

# Segmentation of Circular and Rectangular Shapes in an Image Using Helmholtz Principle

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## Abstract

**We propose algorithms to extract groups of meaningful image level lines using Helmholtz perception principle. In this paper, the meaningfulness refers to the segmentation of circular and rectangular shapes in an image. We propose an objective assessment of meaningfulness of an image level line when the level line takes the shape of circle or rectangular segment. We have shown that a logical threshold in the meaningfulness value segments images from a variety of applications.**

## 1. Introduction

Recently Gestalt hypotheses are revisited for solving different problems of computer vision [1]. At the same time use of image level lines, which are the boundaries of iso-intensity surfaces in an image, has attracted attention in describing the meaningful content of a scene [2]. In this paper we present techniques to extract meaningful image level lines based on the principles of Gestalt hypotheses. Our contribution in particular is the process of finding image level lines which conforms to a class of shapes, say, circular or rectangular objects. A related contribution is to provide an image segmentation technique that uses fewer parameters.

An important technique in detecting meaningful shapes in an image is the use of Helmholtz principle [1]. Helmholtz principle states that every large deviation corresponding to an a priori fixed list of geometric structures (such as, line, circle, rectangle, etc.) from a random uniform noise image should be perceptible [3].

From the study of visual perception of shape, two important observations are independence of image intensity and regularity of a shape [1]. By

independence of image intensity, we mean that the interpretation of a shape does not depend on individual pixel intensity values, rather on the order and arrangement of these pixel values. By regularity of a shape, we refer to the fact that a specific shape exhibits a regularity of pixel properties, e.g., pixel colors, pixel orientations, contrasts etc. and these regularities are not noticeable in a white noise. More specifically, detection of a circular shape in an image consists of collection of pixels which are not observable in random noise. These above observations are fundamental to the image segmentation presented in this paper.

In this paper we first detect a set of image level lines [4]. Select a few of those level lines which exhibit regularity in terms of image contrasts. Out of these level lines we pick up a further reduced set which exhibit regularity in terms of certain geometric property. That is, we look for the arrangements of pixels along those level lines which exhibit regularity as exhibited by a geometric structure like circle, or rectangle. We have shown how this regularity of pixel contrast or shape may be expressed numerically in terms of *meaningfulness* of that particular event, the event being the regularity of a particular pixel property. Image segmentation results when this meaningfulness is significant and noticeable with respect to white random noise.

Desolneux, Moisan and Morel [1] have extensively studied the Gestalt theory and used it for different image analysis purposes. They have shown that the Helmholtz principle can be effectively used to find meaningful image contents. Cao et al [5] have used this meaningfulness property of image level lines to extract smooth curves. Our work extends this idea to extract different shapes present in a scene. In order to do that we have defined meaningfulness properties of different geometric shapes.

In the next section, we define and detect level lines which exhibit regularity in terms of pixel contrasts. In Section 3, we present technique to pick up level lines

which exhibit certain shape regularity. In Section 4, we present the result of our work on a number of real images and compare them with related segmentation process followed by conclusions in Section 5.

## 2. Image Level Line

Image level lines are the boundary of connected components when the image is thresholded at every intensity level present in the image [4].

For a spatially regular lattice  $S$  having intensity levels  $\Gamma = \{\lambda_1, \lambda_2, \dots, \lambda_l\}$ , the binary intensity slice  $I_\lambda$  at the intensity level  $\lambda \in \Gamma$  is defined as,

$$I_\lambda = \{I(i, j) \geq \lambda, (i, j) \in S, \lambda \in \Gamma\}. \quad (1)$$

Similarly,  $I_\lambda^c$  is the complement of  $I_\lambda$ , where

$$I_\lambda^c = \{I(i, j) \leq \lambda\}. \text{ Henceforth, by } I_\lambda \text{ we refer}$$

$I_\lambda \cup I_\lambda^c$  unless otherwise mentioned explicitly. Fig. 1(b) shows the level lines of the image of Fig. 1(a) when the image is thresholded at every 10<sup>th</sup> intensity level. Naturally, the original image  $I(i, j)$  can be reconstructed from the binary intensity slices  $I_\lambda$  following,

$$I(i, j) = \sup \{\lambda : \lambda \in \Gamma, (i, j) \in I_\lambda\}. \quad (2)$$

The edges of the connected components at the intensity level  $\lambda \in \Gamma$  are the level lines at intensity level  $\lambda \in \Gamma$ . Typically, for an image, collection of all such edges for all the intensity levels forms image level lines.

