Euler Vector: A Combinatorial Signature for Gray-Tone Images *

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Abstract

A new combinatorial characterization of a gray-tone image called Euler Vector is proposed. Euler number of a binary image is a well-known topological feature, which remains invariant under translation, rotation, scaling, and rubber-sheet transformation of the image. Euler vector comprises of a 4-tuple, where each element is an integer representing the Euler number of the partial binary image formed by the four most significant bit planes of the gray-tone image. Experimental results demonstrate robustness of Euler vector under compression and inclusion of noise followed by filtering. The vector is topologically invariant and can be used for image indexing and retrieval.

Index Terms-Euler number, digital image processing.

1. Introduction

Feature extraction of a gray-tone image for fast and robust characterization is a challenging task. Compact image features are always needed for an efficient management of image database, search and retrieval. Defining a good numerical characterization of an image is a fundamental problem in image processing. Further, the characteristic parameters of the image should preferably remain invariant in the presence of various perturbations or transformations, such as translation, rotation, scaling, rubber-sheet shearing, inclusion of noise, compression, etc. Earlier approaches to image characterization include, i) spatial features like amplitude and histogram descriptors; ii) transform features like Fourier descriptor, DCT, iii) shape based features like boundaries , regions, area, Euler number, center of mass, moments, eccentricity, etc., iv) syntactic features based on structural peculiarities, v) statistical and structural texture features [1, 8, 9, 10]. Determination of a compact set of parameters for a gray-tone image which is easy to compute, suitable for efficient database search, and admits robustness against transformations, is now highly needed in the emerging domain of the Internet technology.

In this work, we define a new parameter called Euler vector of a gray-tone image. For a binary image, Euler number (genus) is a well-known geometric feature, which is defined as the difference between the number of connected components (objects) and the number of holes [2, 3, 4, 5, 12, 13]. Euler number is extensively used in optical character recognition, medical diagnosis, skin detection, and in many other applications. Efficient techniques of computing Euler number of a binary image are also well known [2, 4, 14, 15]. However, to the best of our knowledge, nothing is known in the literature on the use of Euler number or similar characterization of a gray-tone image. In an attempt to generalize the concept for a gray-tone image, we consider gray code representation of the intensity values in the pixel matrix, and observe the first 4 most significant bit planes. Each bit plane consists of only 0's and 1's, and hence forms a binary (two-tone) image. We compute Euler number of the partial image formed by each of the 4 bit planes to obtain a 4-tuple of integers, called Euler vector of the original gray-tone image. Bit planes based on gray codes were also used earlier for other bio-medical applications [7]. We give experimental results on a 31-image database. The Euler vector of a image is found to remain near invariant under inclusion of salt and pepper or gaussian noise followed by filtering, and also under JPEG compression. It has a strong discriminatory power and can thus be used to augment other features to facilitate image searching and retrieval.

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2. Combinatorial Feature of Gray Images

2.1. Bit-planes

We assume that a gray-tone image be represented as an $(N \times M)$ matrix, where each element is an integer lying between $[0, 255]$ denoting the intensity of the corresponding pixel. Thus, a 8-bit binary vector, $\{b_7, b_6, b_5, b_4, \ldots, b_0\}$ can represent the intensity value of each pixel. Each b_i is either '0' or '1'. Thus, the i^{th} bit-plane of the image is a binary matrix of size $N \times M$ and hence, can be thought of as a two-tone image. The given image therefore, comprises 8 bit planes.

2.2. Euler vector

To characterize a gray-tone image, we now define a 4-tuple called Euler vector. We retain the first *4 most significant bit planes* (corresponding to (b_7, b_6, b_5, b_4)) as they contain most of the information of the image, and ignore the remaining planes. However, each of these 4-bit binary vectors is converted to its corresponding reflected gray code (g_7, g_6, g_5, g_4)[11], which is defined as: $g_7 = b_7; g_6 = b_7 \oplus b_6; g_5 = b_6 \oplus b_5; g_4 = b_5 \oplus b_4$, where, $\oplus$ denotes XOR (modulo-2) operation. For any binary vector, the corresponding reflected gray code is unique and vice-versa. Consider the first *4 most significant bit planes* of the given image in gray code. Each bit plane now represents a two-tone image. See *Figure 1* for an example.

