

Indian Statistical Institute

Semester-II 2012-2013

M.Tech.(CS) - First Year

Mid-semester Examination (20 February, 2013)

Subject: Automata, Languages and Computation

Maximum marks: 45

Total marks: 50

Duration 3 hrs.

To the extent possible, please be formal in your proofs, arguments, etc.

1. (a) Formally define

- (i) a non-deterministic finite automaton (NFA),
- (ii) the extended transition function δ^* for NFAs,
- (iii) acceptance of a string by an NFA.

(b) When is a relation over strings said to be right invariant?

(c) When is a context free grammar (CFG) said to be ambiguous?

[$(3+3+1) + 2 + 2 = 11$]

2. Draw the state diagram of a deterministic finite automaton (DFA) that recognises the set of all binary strings where the difference between the number of 0s and the number of 1s is even. [6]

3. Let Σ be a finite alphabet, L a language over Σ , and $h : \Sigma \rightarrow \Sigma^*$ a homomorphism. Prove that
(i) $h(h^{-1}(L)) \subseteq L$; (ii) $h^{-1}(h(L)) \supseteq L$. [5 + 5 = 10]

4. Let L be a regular language over an alphabet Σ . Prove that there exists a constant N such that if y is any string of length N and there are strings x and z such that $xyz \in L$, then y can be written as $y = uvw$ such that $|v| \geq 1$ and for each $i \geq 0$, $xuv^i wz \in L$. [7]

5. Let $\Sigma = \{0, 1, \approx, \boxplus\}$, and let

$L = \{a \approx b \boxplus c \mid a, b, c \in (0+1)^* \text{ and } a = b+c \text{ if } a, b, c \text{ are interpreted as unsigned binary integers}\}$.

Note that the string $10 \approx 01 \boxplus 01$ is in L since $2 = 1 + 1$, but the string $10 \approx 11 \boxplus 01$ is not since $2 \neq 3 + 1$. Show that L is not regular. [6]

6. Let L be the set of strings consisting of balanced pairs of square and round brackets. Thus, the strings $()$, $[\]$, $([\])$, and $(()) [\]$ all belong to L , but $([\]) \notin L$.

(a) Give an inductive definition for L . Assume that $\varepsilon \notin L$.

(b) Using (a), construct a context free grammar for L . Your grammar should have only one non-terminal.

[7 + 3 = 10]