The use of this parameter introduces the concept of a unit memory into the algorithm in that the previous iterate is directly involved in the updating process, the degree of involvement being proportional to $1 - p$. This concept may be extended to include two or more previous iterates; this extension has, however, not proved to have produced any significant advantages over the unit memory method.

Example: This example serves to illustrate the voltage update method and, in particular, the speed advantage this method has over Kerr's basic approach. The circuit is that analysed by Kerr[5] and his paper verifies that it may be successfully analysed by the insertion of a transmission line. The voltage update method was successfully used ($p = 0{\cdot}025$) and, as expected, identical waveforms are obtained; however, the voltage update method gives rise to a convergence diagram which differs from that of Kerr (Fig. 2). A comparison between the convergence diagrams of the Kerr method and the voltage update approach indicates that initially the voltage update convergence rate is less than that obtainable with the transmission line approach. However the voltage update method reaches the solution in 350 iterations, compared with the 500 iterations necessary in Kerr's approach. Moreover, the calculations per iteration are smaller for the voltage update technique, thereby resulting in a significantly more efficient analysis program. Following Kerr, convergence was deemed to have occurred when the harmonic impedance ratios were within 0·5% of unity.

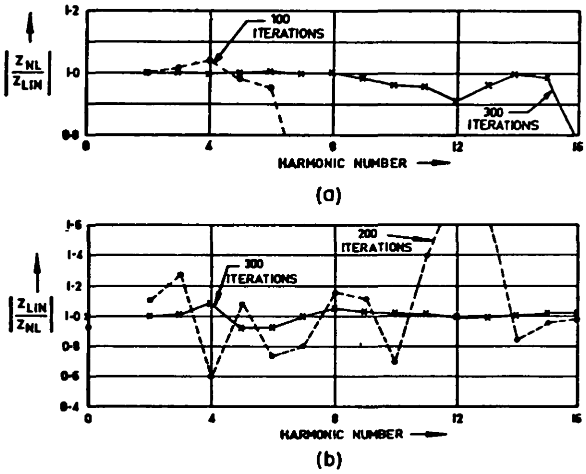

Fig. 2 *Convergence diagrams*

a Kerr's method

b Voltage update method ($p = 0{\cdot}025$)

Z_L is impedance presented by linear network at each harmonic; Z_{NL} is nonlinear harmonic impedance evaluated during each iteration cycle

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R. G. HICKS
P. J. KHAN

Electrical Engineering Department
University of Queensland
St. Lucia, Qld., 4067, Australia

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IMAGE ENHANCEMENT USING FUZZY SET

Indexing terms: Image processing, Logic

A method of image enhancement by computer using the fuzzy set theoretic approach is reported. The algorithm involves extraction of fuzzy properties corresponding to pixels and then successive application of fuzzy operator 'contrast intensification' on the property plane. System performance with different indexes of fuzziness is demonstrated for an English script input.

Introduction: The object of enhancement technique is to process a given image so that the result is more suitable than the original for a specific application. The term 'specific' is, of course, problem oriented. The methods so far developed for image enhancement may be categorised in two broad classes,[1] namely, frequency-domain methods and spatial-domain methods. The technique in the first category is based on modifying the Fourier transform of an image, whereas in spatial domain methods the direct manipulation of the pixel is adopted. The present work is an attempt to illustrate the application of fuzzy set[2,3] theory to the problems of image enhancement. The technique used here is based on the modification of pixels in the fuzzy property plane of an image. The property domain is extracted from the spatial domain using fuzzifiers[4] which play the role of creating different amounts of fuzziness in the plane. The fuzzy operator 'INT' (contrast intensification) is taken as a tool for enhancement. The effectiveness of this algorithm is demonstrated on a picture of handwritten English recursive script. A quantitative measure of quality for different enhanced outputs is indicated by the term 'index of fuzziness'. The digital computer CDC-6500/6400 was used as a processing system.