Definition: The Euler vector of a gray-tone image is a 4-tuple E_7, E_6, E_5, E_4 where E_i is the Euler number of the partial two-tone image formed by the i^{th} bit-plane, $7 \leq i \leq 4$, corresponding to the reflected gray code representation of intensity values. For the gray-tone image (Africa) shown in *Figure 1*, the Euler vector is found to be $\{79, -13, -391, -16\ 24\}$.

Gray code representation of intensity values offers a distinct advantage over standard binary representation in this particular context. Euler vector is found to be more insensitive to noise and other changes, if the gray code is used. This happens because two consecutive numbers have unit hamming distance in gray-code representation, and for most of the cases, a small change in intensity values is not likely to affect all the 4 bit planes simultaneously in gray code representation. Euler vector serves as a fundamental topological feature of a gray-tone image. Like Euler number of a binary image, it remains invariant under translation, rotation, scaling, and rubber-sheet transformation of the image. Since Euler number depends on the combinatorial properties of $0-1$ runs in the binary pixel matrix [12, 13] and is easily computable [14, 15], Euler vector of a gray-tone image also provides a quick combinatorial signature.

2.3. Implementation details

Noise can corrupt the image changing intensity levels and affect the bit-planes. To make the Euler vector more robust, the given image is cleaned successively by median and mean filters. Other sophisticated filters may also be used. Further, we rescale the dynamic range of intensity levels of the image. Visually similar images may differ in their dynamic ranges resulting in different bit-plane representations. To circumvent the problem, the images are rescaled such that their intensity level dynamic range is mapped to [0, 255]. We next describe the proposed algorithm.

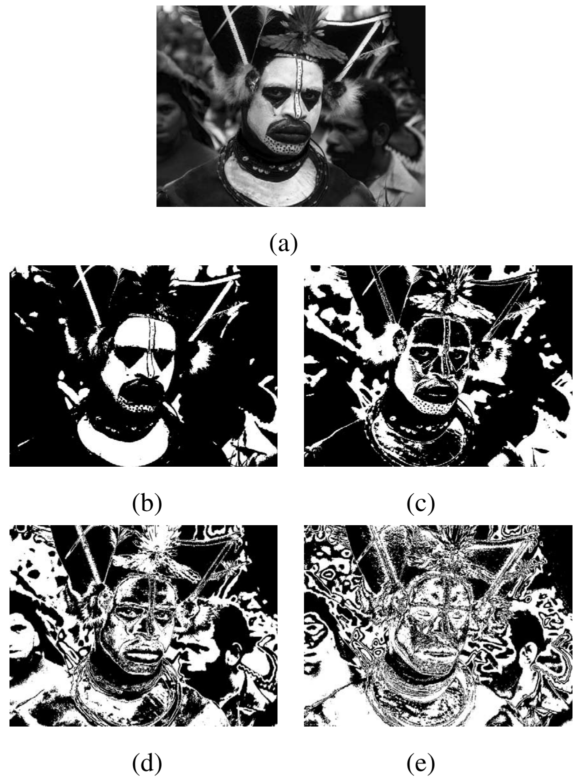

Figure 1. Gray-code bit planes and Euler vector

(a) Gray-tone image, (b) Most significant bit-plane g_7, Euler number $= 79$, (c) bit-plane g_6, Euler number $= -13$, (d) bit-plane g_5, Euler number $= -391$, (e) bit-plane g_7, Euler number $= -1624$.
Euler vector = $\{$79, -13, -391, -1624$\}$

2.4. Algorithm

Method
Input: A pixel matrix for a gray-tone image I.
Output: Euler vector.
Compute_Euler_Vector
Step 1: Apply median filtering followed by mean filtering on I;

Step 2: Linearly rescale the dynamic range of the image to [0-255];

Step 3: Consider the first 4 most significant binary bit planes of I;

Step 4: Convert each 4-bit vector to its corresponding reflected gray code;

Step 5: For each bit plane, compute Euler number;

Step 6: Output the Euler Vector;

3. Results

We have coded our algorithm in C and run on Ultra - 10 Sun Workstation. A database consisting of several gray-tone images is considered. Few samples are shown in *Figure 2*. For each of these images, we computed the Euler vector after cleaning them by median and mean filters. The effects of adding salt and pepper or gaussian noise, and JPEG compression have been studied. Results are shown in Table - 1. It has been observed that Euler vector provides a quick signature with robust behavior in the presence of noise and compression.