The set of connected components  $C$  at different intensity levels  $\Gamma$  has many interesting properties [2]. The image level lines are effectively a Jordan curve. The most valuable property for our subsequent analysis of the segmentation process is that the stacks of  $C$  at different levels of  $\Gamma$  are non-intersecting. This can be explained with the following two observations:

*Observation 1:* Connected components  $C_1, C_2, \dots, C_n$  in  $I_\lambda \cup I_\lambda^c$  at any level  $\lambda \in \Gamma$  are spatially disjoint (i.e. not connected) under 4N or 8N connectivity assumptions in  $Z^2$ .

*Observation 2:* Connected components  $C_a$  in  $I_{\lambda_a}$  and  $C_b$  in  $I_{\lambda_b}$  for  $\lambda_a > \lambda_b$ ,  $\lambda_a, \lambda_b \in \Gamma$  are either completely disjoint or  $C_a$  completely contains  $C_b$ .

Once again, the observation 2 is very important in identifying any shape in the image. However, given that several connected components can represent a single

shape and since from the observation 2 these components are contained within each other, it is a potentially difficult problem to identify the particular connected component which best represents a shape. On the other hand perimeters of connected components not necessarily always represent edge chain delineating objects from the background. It is more likely that a part of the perimeter of a connected component potentially represents the edge of a region of interest. Having detected exhaustive set of level lines, our next task is to pick up those, which convey meaningful information for a specific shape. This is described in the next section.



Fig. 1(a)

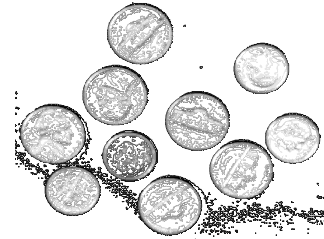


Fig. 1(b)

## 3. Meaningful Level Line

Determination of meaningful level line is motivated by the determination of individual shapes. First we define our basic principle of recognizing a pattern. Next in Section 3.2, we use this principle to extract meaningful level lines for circular shapes. This will be followed by defining meaningfulness properties corresponding to rectangular shapes. Note that while estimating meaningfulness we estimate the deviation of certain regularity properties of the level lines from the randomness.

### 3.1 Basic principle of recognizing a pattern

Given a pattern and a set of features of the pattern (i.e., color, curvature, circularity, rectangularity, etc.), we can estimate the expected number of occurrences of an event, where an event is defined as the occurrences of the features of the pattern mentioned above. Let us

define this expected number of occurrences of the event as the number of false alarms (NFA). If NFA of an event is *insignificant*, then we can assume that the event cannot occur out of a uniform random process. Given the exhaustive set of level lines of an image, if the event of a level line has very small NFA considering any particular feature of the curve, then the curve is *meaningful* or recognizable [6]. Since, NFA of the curve is small, this means that the expected number of times this curve can appear in an image is small. Hence we can conclude that, this curve cannot be the outcome of a uniform random process.

Now we need a threshold to compare the value of NFA below which we can consider the event meaningful. Let the threshold be  $\varepsilon$  below which the curve is meaningful. In that case the curve is called  $\varepsilon$ -meaningful.

In the next section, we present the estimation of NFA for a circular shape.

### 3.2 Detection of circular shape

Here we need to choose those *meaningful* level lines that exhibit circular shape. For each closed level line, we first calculate the centre of gravity (CG) of the co-ordinates of the level line points. Then, for a circular object, it is expected that the absolute difference between the distances between the CG and any two of the points on the level line is close to zero. So, we can calculate the NFA on the difference of those distances. If it is less than a sufficiently small number  $\varepsilon$  then that indicates that the level line maintains the circularity property rather than this specific circularity property of the level line being the outcome of a random process. We come back to the choice of  $\varepsilon$  later.

Once we get only the circular or circle-like level lines, our task is to find out the true boundary of a circular object when a number of concentric level lines represent the shape. We can, of course, select the level line with maximum contrast. For a circular object, the outermost meaningful level line can also represent boundary.