Definitions: A set of events $x_1, x_2 \ldots x_n$ in the universe of discourse U is said to be a fuzzy set A if the transition from membership to nonmembership is gradual rather than abrupt. Such a set is characterised by a membership function $\mu_A(x_i)$, $0 \leq \mu_A(x_i) \leq 1$, which denotes the grade of membership of an event x_1 to set A. This characteristic function can be considered as a weighting coefficient which reflects the ambiguity (fuzziness) in a set. Mathematically A is defined as

$$A = \{\mu_A(x_i)/x_i\} \qquad i = 1, 2, \ldots n \tag{1}$$

Similarly, the property p defined on an event x_i is a function $p(x_i)$ which can have values only in the interval (0, 1). A set of these functions which assigns the degree of possessing some property p by the event x_i constitutes what is called a property set.[5]

Formulation of enhancement algorithm: Let

$$X = \begin{bmatrix} p_{11}/x_{11} & p_{12}/x_{12} & \cdots & p_{1N}/x_{1N} \\ p_{21}/x_{21} & p_{22}/x_{22} & \cdots & p_{2N}/x_{2N} \\ \vdots & \vdots & & \vdots \\ p_{M1}/x_{M2} & p_{M2}/x_{M2} & \cdots & p_{MN}/x_{MN} \end{bmatrix}$$

represent the pattern corresponding to an $M \times N$ image array to be enhanced where p_{mn}/x_{mn}, $0 \leq p_{mn} \leq 1$, $m = 1, 2, \ldots M$, $N = 1, 2, \ldots N$, denotes the fraction of the maximum intensity (grey level) possessed by the (m, n)th picture element x_{mn}. Here $p_{mn} = 0$ denotes dark and $p_{mn} = 1$ bright. Any intermediate value indicates the grade of maximum grey level of the pixel. Now for each p in x, we give a transformation of the form

$$p'_{mn} = T_r(p_{mn}) = \begin{cases} T'_r(p_{mn}) & 0 \leq p_{mn} \leq 0{\cdot}5 \\ T''_r(p_{mn}) & 0{\cdot}5 \leq p_{mn} \leq 1 \end{cases} \tag{2}$$

where $r = 1, 2, \ldots;$

$$T'_1(p_{mn}) = 2(p_{mn})^2; \qquad T''_1(p_{mn}) = 1 - 2(1 - p_{mn})^2 \tag{3}$$

and

$$T'_k(p_{mn}) = T'_1\{T'_{k-1}(p_{mn})\}; \qquad T''_k(p_{mn}) = T''_1\{T''_{k-1}(p_{mn})\}$$
$$k = 1, 2, \ldots$$

The operator $T_1(p_{mn})$ is known as 'contrast intensifier'[2] (INT) of a fuzzy set. $T_r(p_{mn})$ denotes the rth successive application of the operator INT. This transformation reduces the fuzziness of X by increasing the values of p_{mn} which are above 0·5 and decreasing those which are below it. The modified levels p'_{mn} thus produced would therefore result in an enhancement of the image. This is explained graphically in Fig. 1.

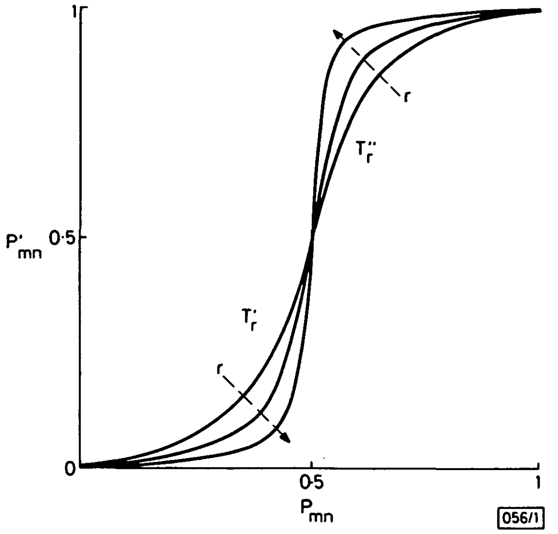

Fig. 1 *INT transformation function for contrast enhancement*

As r increases, the curves tend to be steeper because of the successive application of INT. It is to be noted here that, corresponding to a particular operation of T', one can use any of the multiple operations of T'', and vice versa, to attain a desired amount of enhancement. It is up to the user how he will interpret and exploit this flexibility, depending on the problems to hand.

