Brief Contributions

Fault-Tolerant Routing in Distributed Loop Networks

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Abstract—The ring network is a popular network topology for implementation in local area networks and other configurations. But it has a disadvantage of high diameter and large communication delay. So networks were introduced with fixed-jump links added over the ring. In this paper, we characterize some values for the number of nodes for which the lower bound on the diameter of loop networks is achieved. We also give an $O(\delta)$ time algorithm (where $\delta$ is the diameter of the graph) for finding a shortest path between any two nodes of a general loop network. We also propose a scheme to find a near optimal path (not more than one over the optimal) in case of a single node or link failure.

I. INTRODUCTION

One very common network topology is the ring. The ring has many attractive properties like simplicity of structure, incremental extensibility, low latency, ease of implementation etc. But it has some drawbacks as well. It is highly vulnerable to faults in the network. Also the diameter of a ring of $n$ processors (nodes), is $\frac{n}{2}$. These structures are called double-loop networks or simply loop networks.

Loop networks are special cases of an important class of graphs, called circulants. Circulants have been known in graph theory for a long time. According to Davis [2], they were first introduced by Catalan in 1846. A circulant $C(d, s)$ is a graph with $n$ nodes numbered from 0 to $n-1$ and node $i$ is connected to nodes $(i \pm s) \mod n$ and $(i \pm s^2) \mod n$. There have been several works on their properties [2], [3], [4].

We consider a set of $n$ nodes labeled $V_0, V_1, \ldots, V_{n-1}$. Each node $V_i$ is adjacent to four other nodes, $V_{i-1}, V_{i+1}, V_{i+n}$, and $V_{i-n}$, where $s$ is the length of a chord. Using standard notations [5], let us call this graph $G(N, 1, s)$. Let $d(N)$ denote the diameter of $G(N, 1, s)$ and $d(N) = \text{minimum} \{d(N, 1, s)\}$. Wong and Coppersmith [6] gave a lower bound of $\frac{\sqrt{3s^2} - 1}{2}$ for $d(N)$. Boesch and Wang [4] made the bound tighter to $\text{lb}(N) = \left\lceil \frac{\sqrt{3s^2} - 1}{2} \right\rceil$, where $\lceil x \rceil$ denotes the minimum integer $\geq x$. However, the lower bound $\text{lb}(N)$ may not be achievable for all values of $N$. For example, Du, Hsu, Li, and Xu [5] showed that for $N = 24$, $d(24) = 4$ with $s = 7$, but $\text{lb}(24) = 3$. The graphs whose diameters are equal to $d(N)$ are optimal for the given value of $N$, and those whose diameters are equal to $\text{lb}(N)$ are called tight optimal. Thus, a graph $G(N, 1, s)$ may be optimal for some $s$, but may not be tight optimal if $d(N) > \text{lb}(N)$. Du et al. gave some classes of values of $N$, for which the lower bound $\text{lb}(N)$ can be achieved. They also give some other classes, for which the lower bound cannot be achieved; but an optimal choice was given for such graphs. Tzvieli [7] and Bermond and Tzvieli [8] have classified many more values of $N$ for the optimal design of loop networks. A directed variation of the loop network is the FLBH (forward loop backward hop) network. There are arcs from node to nodes $i+1$ mod $N$ and $(i - s) \mod N$. Some results are available in the literature [9], [10] on the different properties and optimal and quasi-optimal routing in such networks.

About the classification, we suggest some classes of values for $N$, which achieve the lower bound on the diameter. These classes cover a large class of values of $N$. They also include many $N$s not classified by Du et al. Thus, we focus our attention on the problem of routing. Given $\delta$ (the diameter of the network), we propose an $O(\delta)$ time algorithm to find a shortest path between any two nodes. We also propose how to find a near optimal path (not more than one over the optimal) in case of a single node or link failure.

II. OPTIMAL DESIGN CRITERIA

Let $C_b(s, t)$ represent the graph on $n$ nodes labeled $V_0, V_1, V_2, \ldots, V_{n-1}$, such that node $V_j$ is connected to $V_{j+s}$, $V_{j+2s}$, and $V_{j+n}$. Boesch and Wang [4] have found out that for $N > 6$, $d(N)$, $\text{lb}(N)$, $\text{lb}(N) + 1) = \text{lb}(N)$. We try to use the above result, when it is given that one of the jumps is 1 and we have to minimize the diameter over the other jump of length $t$.

