SYLLABUS FOR ONE SEMESTER PRE-PH.D. COURSE WORKS FOR DIFFERENT UNITS OF THE INDIAN STATISTICAL INSTITUTE

Physics and Applied Mathematics Unit

For successful completion of the Pre Ph. D. Course work, all the Ph. D. students of this Unit have to opt for 5 courses and qualify in the examinations. The course is designed according to the guidelines set by the University Grants Commission.

Part I: (100 marks)
A. Research Methodology: (25 marks, 15 lectures)
B. Computer Applications and Statistical Methods: (45 marks, 35 lectures)
(To be conducted centrally)
C. Review Works and Assignment: (30 marks)

Part -II: (150 marks)
Any three (Total Marks-3 X 50= 150)
Course I: Mathematical Methods and Classical Mechanics
Course II : Advanced Quantum Mechanics
Course III : Electrodynamics and Statistical Mechanics
Course IV : Fluid Mechanics
Course V : Advanced Numerical Techniques
Course VI : Quantum Field Theory
Course VII : Particle Physics
Course VIII: General Relativity and Cosmology
Course IX : Condensed Matter Physics
Course X : Nonlinear Dynamics
Course XI : Quantum Information Theory

Detailed Course Structure

Research Methodology (Number of Lectures=15)
1) Meaning of Research Methodology ; Difference(s) between Methods of Research & Research Methodology ; Motivation for Research, Research approaches and Related Tools, Conditions and criteria for good research.
2) Choosing the research area, Factors leading to the choice, Defining the concrete research problem and focusing on it.
3) Importance of communication skill in Research- Development of power of expression in both speaking and writing, Mastery of presentation techniques. Progress-report writing on the Research topic(s).
4) Objectivity, Reality and Ethics in Research, Scientists in the Social World, The Dilemmas and the Decision –Makings vis-à-vis the Reality.

Computer Applications as suggested by Ex Dean of Studies, Prof. A. Bagchi (Number of Lectures=35) (To be conducted centrally)
1) Programming in C :
Constants, variables, data-types; operators : arithmetic, relational, logical, problems involving : assignments, increment / decrement, formatted input and formatted output, decision making and branching, creation of loops : For loop, While loop, and do-while loop. ID and 2D arrays, reading strings from terminal, writing strings to the screen.
2)Numerical Analysis : Significant digits, round-off errors, Finite computational processes and computational errors, Floating- point arithmetic and propagation of errors. Loss of significant digits. Interpolation with one variable finite differences, divided differences. Lagrangian and Newtonian methods. Iterative methods. Aitken Neville's iterative scheme. Spline interpolation. Errors and remainder terms. Inverse interpolation. Interpolation with two variables.

Numerical integration : Newton-Cotes; Orthogonal polynomials and Gaussian quadrature. Accuracy of quadrature formulae. Numerical differentiations; Numerical solution of ordinary differential equations : one step and multistep methods. Euler's, Adam's, Runge- Kutta's methods. Predictor-corrector methods. Errors and accuracy. Numerical solution of nonlinear equation in one variable: Separation of roots and initial approximation. Sturm's theorem.

Improvement of the initial solution using methods of bisection. Regula Falsi and Newton-Raphson, Fixed point iterative schemes. Errors. Order of convergence and degree of precision.
Numerical solution of system of linear equations and matrix inversion: Gaussian elimination, Square Root, L-U methods.

IA. Mathematical Methods (Number of Lectures=25)
1) Complex Variable: Complex Algebra, Cauchy Riemann Conditions, Cauchy's Integral Theorem, Cauchy's Integral Formula, Laurent Expansion, Mapping, Singularities, Calculus of Residues.
2) Differential equations: Partial Differential equations, Separation of Variables, Special functions, Orthogonal Polynomials.
3) Group Theory: Introduction, Generators of Continuous Groups, Orbital Angular Momentum – Ladder Operator Approach.
4) Calculus of Variations: Applications of the Euler Equation, Variation with Constraints – Lagrange Multipliers, Conservation laws and Noether's Theorem.

IB. Classical Mechanics (Number of Lectures=25)
1) Lagrangian Dynamics: Generalized Coordinates, Lagrangian function and Equations of Motion, Conservation Laws.
2) Hamiltonian Dynamics: Phase Space Degrees of Freedom, Hamiltonian Function, Poisson Brackets and Equations of Motion
3) Introduction to Constraint Systems: Classification of Constraints (Primary - Secondary), Dirac Classification of Constraints (First Class – Second Class), Gauge Invariance and First Class Constraints, Solving Second Class Constraints (Dirac Brackets)

II. Advanced Quantum Mechanics (Number of Lectures=50)
1. Formalism of Hilbert space quantum mechanics: Basic postulates, Dynamics (Schrodinger picture, Heisenberg picture and Interaction picture), uncertainty relation, complementary principle.
2. Some exactly solvable problems: Harmonic oscillator, Hydrogen Atom.
3. Theory of Angular momentum, Introduction to spin,
4. Introduction to scattering theory.
5. Symmetries in quantum mechanics : Symmetries, Conservation laws and degeneracy, Parity and space inversion, Permutation symmetry, Introduction to identical particles,
6. Approximate methods: Perturbation theory (degenerate and Non-degenerate cases), WKB approximation,Variational methods.
7. Basics of supersymmetry.
8. Relativistic quantum mechanics: Klein-Gordon equation, Dirac equation- interpretation.
9. Introduction to Foundations of quantum mechanics : Phase space description of quantum mechanics,
Wigner function, The EPR theorem, Bell's theorem, No-cloning theorem

