Differentiation of sets in measure – "fold-up" derivatives

Estáte Khmaladze

Victoria University of Wellington

We will start with one class of spatial statistical problems, so called "change-set problems", and explain how does it lead to set-valued analysis - the field not usually connected with probability and statistics. We will then demonstrate why does this problem require a new type of derivative of a set-valued function, different from the notions used so far.

Then we define this derivative – we call it here "fold up" derivative – and show few properties. In particular we will state the following result:
suppose $K(\varepsilon)$, for each $\varepsilon \in [0,1]$, is bounded Borel subset of $\mathbb{R}^d$ and $K(\varepsilon) \to K(0)$ as $\varepsilon \to 0$. Let $A(\varepsilon) = K(\varepsilon) \triangle K(0)$ be symmetric difference and $\mathbb{P}$ be an absolutely continuous measure on $\mathbb{R}^d$. If $dK(\varepsilon)/d\varepsilon = dA(\varepsilon)/d\varepsilon$ is the fold up derivative, then

$$\frac{d}{d\varepsilon}\mathbb{P}(A(\varepsilon))|_{\varepsilon=0} = \mathbb{Q}(\frac{d}{d\varepsilon}A(\varepsilon)|_{\varepsilon=0}),$$

where $\mathbb{Q}$ is another, explicitly described, measure, although not in $\mathbb{R}^d$.

We briefly compare fold up derivative with other existing notions: the generalised functions, (semi-, quasi-) affine mapping, derivative cones of Auben and Clarke.

We also discuss other possible applications – to the theory of local processes, to extreme value theory, and some other. At the end we will discuss possible generalisations and open problems.

The talk is based on the paper

[1] E. V. Khmaladze (2007). Differentiation of sets in measure. *J. Mathemat. Analysis Appl.* **334**, 1055-1072.

Other references, important for the talk, are:

[2] Z. Artstein (2000). Invariant measures of set-valued maps. *J. Mathemat. Analysis Appl.* **252**, 696-709.

[3] J.-P. Aubin and A. Cellina (1984). *Differential Inclusions.* Springer Verlag.

[4] J.-P. Aubin and H. Frankowska (1990). *Set-valued analysis.* Birkhäuser.

[5] J. H. J. Einmahl and E. V. Khmaladze (2007). Central limit theorems for local empirical processes near boundaries of convex sets. CentER Discussion Papers, nr. 66.

[6] S. Gautier (1990). Affine and eclipsing multifunctions. *Numer. Funct. Anal. Optim.* **11**, 679-699.

[7] E. V. Khmaladze, R. M. Mnatsakanov and N. Toronjadze (2006). Local covering numbers and change-set problem. *Mathemat. Methods Statist.* **15**, 272-293.

[8] E. V. Khmaladze and W. Weil (2007). Local empirical processes near boundaries of convex bodies. *Ann. Inst. Statis. Math.*, to appear.

[9] C.Lemaréchal and J. Zowe (1991). The eclipsing concept to approximate a multi-valued mapping. *Optimization*, **22**, 3-37.

[10] D. Silin (1997). On set-valued differentiation and integration. *Set-Valued Analysis*, **5**, 107-146.

[11] H. Weisshaupt (2001). A Measure-Valued Approach to Convex Set-Valued Dynamics. *Set-Valued Analysis*, **9**, 337- 373.

`List of invited speakers`   `Schedule for December 11`