LEMMA 1. If $\text{gcd}(N, s) = 1$, then the $C_b(s, 1)$ there is a Hamiltonian cycle using only $s$-jumps.

PROOF. See [11].

LEMMA 2. If $\text{gcd}(N, \text{lb}(N)) = 1$ and $\text{gcd}(N, \text{lb}(N) + 1)$ is equivalent to $G(N, 1, s)$ for some $s$.

PROOF. See [11].

LEMMA 3. If $\text{gcd}(N, \text{lb}(N) + 1) = 1$ then $C_b(\text{lb}(N), \text{lb}(N) + 1)$ is equivalent to $G(N, 1, s)$ for some $s$.

PROOF. See [11].

If we combine Lemmas 2 and 3 with Theorem 5 of Boesch and Wang [4], we have the following result.

THEOREM 1. For $N > 6$, if $\text{gcd}(N, \text{lb}(N)) = 1$ or $\text{gcd}(N, \text{lb}(N) + 1) = 1$, then $d(N) = \text{lb}(N)$.

EXAMPLE 1. Consider the case of $N = 14$ nodes. Here we see that $\text{lb}(14) = 3$, i.e., $\text{gcd}(N, \text{lb}(N)) = 1$. So, by Theorem 1, $G(14; 3, 4)$ has diameter 3 and it can be redrawn as $G(14; 1, s)$ for $s = 6$.

In Fig. 1a, we see $G(14; 3, 4)$ with the 4-jumps shown in broken lines. Here nodes 0, 3, 6, 9, 12, 1, 4, 7, 10, 13, 2, 5, 8, 11, 0 form a Hamiltonian cycle. In Fig. 1b these have been relabeled as 0, 1, 2, ..., 13, 0, respectively. The 4-jumps are converted into 6-jumps.
Using Theorem 1, we get an excellent coverage over the possible values of $N$, the number of nodes in the network. We have exhaustively searched the optimal designs up to $N = 16,000$ and found that the tight optimal designs can be obtained for more than 80% of the values of $N$ by following the scheme of Theorem 1. For $7 \leq N \leq 5,305$, the classes given by Du et al. [5] cover only about 13% of the values of $N$, whereas Theorem 1 covers about 88.6%; nearly 10% of the values remain unclassified by either scheme. As $N$ increases, the classes defined by Du et al. give lesser coverage.

**III. SHORTEST-PATH ROUTING**

For two nodes with a link connecting them, communication is carried out through that link. In absence of a direct link, the message is transmitted through some intermediate nodes. The number of links traversed in such a path represents the transmission delay. So for any two nodes, it is important to find a path with minimum number of links. Consider a path involving $w$, $x$, and $z$, all non-negative integers) number of $[+s]$, $[-s]$, $[+1]$, and $[-1]$ links, respectively. Let the endpoints of the path be $V_i$ and $V_j$. Then, the relation $j = (w.s - x.s + y - z) \mod N$ holds irrespective of the order in which the links appear in the path. Since we are interested only in the lengths of the paths, we shall denote such a path by $(w)[+s] + (x)[-s] + (y)[+1] + (z)[-1]$.

**LEMMA 4.** Let $(w)[+s] + (x)[-s] + (y)[+1] + (z)[-1]$ be a shortest path from $V_i$ to $V_j$. Then at most one of $w$ and $x$ and at most one of $y$ and $z$ is nonzero.

**PROOF.** Let both $w$ and $x$ be nonzero. Without loss of generality, let $w \geq x$. Consider the path $(w - x)[+s] + (0)[-s] + (y)[+1] + (z)[-1]$. As $(w)[+s] + (x)[-s] + (y)[+1] + (z)[-1] = (w - x)[+s] + (0)[-s] + (y)[+1] + (z)[-1]$ is also a path from $V_i$ to $V_j$ and it is shorter than $(w)[+s] + (x)[-s] + (y)[+1] + (z)[-1]$, which contradicts the hypothesis that $(w)[+s] + (x)[-s] + (y)[+1] + (z)[-1]$ is a shortest path. Similarly, at most one of $y$ and $z$ may be nonzero. \[ \square \]

In view of Lemma 4, at most two of $w, x, y, z$ can be nonzero. From now on, we shall drop the terms with zero coefficient.