IIIA. Electrodynamics (Number of Lectures=25)
1. Electromotive Force: Ohm's law, Motional e.m.f.
2. Electromagnetic Induction: Faraday's law, Induced electric field, Inductance, Energy in magnetic field
3. Maxwell's Equations: Electrodynamics before Maxwell's equations, Magnetic Charge, Maxwell's equation in vacuum and matter, Boundary Conditions

4. Conservation Laws: Charge, Energy and Momentum, Continuity equation, Poynting's Theorem
5. Electromagnetic Waves: The wave equation, Boundary Conditions, Polarization, Electromagnetic waves in vacuum, matter and conductors
6. Potential and Fields: Scalar and Vector Potentials, Gauge transformations, Retarded Potentials, Lienard-Wiechert Potentials
7. Dipole Radiation: Electric and Magnetic, Radiation from an arbitrary source

IIIB. Statistical Mechanics (Number of Lectures=25)
1. Recapitulation of Classical Statistical Mechanics: Phase Space, Entropy, Gibb's Paradox
2. Ensemble Theory: Liouville's theorem, Microcanonical, Canonical and Grand canonical ensembles, Partition Function
3. Density Matrix Formulation of quantum statistical mechanics: Density operators, Pure and Mixed states
4. Ideal Gases: Ideal Bose gas, Ideal Fermi gas
5. Variational Principles
6. Bose and Fermi Systems
7. Ising Model

IV. Fluid Mechanics (Number of Lectures=50)
1) Fundamental Laws of Viscous Fluid: Real and perfect fluids, Principle of similarity, Boundary layer concepts, Separation and vortex formations.
2) Equations of motions of a compressible viscous fluid (Navier-Stokes equations). Fundamental equations of continuity and motion for a fluid flow; Reynolds' principles of similarity from the Navier- Stokes equations; Limiting cases of large and small viscous forces.
3) Exact solutions of Navier-Stokes Equations in some special cases, like flow through a straight channel and Couette flow; Poiseuille flow through a pipe; Stokes' first and second problems; Stagnation in plane flow; Slow motion.
4) Laminar boundary layers: Approximations and derivation of boundary layer equations; Separation of boundary layers; Integration of boundary layers; Skin friction; General properties of boundary layer equations.

V. Advanced Numerical Techniques (Number of Lectures=50)
Iterative methods for linear systems: Classical iterative methods (Jacobi, Gauss-Seidel and successive over relaxation (SOR) methods), Krylov subspace methods; GMRES, Conjugate-gradient, biconjugate- gradient (BiCG), BiCGStab methods, preconditioning techniques, parallel implementations.
Finite difference method: Explicit and implicit schemes, Crank-Nicolson schemes, consistence, stability and convergence, Lax's equivalence theorem, numerical solutions to elliptic, parabolic and hyperbolic partial differential equations. Dirichlet, Neumann and Mixed problems. Sparseness and the ADI method.
Approximate method of solution: Galerkin method, properties of Galerkin approximations, Petrov-Galerkin method, generalized Galerkin method.
Finite Element Method (FEM): FEM for second order problems, one and two dimensional problems, finite elements (elements with a triangular mesh and a rectangular mesh and three dimensional finite elements), fourth-order problems, Hermite families of elements, isoparametric elements, numerical integration.

VI. Quantum Field Theory (Number of Lectures = 50)
i. (a) Meaning and objective of Quantization of Fields, Types of Fields .
(b) (i) Quantization of Scalar Fields, (ii) Quantization of Spinorial (Dirac) Fields, (iii) Quantization of Vector (Electromagnetic) Fields.
ii). The Interaction of Fields, The S-Matrix : Idea and properties, the unitarity of the S-Matrix and

the Optical Theorem.
iii). Iteration solution of the S-Matrix, Ordering theorems and the Introduction to Feynman Diagram Techniques.
iv) Some Applications : Calculations of (i) Compton scattering, (ii) e - e + Annihilition.v) Divergences and the Self-Energy of some particles, Ideas and calculations of charge and Mass Renormalisation.

VII. Particle Physics (Number of Lectures = 50)
i) A preview of particle physics and some basics.
ii) Symmetries and Quarks : SU(2), SU(3) etc.
iii) The structure of Hadrons : Form factors, e-p Scatterings, Protonic form factor, Inelastic ep→ex Scatterings [Properties in some detail].
iv) Partons : Bjorken scaling, The Quarks within the proton, what and where the 'gluon's are ?
v) QCD : Role of gluon's. Introduction to parton model, Scaling violations. The Altarelli-Parisi equation and complete evolution equations for the parton densities.
vi) e + e - Annihilation and QCD
a)Two-Jet structures in e - e + → qq (quark-antiquark)
b)Three-Jet events in e - e + → qq g (quark-antiquark-gluon)
c)A derivation of the e - e + → qq g cross-section
d)Perturbative QCD and Multiparticle production phenomena : an outline.
vii) Weak interactions : properties.
viii) Electroweak interactions : properties.
ix) The Weinberg-Salam Model and Beyond.
x) Gauge symmetries, Symmetry breaking and 'Hidden' symmetry, The 'Higgs' Mechanism.

