Poster Snapshot

Second Order Convergence, Gaussian Approximation and Law of Iterated Logarithm for Interacting Urn Schemes on FA-DAGs

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Consider an infinite collection of urns each containing balls of k-colors interacting among each other through an infinite directed acyclic graph. More precisely, suppose urns are placed at each vertex of an infinite "Finite-Ancestral Directed Acyclic Graph (FA-DAG)". An FA-DAG is a DAG where every vertex has only finitely many ancestors. We assume that the oldest ancestral vertices (i.e., vertices with in-degree at most one) are of two types: one with a directed self loop on it which we call "Generators" and others with in-degree 0 are called "Stubborn Vertices". We consider a discrete time stochastic process which is described as follows: at each discrete time, balls are simultaneously drawn at random from each of the urns independently, and then reinforce the urns based on the colors drawn from their respective neighboring urns. In an earlier work we showed that the process has an almost sure limit (possibly random), which can be described as a solution of an appropriate Dirichlet problem on a suitable configuration space. In this work, we will discuss the asymptotic behavior of the urn configurations around their respective limits when the limit is non-random. We establish that like in the classical urn models, for the interactive process also, there are three phases of second order asymptotics for each urn composition. We further show that the phases where normal limit holds they further admit respective Gaussian process approximations which are a solutions of certain SDEs. We also prove Law of Iterated Logarithm for the phase with $\sqrt{n}$-convergence regime.

[Some parts are joint work with Antar Bandyopadhyay]