As a consequence of Lemma 4, we note that a shortest path from $V_0$ to $V_s$ would be using either $(+s, +1)$ or $(+s, -1)$ or $(-s, +1)$ or $(-s, -1)$ links. So, if we find the shortest of the paths of each combination of links, that path will be the required shortest path. We shall discuss in detail a method for finding a shortest path using $+s$ and $+1$ links. The case of $+s$ and $-1$ links would be very similar. The other two cases would be the same as finding a shortest path from $V_0$ to $V_s$ using $(+s, -1)$ and $(+s, +1)$ links. From now on, by a $(+s, +1)$-shortest path we shall mean a shortest path among the paths using $+s$ and $+1$ links only.

**LEMMA 5.** Let $(w)[+s] + (x)[+1]$ be a $(+s, +1)$-shortest path from $V_0$ to $V_s$. Then $x < s$.

**PROOF.** If $x \geq s$ then $(w + 1)[+s] + (x - s)[+1]$ is a shorter $(+s, +1)$-path from $V_0$ to $V_s$. \[ \square \]

**LEMMA 6.** A $(+s, +1)$-shortest path from $V_0$ to $V_s$ has at least $\left[ \frac{N}{s} \right]$ numbers of $+s$-links.

**PROOF.** If we use less than $\left[ \frac{N}{s} \right]$ number of $+s$-links, then we have to use more than $s$ number of $+1$-links. But a group of $s$ number of $+1$-links can always be replaced by one $+s$-link. \[ \square \]

Let $S_1 = s$ and $W_1$ be the cost of reaching the node at $S_1$ from $V_0$ using $+s$-links only. That is, $W_1 = 1$. For $u > s$, we can use Lemma 6 and reduce it to a problem of reaching $V_s$ from $V_0$ with $u < S_1$.

**LEMMA 7.** For $u < s$, the number of $+s$-links in a $(+s, +1)$-shortest path is either zero or at least $W_2 = \left\lfloor \frac{N}{s} \right\rfloor + 1$.

**PROOF.** Let $(w)[+s] + (x)[+1]$ be a shortest path from $V_0$ to $V_s$ for some $w, 0 \leq w < W_2 = \left\lfloor \frac{N}{s} \right\rfloor + 1$. Then $s \leq w.s \leq N$. The length of this path is $w + (u + N - w.s) > u$. But $(0)[+s] + (u)[+1]$ is a $(+s, +1)$-path between $V_0$ and $V_s$ and its length is $u$. \[ \square \]

**REMARK.** $W_2$ is the cost of reaching the node at $S_2 = (W_2, s)$ mod $N = s \left( \left\lfloor \frac{N}{s} \right\rfloor + 1 \right)$ - $N$, from $V_0$ by using $+s$-links only. Clearly, $S_2 = s - N$ mod $s \leq s = S_1$.

Now, if $S_1 > u \geq S_2$, and $W_2 < S_2$, then we may use $W_2$ $+s$-links from $V_0$ to reach the node number $S_2$. We may use groups of $W_2$ $+s$-links repeatedly, until we would reduce the problem to one of routing to a node within $S_2$ distance. If, however, $W_2 > S_2$, then $(+s, +1)$-
shortest path will not have any +s-link at all. Because, if a (+s, +1)-shortest path has any +s-link, in view of Lemma 7, it will have at least \( W_2 \) +s-links. But we can replace a group of \( W_2 \) +s-links by a group of +1-links and get a shorter path. So we may refine our Lemma 7 as follows.

**LEMMA 8.** For \( u < s \), the number of +s-links in a (+s, +1)-shortest path is

\[ \begin{align*}
&1) \text{zero,} \\
&2) \text{at least} \left\lceil \frac{u}{s} \right\rceil W_2, & \text{if} \ W_2 > S_j \\
&\qquad \quad \text{or, if} \ W_2 < S_j \\
\end{align*} \]

**REMARK.** If \( W_2 = S_j \), then (+s, +1)-shortest path may be found in the same way as for \( W_2 < S_j \), with the only exception that any group of +1-links may very well be replaced by \( W_2 \) (+s, +1)-links.