Property plane and fuzzification: All the operations discussed above are restricted to only the property plane. To enter this domain from the x_{mn}-plane, we define an expression of form similar to that defined by one of the authors in speech recognition:[4]

$$p_{mn} = G(x_{mn}) = \left[1 + \frac{(x_{max} - x_{mn})}{F_d}\right]^{-F_e} \tag{4}$$

where x_{max} denotes the maximum grey level desired, F_e denotes the exponential fuzzifier, F_d denotes the denominational fuzzifier, $m = 1, 2, \ldots M$ and $n = 1, 2, \ldots N$. Also p_{mn} denotes the degree of possessing maximum brightness (x_{max}) by the (m, n)th pixel. The fuzzifiers have the effect of altering ambiguity in the p-plane. As a result, it is these two positive constants whose values control the placing of threshold (0·5) and hence the ultimate performance of the system.

Eqn. 4 shows that $p_{mn} \to 1$ as $(x_{max} - x_{mn}) \to 0$ and p_{mn} decreases as $(x_{max} - x_{mn})$ increases. For $x_{mn} = 0$, p_{mn} holds a finite positive value α, say. So the p_{mn}-plane becomes restricted in the interval (α, 1) instead of (0, 1). After processing, the modified p'_{mn}-plane may contain some region where $p'_{mn} < \alpha$ due to the transformation T'. The algorithm, for those values, did include a boundary at $p'_{mn} = \alpha$ so that the inverse transformation

$$x'_{mn} = G^{-1}(p'_{mn}) \qquad \alpha \leq p'_{mn} \leq 1 \tag{5}$$

will allow those corresponding x'_{mn}-values to have zero grey level. Of course, one can change α to some other value depending on the contrast or background level desired.

The amount of fuzziness present in both the original and modified property planes of the pattern X can be measured by the term 'linear index of fuzziness', which is defined as

$$\nu_1(X) = \frac{2}{MN} \sum_m \sum_n \mu_{X \cap \bar{X}}(x_{mn}) \tag{6}$$
$$m = 1, 2, \ldots M; \quad n = 1, 2, \ldots N$$

This expression is a version extended in the 2-dimensional plane from that developed for a fuzzy set.[3] μ corresponds to p_{mn} and p'_{mn} in the respective cases.

Implementation and results: The above designed algorithm was implemented on a picture of handwritten script as shown in Fig. 2. The digitised version of the image of this picture is represented by an 96 × 99 array where each pixel can have one of the 32 (0, 1, 2, ... 9, A, B, ... V) grey levels. Thus in our algorithm $M = 96$, $N = 99$ and $x_{max} = 31$.

