The use of this parameter introduces the concept of a unit memory into the algorithm in that the previous iterate is directly involved in the updating process, the degree of involvement being proportional to $1 - p$. This concept may be extended to include two or more previous iterates; this extension has, however, not proved to have produced any significant advantages over the unit memory method.

**Example**: This example serves to illustrate the voltage update method and, in particular, the speed advantage this method has over Kerr’s basic approach. The circuit is that analysed by Kerr and his paper verifies that it may be successfully analysed by the insertion of a transmission line. The voltage update method was successfully used $(p = 0.025)$ and, as expected, identical waveforms are obtained; however, the voltage update method gives rise to a convergence diagram which differs from that of Kerr (Fig. 2). A comparison between the convergence diagrams of the Kerr method and the voltage update approach indicates that initially the voltage update convergence rate is less than that obtainable with the transmission line approach. However the voltage update method reaches the solution in 350 iterations, compared with the 500 iterations necessary in Kerr’s approach. Moreover, the calculations per iteration are smaller for the voltage update technique, thereby resulting in a significantly more efficient analysis program. Following Kerr, convergence was deemed to have occurred when the harmonic impedance ratios were within $0.5\%$ of unity.

**Acknowledgments**: This project was supported by the United States ARO under Grant DAA G29-76-G-0279 and the Australian Research Grants Committee under Grant F76/15147.

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**Fig. 2** Convergence diagrams  
(a) Kerr’s method  
(b) Voltage update method $(p = 0.025)$

**Definitions**: A set of events $x_1, x_2, \ldots, x_n$ in the universe of discourse $U$ is said to be a fuzzy set $A$ if the transition from membership to nonmembership is gradual rather than abrupt. Such a set is characterised by a membership function $\mu_A(x)$, $0 \leq \mu_A(x) \leq 1$, which denotes the grade of membership of an event $x_i$ to set $A$. This characteristic function can be considered as a weighting coefficient which reflects the ambiguity (fuzziness) in a set. Mathematically $A$ is defined as

$$A = \{ \mu_A(x_i) / x_i \} \quad i = 1, 2, \ldots, n$$

Similarly, the property $p$ defined on an event $x_i$ is a function $p(x_i)$ which can have values only in the interval $(0, 1)$. A set of these functions which assigns the degree of possessing some property $p$ by the event $x_i$ constitutes what is called a property set.

**Formulation of enhancement algorithm**: Let

$$X = \begin{bmatrix} p_{11} / x_1 & p_{12} / x_2 & \ldots & p_{1N} / x_N \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} / x_M & p_{M2} / x_M & \ldots & p_{MN} / x_M \end{bmatrix}$$

represent the pattern corresponding to an $M \times N$ image array to be enhanced where $p_{mn} / x_m$; $0 \leq p_{mn} \leq 1, m = 1, 2, \ldots, M, N = 1, 2, \ldots, N$, denotes the fraction of the maximum intensity (grey level) possessed by the $(m, n)$th picture element $x_{mn}$. Here $p_{mn} = 0$ denotes dark and $p_{mn} = 1$ bright. Any intermediate value indicates the grade of maximum grey level of the pixel. Now for each $p$ in $x$, we give a transformation of the form

$$p'_{mn} = T(p_{mn}) = \begin{cases} T_1(p_{mn}) & 0 \leq p_{mn} \leq 0.5 \\ T_2(p_{mn}) & 0.5 \leq p_{mn} \leq 1 \end{cases}$$

**IMAGE ENHANCEMENT USING FUZZY SET**

**Indexing terms**: Image processing, Logic

A method of image enhancement by computer using the fuzzy set theoretic approach is reported. The algorithm involves extraction of fuzzy properties corresponding to pixels and then successive application of fuzzy operator 'contrast intensification' on the property plane. System performance with different indexes of fuzziness is demonstrated for an English script input.