**EXAMPLE 2.** Consider \( G(258; 1, 100) \). Suppose we have to find a shortest path from \( V_i \). Let \( W_i \) be the number of +s-links we may use in reaching \( V_i \), and use one -s-link (i.e., use 9 = \( \left\lceil \frac{u}{s} \right\rceil \)) to reach +1-links from \( V_i \). So we have to consider only \( W_i < p < W_{i+1} \), with the only exception that any group of +1-links may very well be replaced by \( W_2 \) (+s, +1)-links.

Now we proceed to generalize the results in Lemmas 6-8. In order to do that, we define some terms:

1) As \( S_0 = N, S_1 = s, \ldots S_k = S_{k-1} \left( \left\lceil \frac{u}{s} \right\rceil + 1 \right) - S_{k-2} \)
2) \( W_0 = 0, W_1 = 1, \ldots, W_k = W_{k-1} \left( \left\lceil \frac{u}{s} \right\rceil + 1 \right) - W_{k-2} \)

We now describe some properties of the sequences \( \{ S_k \} \) and \( \{ W_k \} \).

**LEMMA 9.** The sequence \( \{ S_k \} \) satisfies the following properties.

1) \( S_k \geq 0 \)
2) \( S_0 \geq S_1 \geq S_2 \geq \ldots 
3) \text{if} \ S_i = S_{i+1}, \text{for some} \ i \geq 0, \text{then} \ S_{i+1} = S_i \text{for all} \ k \geq 0.

**PROOF.**

1) As \( S_{k-1} \left( \left\lceil \frac{u}{s} \right\rceil + 1 \right) \geq S_{k-2} \), the result follows from the definition of \( S_k \).
2) Again the result follows from the definition and the observation that \( S_{k-1} \left( \left\lceil \frac{u}{s} \right\rceil + 1 \right) \geq S_{k-2} - S_{k-1} \).
3) If for some \( k, S_k = S_{k+1}, \) then \( S_{k+1} = S_k \left( \left\lceil \frac{u}{s} \right\rceil + 1 \right) - S_{k-1} = 2S_k - S_{k-1} = S_k \), and so on.

**LEMMA 10.** \( W_0 \leq W_i \leq W_{i+1} \)

**PROOF.** From definition, \( W_0 \leq W_i \). Let \( W_0 \leq W_i \leq \ldots \leq W_{k+1} \). As \( S_{k-1} \geq S_k \) (Lemma 9), \( W_0 \geq W_{k+1} - W_{k+2} \geq W_{k+1} \).

**LEMMA 11.** \( (w, s) \) mod \( N = S_i \) for \( i = 0, 1, 2, \ldots 

**PROOF.** The result is easily verified to hold for \( i = 0 \) and 1. Let the result be true for \( i = k \), for some \( k > 0 \).

Then, \( (W_0, s) \) mod \( N = S_{k+1} \left( \left\lceil \frac{u}{s} \right\rceil + 1 \right) - W_{k-1} \) \[ \text{mod} \ N \] [From definition of \( W_{k+1} \)]

\[ = S_{k+1} \left( \left\lceil \frac{u}{s} \right\rceil + 1 \right) - S_{k-1} \text{mod} \ N \] [By induction hypothesis]

\[ = S_{k+1} \]

**THEOREM 2.** For \( 1 \leq p < W_i, p \text{ mod } N \geq S_{i+1} \), \( i = 2, 3, \ldots 

**PROOF.** For \( i = 2 \), \( W_2 = \left\lceil \frac{u}{s} \right\rceil + 1 \). Take \( p \) such that, \( 1 \leq p \leq \left\lceil \frac{u}{s} \right\rceil \). So, \( s \leq p \leq N \), i.e., \( p \) mod \( N \geq S_{i+1} \).

Let the result be true for \( 2 \leq i \leq k \), and \( \leq W_{i+1} \).

Now consider the case when \( S_{i+1} > W_{i+1} \). If we do not use any +s-link, the length of the path is \( u \). By Theorem 2, if we use any +s-link,

\[ \text{THEOREM 3.} \text{ For } u < S_0, \text{ the number of +s-links in a (+s, +1)-shortest path is} \]

\[ \begin{align*}
&1) \text{zero,} & \text{if} \ W_{i+1} \geq S_{i+1} \\
&2) \text{at least} \left\lceil \frac{u}{s} \right\rceil W_{i+1}, & \text{if} \ W_{i+1} < S_{i+1} \\
&3) \text{at least} \left\lceil \frac{u}{s} \right\rceil W_{i+1}, & \text{if} \ W_{i+1} = S_{i+1} \\
\end{align*} \]

**PROOF.** Let \( (p \text{ mod } N) \) be a (+s, +1)-shortest path from \( V_0 \) to \( V_i \). Since \( (0) \text{ mod } N \) is a (+s, +1)-path from \( V_0 \), \( p \leq u \). In particular, \( p \leq S_i \), So, \( p \geq S_{i+1} \). By Theorem 2, we must have \( p \geq W_{i+1} \).