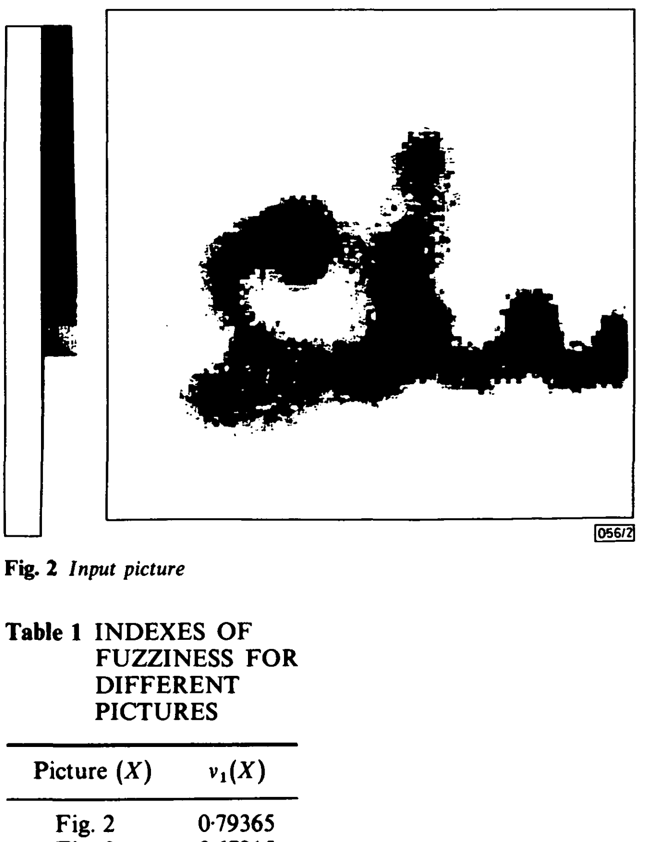

Fig. 2 *Input picture*

Table 1 INDEXES OF FUZZINESS FOR DIFFERENT PICTURES

Picture (X)	$\nu_1(X)$
Fig. 2	0·79365
Fig. 3*a*	0·67215
Fig. 3*b*	0·66880
Fig. 3*c*	0·66396

Figs. 3*a* through 3*c* demonstrate the three different enhanced picture outputs for different values of F_e and F_d when Fig. 2 was used as an input to the system. In Fig. 3*a* we considered $F_e = 2$, $F_d = 45$ and $\alpha = 0{\cdot}3506$ so that the threshold lay between the grey levels C and D. The corresponding levels in Figs. 3*b* and 3*c* were allowed to be D and E and E and F using $F_d = 43$ and 40 and $\alpha = 0{\cdot}3377$ and 0·3174. F_e was kept constant at a value of 2. All these enhanced outputs correspond to $r = 2$, i.e., operator INT(INT) as enhancement tool.

Fuzziness in picture is reduced by lowering the value of F_d. The change in enhancement between Figs. 3*b* and 3*c* is seen to be insignificant. Use of $F_d = 40$ made the output overcorrected and thinner. It therefore appears that further reduction in the value of F_d would result in automatic 'thinning' which is a most important part of a picture recognition system.

The linear indexes of fuzziness of these images were measured with respect to $F_d = 40$ and $F_e = 2$. The index values as listed in Table 1 indicate that the index of fuzziness reflecting the ambiguity in a picture is reduced through enhancement.

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S. K. PAL*
R. A. KING

Electrical Engineering Department
Imperial College of Science & Technology
London, SW7 2BT, England

