}^{Tag}$
  - $m \rightarrow \tau \rightarrow \text{Enc}_{k_E}$
  - $c$
Outline

- Background
- Problem
- Results
- Attack on N4
Problem

NRS14 left 3 ivE and 1 nE elusive schemes
Problem

NRS14 left 3 ivE and 1 nE elusive schemes
Privacy | nAE-E is easy to prove.
---|---
Unforgeability | Difficult to produce a *valid and fresh* \((n, a, c)\)
**Privacy** nAE-E is easy to prove.

**Unforgeability** Difficult to produce a *valid and fresh* \((n, a, c)\)

- \(c\) is (pseudo)random
- if the \(iv\) or the tag \(\tau\) are fresh then it is unforgeable

**Problem** When \(iv\) and \(\tau\) are not fresh, it may be used to forge a fresh valid \(c\)
Outline

- Background
- Problem
- Results
- Attack on N4
Results

- A10, A11, A12 are either all secure or insecure
- A10, A11, A12 are secure if
  - Enc is “message-malleable”
  - Enc is “misuse-resistant”
- A10, A11, A12 are insecure if
  - Enc is stateful
  - Enc is not tidy
- N4 is insecure
A10 with Message malleability

A10 is secure if ivE is message malleable. Example CTR
A10 with Message malleability

A10 is secure if ivE is message malleable. Example CTR
A10 with Message malleability

A10 is secure if $ivE$ is message malleable. Example CTR

$$c' = c \oplus m \oplus m'$$
A10 with Message malleability

A10 is secure if ivE is message malleable. Example CTR

\[ c' = c \oplus m \oplus m' \]

A10 is secure if ivE is implemented with CTR.

**Idea:** an adversary vs CTR may distinguish valid from invalid ciphertexts.
A10 with Message malleability

A10 is secure if $ivE$ is message malleable. Example CTR

\[ c' = c \oplus m \oplus m' \]

A10 is secure if $ivE$ is implemented with CTR.

**Idea:** an adversary vs CTR may distinguish valid from invalid ciphertexts.

**Rember** the only possible forgery is with $iv$ and $\tau$ repeated.
Message-malleability

We can generalize:
Message-malleability

We can generalize:

**Definition**

An IV-based encryption scheme is *message malleable* if given \((iv, m, c)\) with \(c \leftarrow \text{Enc}^{iv}(m)\) then \(\forall c'\) an adversary can compute \(m' \leftarrow \text{Dec}^{iv}(c')\).
Message-malleability

We can generalize:

**Definition**
An ivE scheme is *message malleable* if given \((iv, m, c)\) with \(c \leftarrow Enc^iv(m)\) then 
\(\forall c'\) an adversary can compute \(m' \leftarrow Dec^iv(c')\).

If ivE is message malleable then A10 is secure.
**Misuse-resistant**

Indistinguishable even if $n$ (or $iv$) is repeated.
Misuse-resistant

Indistinguishable even if $n$ (or $iv$) is repeated. In such a case A10 is secure.
Misuse-resistant

Indistinguishable even if $n$ (or $iv$) is repeated. In such a case A10 is secure.
Interestingly message malleability and nonce-misuse are opposite.
Misuse-resistant

Indistinguishable even if \( n \) (or \( iv \)) is repeated. In such a case A10 is secure.
Interestingly message malleability and nonce-misuse are opposite.
Outline

- Background
- Problem
- Results
- Attack vs N4
  - PRF and TPRF
  - Attack.
PRF / TPRF

The output should be *indistinguishable* from random.
Attack N4 (1/2)

We give a counterexample.
We give a counterexample.
Let $\mathcal{M} = \{0, 1\}^l$ for AEnc, thus $\text{Enc} (m || \tau)$. 