If \( W_{i+1} > S_{i+1} \), then (+s, +1)-shortest path will not have any +s-link at all, because instead of using \( W_{i+1} \) +s-links, we can use \( S_{i+1} \) +1-links and get a shorter path. If the distance of \( V_{i+1} \) from \( V_i \) is at least \( S_{i+1} \), we can repeat the replacement of \( W_{i+1} \) +s-links by \( S_{i+1} \) +1-links and, repeating this, we can reach a node within \( S_{i+1} \) distance from \( V_i \), by using only +1-links. As \( S_{i+1} \leq S_{i+1} \), again by Theorem 2, the number of +s-links is either zero or at least \( W_{i+1} \); but a direct +1-path has length \( u \leq S_{i+1} < W_{i+1} \).

Now consider the case when \( S_{i+1} > W_{i+1} \). If we do not use any +s-link, the length of the path is \( u \). By Theorem 2, if we use any +s-link,

\[ \text{THEOREM 4.} \text{ For } u < S_0, \text{ the number of +s-links in a (+s, +1)-shortest path is} \]

\[ \begin{align*}
&1) \text{zero,} & \text{if} \ W_{i+1} \geq S_{i+1} \\
&2) \text{at least} \left\lceil \frac{u}{s} \right\rceil W_{i+1}, & \text{if} \ W_{i+1} < S_{i+1} \\
&3) \text{at least} \left\lceil \frac{u}{s} \right\rceil W_{i+1}, & \text{if} \ W_{i+1} = S_{i+1} \\
\end{align*} \]
THEOREM 4. The links of any shortest path \((p)[+s] + (q)[+1]\) with \(p, q > 0\), can always be ordered in a way such that it does not pass through a specified node.

PROOF. Without loss of generality, let the path be \((p)[+s] + (q)[+1]\). We also assume that one of the endpoints of the path is \(V_0\). First consider the following realization \(R\) of the path, where we traverse all the \(p\) \([+s]\)-links and then the \(q\) \([+1]\)-links. If the faulty node \(V_f\) is not on \(R\), then \(R\) gives us the path bypassing the faulty node. Suppose \(V_f\) is a node on \(R\).

Case I: \(f = z \mod N\), \(0 < z < p\). We consider another realization \(R'\) of the path \((p)[+s] + (q)[+1]\), where we first traverse a \([+1]\)-link, then \(p\) \([+s]\)-links and lastly the remaining \((q - 1)\) \([+1]\)-links. We claim that \(V_f\) is not a node on \(R'\). Suppose \(V_f\) is a node on \(R'\). The segments of \(R\) and \(R'\) from \(V_0\) to \(V_f\) must have equal length. Otherwise, we can replace the last segment of the shorter one to get a shorter path. Note that a typical node on \(R'\) is \(V_i\) where \(i = (t + s + 1) \mod N\), \(0 \leq t \leq p\). From the equality of path lengths from \(V_0\) to \(V_f\), we have, \(z = t + 1\).

Subcase Ia: \(f = (t + 1) \mod N\).

Subcase Ib: \(f = (t + s + 1) \mod N\), \(0 < s < p\). From the equality of path lengths from \(V_0\) to \(V_f\), we have \(z = p + j\). But we know that \(z = t + 1\).

Case II: \(f = (p + s + j) \mod N\), \(1 < j < s - 1\). Here we take \(R'\) to be the realization of the path on which we traverse \((j + 1)\) \([+1]\)-links, then the \(p\) \([+s]\)-links and then the rest of the \((q - 1)\) \([+1]\)-links. A typical node on \(R'\) is \(V_i\) where \(i = (t + s + 1) \mod N\), \(1 < t < p\) or \(i = (p + s) \mod N\), \(j + 2 < s \leq q\).