Each image is of size (480×640). For each image, the *Euler vector* $\{E_7, E_6, E_5, E_4\}$ is shown, the top one corresponds to E_7. Results shown in the *Table 1* for images shown in *Figure 2* were obtained after filtering the images successively by median and mean filter each of size (3×3). The *salt & pepper* noise was a random noise which affected 5% of the image ($=$ 15360 pixels approximately). The gaussian noise is a white noise with $'0'$ mean and variance $'0.05'$.

4. Conclusions

We have introduced a new combinatorial parameter for gray-tone images called Euler vector. It derives its definition from the concept of Euler number of a binary image. Euler vector retains the topological properties of Euler number and serves as a numerical signature of a gray-tone image. Results show that Euler vector is also robust against noise effects and compression.

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Table 1. Computation of Euler vector

Image name	Original image	Image corrupted by salt & pepper noise	Image corrupted by gaussian noise	JPEG compressed image
africa.g	79 -13 -391 -1624	78 -13 -406 -1624	83 3 -364 -1484	74 -10 -403 -1608
army.g	82 -150 -953 -1543	83 -145 -923 -1562	88 -85 -976 -1856	86 -167 -833 -1644
blaze.g	21 -44 -250 -880	20 -50 -245 -857	16 55 -1089 -1576	22 -41 -201 -974
castle.g	135 -297 -259 -1748	115 -341 -206 -1765	392 -360 177 -1657	121 -315 -172 -1768
cathed.g	401 -393 -429 -1840	407 -358 -376 -2097	441 -760 -174 -1858	405 -367 -368 -1753
cattle.g	116 -91 -579 -1119	116 -97 -579 -1103	155 -192 -672 -1388	120 -121 -528 -1051
chimp.g	-67 -82 -530 -1737	-72 -79 -557 -1550	-85 -113 -589 -1564	-73 -93 -526 -1711
choper.g	53 -201 141 -1155	58 -205 117 -1207	12 -279 222 -1303	37 -194 129 -1123
couple.g	76 -75 -480 -1882	80 -76 -479 -1894	46 -122 -78 -2016	84 -65 -443 -1727
fish.g	20 -122 -117 -972	19 -123 -111 -978	29 -89 -182 -773	20 -118 -106 -970

Image name	Original image	Image corrupted by salt & pepper noise	Image corrupted by gaussian noise	JPEG compressed image
goldfish.g	135 -2 -331 -842	140 -4 -312 -885	150 -51 -306 -854	137 -2 -294 -852
hawk.g	42 0 -49 -247	36 4 -46 -268	277 -20 225 -469	33 -2 -43 -230
ice.g	-10 -77 7 -10	-12 -74 6 -26	-57 -223 -163 -187	-8 -66 6 -4
insect.g	13 -185 -287 -1431	11 -186 -284 -1442	12 -402 -731 -1736	16 -161 -268 -1342
kid1.g	42 -80 -102 -539	50 -75 -104 -550	72 -112 -205 -35	23 -81 -106 -474
kid2.g	13 -86 -31 -734	11 -78 -47 -746	-316 -181 -274 -433	9 -84 -56 -586
kid3.g	11 -62 -143 -281	7 -57 -154 -278	-2 -274 -194 -184	8 -50 -140 -295
leaf.g	-44 -554 -337 -6735	-38 -591 -317 -6363	-94 -1174 145 -4284	-47 -564 -279 -6568
neweng.g	201 2 -376 -471	216 14 -412 -507	207 33 -514 -412	206 21 -384 -305
photogra.g	143 31 -480 -2510	140 40 -496 -2483	126 84 -410 -2684	150 51 -470 -2396

Figure 2. Some Sample Images