1. Fill in the blanks below with the correct expressions. \[1 \times 5 = 5\]

(a) Given a grammar \(G = (V, T, P, S)\), a symbol \(X \in V \cup T\) is said to be

(i) **useful** if \(S \Rightarrow^{*} \alpha X \beta \Rightarrow^{*} w\) for some \(\alpha, \beta \in (V \cup T)^{+}\) and \(w \in T^{*}\);

You get \(\frac{1}{2}\) if the two conditions are given separately; 0 if only 1 part is given.

(ii) **nullable** if \(X \Rightarrow^{*} \varepsilon\)

Full credit was given for “\(X \rightarrow \varepsilon\) or \(X \rightarrow X_{1} \ldots X_{n}\) and each \(X_{i}\) is nullable.”

(b) Given a grammar \(G = (V, T, P, S)\), a production \(A \rightarrow X_{1} \ldots X_{n}\) is said to be a unit production if \(n = 1\) and \(X_{1} \in V\).

(c) Given a pushdown automaton (PDA) \(P = (Q, \Sigma, \Gamma, \delta, q_{0}, Z_{0}, F)\), the language accepted by \(P\)

(i) by **final state** is given by \(L(P) = \{w \mid (q_{0}, w, Z_{0}) \vdash^{*}_{P} (q, \varepsilon, \alpha), q \in F, \alpha \in \Gamma^{*}\}\);

(ii) by **empty stack** is given by \(N(P) = \{w \mid (q_{0}, w, Z_{0}) \vdash^{*}_{P} (q, \varepsilon, \varepsilon), q \in Q\}\).

2. Recall that **postfix notation** is a method for writing arithmetic expressions in which every operator is written after all of its operands. For example, the postfix equivalent of \(A \times B + C/D\) is \(AB \times CD/+\). Write a context-free grammar (CFG) for arithmetic expressions in postfix notation involving variables and the operators +, −, × and /. You may assume that variable names consist of single letters only (as in the example above). You should use a single non-terminal \(S\). \[3\]

**Answer:** \(S \rightarrow SS + \mid SS - \mid SS \times \mid SS/ \mid A \mid \ldots \mid Z \mid a \mid \ldots \mid z\)

P.T.O.
3. Let \( L = \{ w | w \) is obtained by taking a syntactically correct C program and removing everything other than the keywords \textbf{if} and \textbf{else} from it\}. Draw the state diagram of a PDA that accepts \( L \) by empty stack. You may assume that \textbf{if} and \textbf{else} are single symbols. 

\[
\begin{array}{c}
\text{if, } Z_0 / \text{if } Z_0 \\
\text{if, if } / \text{if if} \\
\text{start } q_0 \\
\text{else, if } / \varepsilon \\
\varepsilon, \text{if } / \varepsilon
\end{array}
\]

-1 if the empty string is not handled correctly

-1 if excess if\( s \) are not popped at the end

4. Let \( G = (V, T, P, S) \) be a CFG in Greibach Normal Form. Let \( |V| = n, \; |T| = m, \; |P| = p \). Suppose that \( p_0 \) of the productions are of the form \( A_0 \rightarrow aA_1A_2 \ldots A_k \) where \( k \) is a fixed number, \( A_i \in V \) for \( 0 \leq i \leq k \) and \( a \in T \). The remaining productions are of the form \( A \rightarrow a \) where \( a \in T \). Let \( G' = (V', T, P', S) \) be a CFG in Chomsky Normal Form (CNF) obtained from \( G \) using the standard algorithm for conversion to CNF. Then:

\[
|V'| \leq n + p_0(k - 1) + \min(p_0, m) \quad |P'| \leq p + p_0(k - 1) + \min(p_0, m).
\]

Your bounds should be tight. Briefly justify your answer. 

Full credit was given for \( n + kp_0 \) and \( p + kp_0 \).

For each calculation error (while calculating \( |V'| \) or \( |P'| \)), you loose \( \frac{1}{2} \).