**Attack N4 (1/2)**
Attack N4 (2/2)

**Key:** $k$ (key TPRF $F$), $v^* \in \mathcal{M}$
**Attack N4 (2/2)**

**Key:** $k$ (key TPRF $F$), $v^* \in \mathcal{M}$

$\text{Enc}_k^{n,v^*}(m\|\tau)$:
**Attack N4 (2/2)**

**Key:** $k$ (key TPRF $F$), $\nu^* \in \mathcal{M}$

$\text{Enc}_{k,\nu^*}^n(m||\tau)$:

- if $n = 1$, $c_0 = \nu^*$,
Attack N4 (2/2)

**Key:** \( k \) (key TPRF \( F \)), \( v^* \in \mathcal{M} \)

\[
\text{Enc}^n_{k,v^*}(m||\tau) :
\]

- if \( n = 1 \), \( c_0 = v^* \), else \( c_0 = F_k^{0,0}(n) \)
**Attack N4 (2/2)**

**Key:** $k$ (key TPRF $F$), $v^* \in \mathcal{M}$

$\text{Enc}^n_{k,v^*}(m\|\tau)$:
- if $n = 1$, $c_0 = v^*$, else $c_0 = F^{0,0}_k(n)$
  - $c_1 = F^{1,0}_k(n) \oplus m$
Attack N4 (2/2)

**Key:** \( k \) (key TPRF \( F \)), \( v^* \in \mathcal{M} \)

\( \operatorname{Enc}^n_{k,v^*}(m\|\tau) : \)

- if \( n = 1 \), \( c_0 = v^* \), else \( c_0 = F_{k}^{0,0}(n) \)
  
  \( c_1 = F_{k}^{1,0}(n) \oplus m \)

- if \( n = 1, 2 \land m = v^* \), \( c_2 = F_{k}^{2,1}(v^*) \oplus \tau \)
**Attack N4 (2/2)**

**Key:** $k$ (key TPRF $F$), $v^* \in \mathcal{M}$

$\text{Enc}_{k,v^*}^n(m\|\tau)$:

- if $n = 1$, $c_0 = v^*$, else $c_0 = F_{k}^{0,0}(n)$
  
  $c_1 = F_{k}^{1,0}(n) \oplus m$

- if $n = 1$, $2 \land m = v^*$, $c_2 = F_{k}^{2,1}(v^*) \oplus \tau$
  
  else $c_2 = F_{k}^{2,0}(n) \oplus \tau$
**Attack N4 (2/2)**

**Key**: $k$ (key TPRF $F$), $\nu^* \in \mathcal{M}$

$\text{Enc}^n_{k,\nu^*}(m||\tau) :$

- if $n = 1$, $c_0 = \nu^*$, else $c_0 = F^{0,0}_k(n)$
  
  $c_1 = F^{1,0}_k(n) \oplus m$

- if $n = 1, 2 \land m = \nu^*$, $c_2 = F^{2,1}_k(\nu^*) \oplus \tau$
  else $c_2 = F^{2,0}_k(n) \oplus \tau$

**Forgery:**

- $(1, a, m)$ obtaining $c^1 = (\nu^*, c^1_1, c^2_1) [\nu^*, F^{1,0}_k(1)]$

- $(2, a, \nu^*)$ obtaining $c^2 = (c^2_0, c^1_1, c^2_2) [F^{2,1}_k(\nu^*) \oplus \tau]$

**Output** $(1, a, c = [\nu^*, c^1_1 \oplus m \oplus \nu^*, c^2_2])$. 
**Attack N4 (2/2)**

**Key:** \( k \) (key TPRF \( F \)), \( v^* \in \mathcal{M} \)

**Enc\(_n\)\(_{k,v^*}(m\|\tau)\):**

- if \( n = 1 \), \( c_0 = v^* \), else \( c_0 = F_{k}^{0,0}(n) \)
  
  \[ c_1 = F_{k}^{1,0}(n) \oplus m \]

- if \( n = 1, 2 \lor m = v^* \), \( c_2 = F_{k}^{2,1}(v^*) \oplus \tau \)
  
  else \( c_2 = F_{k}^{2,0}(n) \oplus \tau \)

**Forgery:**

- \((1, a, m)\) obtaining \( c_1 = (v^*, c_1^1, c_1^2) \) \([v^*, F_{k}^{1,0}(1)]\)

- \((2, a, v^*)\) obtaining \( c_2 = (c_0^2, c_1^1, c_2^2) \) \([F_{k}^{2,1}(v^*) \oplus \tau]\)