Subcase IIa: \(f = (t + s + 1) \mod N\), \(1 < j < s - 1\). Again, from the equality of path lengths from \(V_0\) to \(V_f\), we have, \(p + j = t\). So, \(p + j = (p + s + j) \mod (s - 1) \mod N = 0\). Contradiction!

Subcase IIb: \(f = (p + s + j) \mod N\), \(1 \leq j < s\). From the equality of path lengths from \(V_0\) to \(V_f\), we have, \(p + j = t + j + 1\). From the equality of path lengths from \(V_0\) to \(V_f\), we have, \(p + j = p + t + j\). Contradiction!

IV. ROUTING UNDER FAULT

In this paper, we have classified many values of \(N\) for which tight loop networks exist. Though this gives a much wider coverage than the classes defined by Du et al., some values of \(N\) remain yet to be classified. Some further works on the classification have also been reported in the literature [7], [8]. We also give an algorithm to find a shortest path between any pair of nodes and a near optimal routing in the presence of single node or link failure.

For improvement over the ring we have considered the addition two chords from every node. One may consider a further generalization where there are \(2k\) chords from every node. Let \(G(N, 1, s_1, s_2, \ldots, s_k)\) denote the supergraph of ring where from each node \(V_i\) there are links to the nodes \(V_{i+1}, V_{i+s_1}, V_{i+s_2}, \ldots, V_{i+s_k}\). Below, we list some of the problems which remain to be solved.
Modular Asynchronous Arbiter
Insensitive to Metastability

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Abstract—The purpose of this paper is to present a novel modular N-user asynchronous arbiter circuit which is insensitive to metastable operation (i.e., the new arbiter cannot fail because of metastability), operating asynchronously and incorporating a modular architecture. A 1.5µm CMOS prototype arbiter has been designed and tested. Laboratory tests demonstrate the arbiter operates correctly.

Index Terms—Asynchronous circuits, arbitration, metastability, modular design, Q-flop resolver.

I. INTRODUCTION

An N-user arbiter is a module designed to control access to a single common resource through the arbitration of contending request signals coming from N systems. This is a circuit component which has received a lot of attention for many years. Its key role in the control mechanism of asynchronous computer interactions has justified the proposal of many solutions for the problem of designing efficient arbiter implementations. This problem is still open for new ideas. Many different aspects have been addressed by the reported arbiters: programmability, priority schemes, hardware simplicity, modularity, resource granting speed. They have been some of the topics covered by researches [1], [2], [3]. One of the most relevant problems in arbiters is caused by the indetermination due to metastability [4], [5], [6].

The nondeterministic evolution of metastability may provoke diverse arbitration failures (the resource is not granted to any requesting device, the priority scheme is violated, the resource is granted to more than one user simultaneously, etc). Arbitration failures due to metastability have been reported (VMEbus controller and Multibus II [7]). Arbiters design must include, at least, a previous analysis of failure probability when entering to its metastable state. When the N users operate asynchronously (i.e., without a common clock), the metastable operation brings, as a consequence, indetermination in both output logic levels and time elapsed to reach a stable and correct state. However, there are some circuits (such those presented in [8]) which detect metastable operation and generate well-defined outputs at a logic level, even during metastability. This means that using circuit techniques, indetermination due to metastability is restricted to resolution time to the stable state, and arbitration failures (in a logical sense) can be avoided.

The purpose of this paper is to present a novel arbiter circuit insensitive to metastable operation, in the sense that it does not provoke any arbitration failure (i.e., the arbiter grants the resource according to the priority scheme and it guarantees mutually-exclusive access). Furthermore, the proposed arbiter incorporates a N-user modular architecture, operating asynchronously. The arbiter samples the request lines even if no request is present and grants the resource on a fix priority policy.

The paper is organized into five more sections. Section II deals with the overall structure of the new arbiter formed by interconnecting a few basic modules; Section III discusses the hardware implementation of these modules, while Section IV considers the essential issue of control signals and timing. In Section V, we give practical results validating the approach. Finally, we draw some conclusions in Section VI.

Manuscript received Nov. 23, 1992; revised Dec. 14, 1994.

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1) Deriving an analytical formula for the diameter of $G(N; 1, s)$,
2) Design of optimal loop networks for all values of $N$,
3) Optimal routing under single as well as multiple faults,
4) Analysis of generalized loop networks $G(N; 1, s_1, s_2, \ldots, s_k)$, etc.

REFERENCES