VIII: General Relativity and Cosmology (Number of Lectures = 50)
1) Preliminaries: Why GR, Space-time curvature, Einstein's equations, Spherically symmetric geometry, Static spherically symmetric pathology.
2) Advanced topics: Gravitational collapse; Gravitational lensing: lensing geometry, magnification and shear, applications; Gravitational waves: propagation of fluctuations, radiation emission, radiated power, indirect and direct detection techniques.
3) Basic cosmology: Friedmann equations, Observables and measurements, Survey of present status of observational cosmology.
4) Early universe: Inflationary paradigm, Slow roll technique and model building algorithm, Quantum fluctuations and observable parameters, Construction of inflationary models and typical examples.
5) Cosmic Microwave Background radiation: Generation of temperature anisotropies and reflection on CMB multipoles, Adiabatic and isocurvature modes, CMB polarization, Lensing in CMB, Cosmological parameters from CMB: observational status.
6) Post-inflationary perturbations: Perturbations in geometry and matter, Evolution of super-Hubble and sub-Hubble modes, Sachs Wolfe (and Integrated Sachs Wolfe) effect, Baryon acoustic oscillations, Transfer function and numerical fit.
7) Estimation of power spectra: Gaussian and non-gaussian distributions, Links to CMB and galaxy redshift surveys.
8) Dark energy: Different dark energy models: pros and cons; Reconstruction of parameters; Observational status: reflection inSNIa, CMB and growth of structures; Chi-square analysis; Dark energy perturbations.
9) Statistical tools: Bayesian methods and maximum likelihood analysis, Cross-correlation of data, Evidence calculation and model discrimination techniques, Application to CMB and Dark energy surveys.

IX: Condensed Matter Physics (Number of Lectures = 50)
1. Chemical bonding
2. Crystal structures and symmetries
3. Classical theory of crystal diffraction
4. Electronic band structure of solids
5. Transport properties
6. Magnetism
7. Superconductivity
8. Many-body theory

X: Nonlinear Dynamics (Number of lectures =50)
1. Dynamical Systems: Maps and Continuous Systems, fixed point analysis, local stability and global stability of fixed points, normal form analysis.
2. Chaotic dynamics: Poincaŕe map, Bifurcations, Lyapunov exponent, route to chaos, Homolcinic and heteroclinic chaos.
3. Delay differential equations: local stability analysis, bifurcation and numerical simulations.
4. Synchronization: Complete, phase, lag and generalized synchronization. Coupling: Linear, nonlinear and design of coupling, time varying coupling.
5. Cryptography using chaos: Encryption, decryption and secure communication using chaos synchronization.
6. Complex dynamical network: stability behavior of regular network, small-world and scale free network (with and with-out time delayed).

XI: Quantum Information Theory (Number of lectures =50)
1. **Fundamental Concepts**: Basic linear algebra, postulates of quantum mechanics, qubits, composite quantum systems, entanglement, Bell states.
2. **Density Operator**: General properties, Bloch sphere representation, projective measurement, POVM, Schmidt decomposition, reduced density operator, noisy evolution.
3. **Quantum Operations**: Classical noise, quantum system – environment interaction, operator - sum representation, axiomatic approach, dynamical evolution of the density operator under some quantum channels like Bit flip, Phase flip, Depolarizing and Amplitude Damping.
4. **Entanglement Theory**: Separability problem, completely positive maps, witnesses, entanglement measures.
5. **Purification**: Purification of a density operator, isometric extension of single qubit channels, Quantum instrument.
6. **Unit Quantum Protocols**: Entanglement distribution, super dense coding, quantum teleportation, optimality of these protocols, unit source capacity region.
7. **Coherent Protocols**: Coherent Communication, coherent dense coding and teleportation.
8. **Distance Measures**: Norms, Trace Distance and its properties, Hilbert – Schmidt distance, Fidelity and its properties, relation between Trace Distance and Fidelity.
9. **Classical Information**: Shannon Entropy and it's basic properties, conditional and joint entropy, mutual information, relative entropy, the Fundamental Information Inequality, Data Processing Inequality, Fano's Inequality, Accessible information, Shannon's source coding theorem, channel capacity, Shannon's channel coding theorem,
10. **Quantum Information**: von Neumann Entropy and it's properties, joint and marginal entropies, conditional quantum entropy and it's operational interpretation, coherent information, quantum mutual information, Holevo information, quantum relative entropy, some important quantum informational inequalities.
11. **Quantum Channels:** Mutual information of a classical channel, private information of a wiretap channel, Holevo information and mutual information of a quantum channel, coherent and private information of a quantum channel, quantum data compression.