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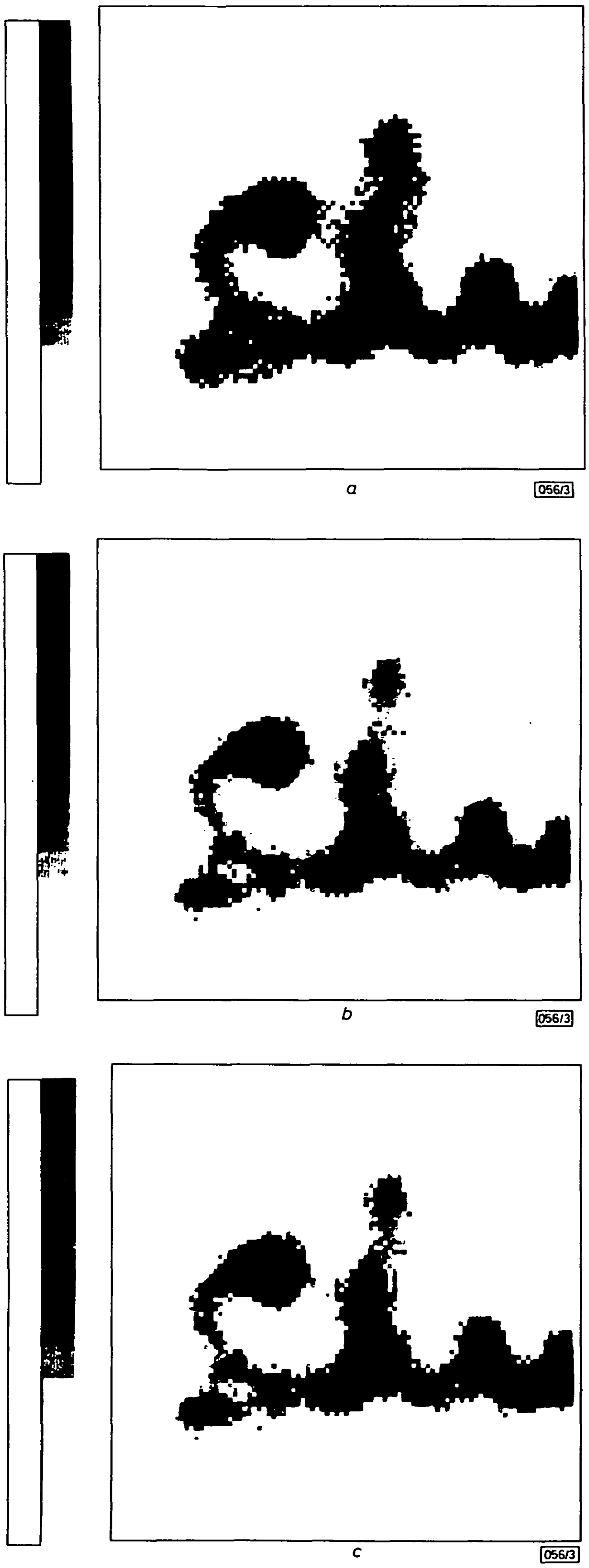

Fig. 3 *Enhanced picture output*

a $F_d = 45, F_e = 2$
b $F_d = 43, F_e = 2$
c $F_d = 40, F_e = 2$

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* On leave from Electronics & Communications Sciences Unit, Indian Statistical Institute, Calcutta 700035, India

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NOVEL ACTIVE FILTERS USING AMPLIFIER POLE

Indexing terms: Active filters, Poles and zeros

Active *R* realisation of bandpass and lowpass filters using lossy simulated inductors and capacitors is described.

Recently, inductance and capacitance simulation using the amplifier pole has received considerable attention.[1–4] The simplest lossy grounded inductance simulator is shown in Fig. 1*a*. In this letter, we consider a lossy simulated grounded capacitor using the amplifier pole and use it to replace the external capa-

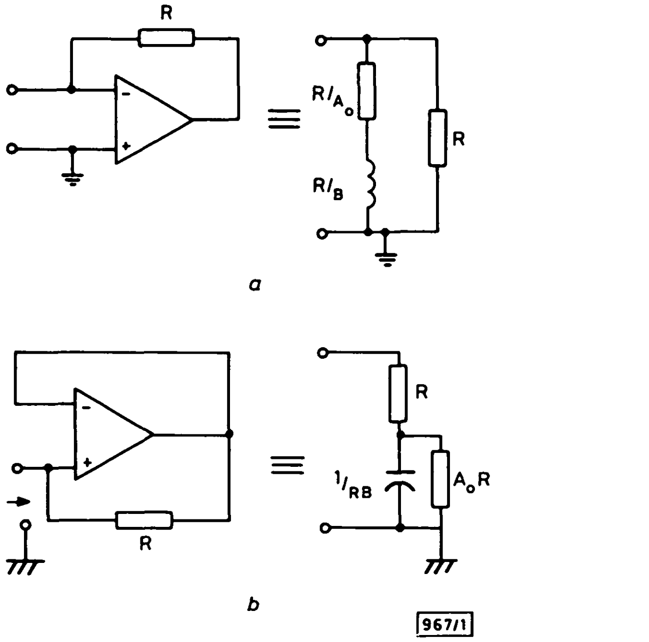

Fig. 1 *Lossy simulated inductor (a) and lossy simulated capacitor (b) using the amplifier pole*