**Introduction**: The object of enhancement technique is to process a given image so that the result is more suitable than the original for a specific application. The term 'specific' is, of course, problem oriented. The methods so far developed for image enhancement may be categorised in two broad classes, namely, frequency-domain methods and spatial-domain methods. The technique in the first category is based on modifying the Fourier transform of an image, whereas in spatial domain methods the direct manipulation of the pixel is adopted. The present work is an attempt to illustrate the application of fuzzy set theory to the problems of image enhancement. The technique used here is based on the modification of pixels in the fuzzy property plane of an image. The property domain is extracted from the spatial domain using fuzzifiers which play the role of creating different amounts of fuzziness in the plane. The fuzzy operator 'INT' (contrast intensification) is taken as a tool for enhancement. The effectiveness of this algorithm is demonstrated on a picture of handwritten English recursive script. A quantitative measure of quality for different enhanced outputs is indicated by the term 'index of fuzziness'. The digital computer CDC-6500/6400 was used as a processing system.
where \( r = 1, 2, \ldots; \)

\[
T_1(p_{mn}) = 2(p_{mn})^2; \quad T_1(p_{mn}) = 1 - 2(1 - p_{mn})^2
\]

and

\[
T_k(p_{mn}) = T_1(T_{k-1}(p_{mn})); \quad T_k(p_{mn}) = T_1(T_{k-1}(p_{mn}));
\]

\[ k = 1, 2, \ldots \]

The operator \( T_k(p_{mn}) \) is known as 'contrast intensifier' \( \text{INT} \) of a fuzzy set. \( T_k(p_{mn}) \) denotes the \( k \)th successive application of \( \text{INT} \) of the operator INT. This transformation reduces the fuzziness of \( X \) by increasing the values of \( p_{mn} \) which are above 0.5 and decreasing those which are below it. The modified levels \( p'_{mn} \) thus produced would therefore result in an enhancement of the image. This is explained graphically in Fig. 1.

![INT transformation function for contrast enhancement](image)

As \( r \) increases, the curves tend to be steeper because of the successive application of INT. It is to be noted here that, corresponding to a particular operation of \( T^r \), one can use any of the multiple operations of \( T^r \), and vice versa, to attain a desired amount of enhancement. It is up to the user how he will interpret and exploit this flexibility, depending on the problems to hand.

**Property plane and fuzzification:** All the operations discussed above are restricted to only the property plane. To enter this domain from the \( p^-\)-plane, we define an expression of form similar to that defined by one of the authors in speech recognition.

\[
p_{mn} = G(x_{mn}) = \left[ 1 + \frac{(x_{mn} - x_{max})}{F_d} \right]^{-F_e}
\]

where \( x_{max} \) denotes the maximum grey level desired, \( F_e \) denotes the exponential fuzzifier, \( F_d \) denotes the denominational fuzzifier, \( m = 1, 2, \ldots M \) and \( n = 1, 2, \ldots N \). Also \( p_{mn} \) denotes the degree of possessing maximum brightness \( x_{max} \) by the \((m, n)\)th pixel. The fuzzifiers have the effect of altering ambiguity in the \( p^-\)-plane. As a result, it is these two positive constants whose amount of enhancement. It is up to the user how he will inter-

[Credit: ELECTRONICS LETTERS 8th May 1980 Vol. 16 No. 10]
reflecting the ambiguity in a picture is reduced through enhancement.

Acknowledgment: The authors acknowledge the valuable help rendered by Programme Advisory of ICCC, B. Singh and P. Cheung in programming, P. Saraga for provision of data and V. Hikel in typing the manuscript. The financial assistance of the Association of Commonwealth Universities in the UK and Indian Statistical Institute, Calcutta, is also gratefully acknowledged by one of the authors (S. K. Pal).

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2nd April 1980

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NOVEL ACTIVE FILTERS USING AMPLIFIER POLE

Indexing terms: Active filters, Poles and zeros

Active R realisation of bandpass and lowpass filters using lossy simulated inductors and capacitors is described.

Recently, inductance and capacitance simulation using the amplifier pole has received considerable attention.1-4 The simplest lossy grounded inductance simulator is shown in Fig. 1a. In this letter, we consider a lossy simulated grounded capacitor using the amplifier pole and use it to replace the external capa-

Fig. 1 Lossy simulated inductor (a) and lossy simulated capacitor (b) using the amplifier pole

Fig. 3 Enhanced picture output
a $F_x = 45, F_y = 2$
b $F_x = 43, F_y = 2$
c $F_x = 40, F_y = 2$