1. Suppose the following non-deterministic finite automaton (NFA) is converted to an equivalent deterministic finite automaton (DFA) using the standard algorithm. \[4\]

Determine whether each of the following statements is true or false.

(a) \( \delta(\{q_1\}, 0) = \{q_1, q_2\} \). \text{FALSE}

(b) \( \delta(\{q_2\}, 0) = \{\emptyset\} \). (This was a typo; it should read \( \delta(\{q_2\}, 0) = \emptyset \).) \text{TRUE}

(c) The state \( \{q_0, q_2\} \) is unreachable. \text{TRUE}

(d) The state \( \{q_0, q_1, q_2\} \) is a final state. \text{TRUE}

2. Write down the regular expression for hexadecimal numbers in C. \[4\]

\text{Answer: } \begin{align*} 0 & \{xX\} \begin{bmatrix} 0-9 \begin{bmatrix} a-f \begin{bmatrix} A-F \end{bmatrix} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \text{+} \end{bmatrix} \text{(-1/2 if you used * instead of +)} \end{align*}

3. The language \( L = \{0^p \mid p \text{ is prime} \} \) is not regular. If you have to prove this using the Pumping Lemma, how many times should you pump \( v \)? Your answer should be in terms of the lengths of \( u, v, w \) (\( u, v, w \) have their usual significance). \[6\]

\text{Answer: } \text{Let } x = uvw \in L. \text{ Then } uv^{|x|+1}w \not\in L. \\
(\text{Length of } uv^{|x|+1}w = |uvw| + |x||v| = |x|(1 + |v|), \text{ where } |v| \geq 1.) \\
\text{For just the correct answer (proof missing / incorrect), you get 2 marks.}
4. Let $M_1 = (Q_1, \Sigma, \delta_1, q_0^{(1)}, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_0^{(2)}, F_2)$ be two DFAs. Describe DFAs $M_\cup$ and $M_\cap$ that accept, respectively, $L(M_1) \cup L(M_2)$ and $L(M_1) \cap L(M_2)$.

$M_\cup$  

$M_\cap$

<table>
<thead>
<tr>
<th>States (1 mark)</th>
<th>$(Q_1 \times Q_2)$ for both</th>
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</thead>
<tbody>
<tr>
<td>Alphabet</td>
<td>$\Sigma$ for both</td>
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<tr>
<td>Transition (1 mark)</td>
<td>$\delta((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a))$ for both</td>
</tr>
<tr>
<td>Initial state (1 mark)</td>
<td>$(q_0^{(1)}, q_0^{(2)})$ for both</td>
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<tr>
<td>Final states (1.5 marks $\times$ 2)</td>
<td>$(Q_1 \times F_2) \cup (F_1 \times Q_2)$</td>
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