**Output** \((1, a, c = [v^*, c_1^1 \oplus m \oplus v^*, c_2^2])\).
**Attack N4 (2/2)**

**Key:** $k$ (key TPRF $F$), $v^* \in \mathcal{M}$

$\text{Enc}^n_{k,v^*}(m||\tau) :$

- if $n = 1$, $c_0 = v^*$, else $c_0 = F^{0,0}_k(n)$
  
  $c_1 = F^{1,0}_k(n) \oplus m$

- if $n = 1$, $2 \lor m = v^*$, $c_2 = F^{2,1}_k(v^*) \oplus \tau$

  else $c_2 = F^{2,0}_k(n) \oplus \tau$

**Forgery:**

- $(1, a, m)$ obtaining $c^1 = (v^*, c^1_1, c^1_2) [v^*, F^{1,0}_k(1)]$

- $(2, a, v^*)$ obtaining $c^2 = (c^2_0, c^1_1, c^2_2) [F^{2,1}_k(v^*) \oplus \tau]$

**Output** $(1, a, c = [v^*, c^1_1 \oplus m \oplus v^*, c^2_2])$. 
**Attack N4 (2/2)**

**Key:** $k$ (key TPRF $F$), $v^* \in \mathcal{M}$

$\text{Enc}_{k,v^*}^n(m || \tau)$:

- if $n = 1$, $c_0 = v^*$, else $c_0 = F_{k}^{0,0}(n)$
  
  $c_1 = F_{k}^{1,0}(n) \oplus m$

- if $n = 1, 2 \lor m = v^*$, $c_2 = F_{k}^{2,1}(v^*) \oplus \tau$
  
  else $c_2 = F_{k}^{2,0}(n) \oplus \tau$

**Forgery:**

- $(1, a, m)$ obtaining $c^1 = (v^*, c_1^1, c_2^1) [v^*, F_{k}^{1,0}(1)]$

- $(2, a, v^*)$ obtaining $c^2 = (c_0^2, c_1^2, c_2^2) [F_{k}^{2,1}(v^*) \oplus \tau]$

**Output** $(1, a, c = [v^*, F_{k}^{1,0}(1) \oplus v^*, c_2^2 = F_{k}^{2,1}(v^*) \oplus \tau])$. 
Observation of the attack

- N4 is secure if message malleable
- N4 is secure if nonce-misuse resistant
- The nE scheme just used is message malleable if the following does not happen:
  - $n = 1$ or $2$ AND
  - $m = v^*$
Conclusions

Are A10, A11 and A12 secure?

Stateful generic composition
Questions?
Results

- A10, A11, A12 are either all secure or insecure
- A10, A11, A12 are secure if
  - Enc is “message-malleable”
  - Enc is “misuse-resistant”
- A10, A11, A12 are insecure if
  - Enc is stateful
  - Enc is not tidy
- N4 is insecure
Stateful

The encryption algorithm uses a *state* which is

- used only by Enc
- kept in memory by Enc after executions
- updated at every execution

A10 is not secure if ivE is stateful

Attack similar to N4
Stateful

- **Stateful** The encryption algorithm uses a *state* which is
  - used only by Enc
  - kept in memory by Enc after executions
  - updated at every execution
- A10 is not secure if ivE is stateful
  - Attack similar to N4
Unitidy

Untidy means

\[ \text{Dec}_k(\text{iv}, c) = m \Leftrightarrow \text{Enc}_k(\text{iv}, m) = c \]

A10 in this case is not secure:

- \[ \text{Enc}_k(\text{iv}, m) := (\text{Enc}'_k(\text{iv}, m) || \text{Enc}''_k(\text{iv}, m)) \]
- \[ \text{Dec}_k(\text{iv}, c', c'') = \text{Dec}_k(\text{iv}, c') \]
We prove that: either all 3 are secure or all 3 are not secure.
We prove that: either all 3 are secure or all 3 are not secure.

**Idea:**

- Observe that $\text{Mac}^{iv}(n) = \text{Mac}^{iv}(n, a)$ since $n$ not repeated.
- Replace $n$ with $n' = n||H(a)$
- Replace $m$ with $m' = H(a)||m$