Let  $C_\lambda$  be the set of all closed level lines for level  $\lambda$  using equation (1). We first sample the curve with sample length  $\delta$  apart and get  $(n+1)$  number of points on the curve, say,  $p_0, p_1, p_2, \dots, p_n$  as shown in Fig. 2. The criterion for any level line  $C$  of the set  $C_\lambda$  to be circular is that the differences between the distances of any two sample points on  $C$

from the CG of  $C$  are sufficiently small for each of the points  $p_i$  on  $C$ . Let us call this criterion as circularity property. At each  $p_i$ , we have first calculated the distance  $d_i$  from the CG of that curve to  $p_i$  (i.e., the radius in case of a circular shaped object). Now, for each  $i$  in  $0 \leq i \leq n$  we calculate  $k_i = d_i - d_{i-1}$ , the difference between two consecutive distances. Here, for  $i=0$ ,  $d_{i-1}$  is the distance of  $p_n$  from CG of  $C$ . For a circular or circle like object,  $k_i$  should be close to zero. Clearly,  $|k_i|$  achieves its maximum possible value when the two consecutive points  $p_{i-1}$ ,  $p_i$  and the CG are collinear. Since, the points on the level line are sampled  $\delta$  distance apart, the maximum value that  $|k_i|$  can achieve is  $\delta$ . The  $k_i$  values are independent to each other and an *i.i.d.* sequence of random variables with uniform distribution in  $[0, \delta]$ . In reality, the maximum allowable value of  $|k_i|$ ,  $k_{\max}$  should be far less than  $\delta$  for the level line to be circular. We compute the probability  $P(k, n)$  for  $|k_i| < k_{\max}$ ,  $0 \leq i \leq n$  as,

$$P(k, n) = (k_{\max} / \delta)^n. \quad (3)$$

Equation (3) represents the probability that,  $|k_i|$  is less than  $k_{\max}$ , hence the circularity property is maintained at the point  $p_i$ . Since this probability is independent for all  $i$ , so extending (3) for all points on the curve upto the point  $p_i$ , we can say that the probability that the circularity property is maintained upto the point  $p_i$  is,  $(k_{\max} / \delta)^i$ .

Now, let  $N_{||}$  be the number of level lines obtained for the level  $\lambda$  for which the level line  $C$  has been obtained. Then, in this case we define NFA as follows:

$$\begin{aligned} NFA(C) &= N_{||} (k_{\max} / \delta)^i \text{ for } |k_i| < k_{\max} \text{ and} \\ NFA(C) &= N_{||} \text{ otherwise} \end{aligned} \quad (4)$$

For each point  $p_i$  we calculate the NFA. As mentioned earlier that if NFA is less than  $\varepsilon$  upto the point  $p_i$ , then the subcurve is  $\varepsilon$ -meaningful upto the

point  $p_i$ . If for each point  $p_i$ , the curve becomes  $\varepsilon$ -meaningful, then we conclude that, the level line is  $\varepsilon$ -meaningful, which alternately means that the level line satisfies circularity property. Otherwise, we can reject the curve as it may appear as an outcome of a uniform random process.

Besides the circularity checking, we have also made a contrast checking. That is, we want to select only those level lines whose contrast  $c_i$  is more than at least a minimum contrast, say,  $c_{\min}$ . Clearly,  $c_i$  is an *i.i.d.* sequence of uniformly distributed random variables in  $[0,255]$  for an 8-bit image. Then, in this case, the probability that all the pixels upto the point  $p_i$  on the level line has a contrast greater than  $c_{\min}$  is  $(1 - c_{\min} / 255)^i$ . So, considering minimum contrast of the level line we define the NFA as,

$$NFA(C) = N_{ll} \left(1 - \frac{c_{\min}}{255}\right)^i \text{ for } c_i > c_{\min} \text{ and} \quad (5)$$

$$NFA(C) = N_{ll} \text{ otherwise}$$

Here also we calculate NFA for each point  $p_i$ , and check the meaningfulness of the subcurve upto the point  $p_i$ . If the NFA given by the equations (4) and (5) are both less than  $\varepsilon$  for all  $i$  in  $0 \leq i \leq n$ , then the subcurve is  $\varepsilon$ -meaningful where both the circularity property and the contrast are considered simultaneously. So, we have given the definition of  $\varepsilon$ -meaningfulness such that both inequalities simultaneously hold.

Now, the question may arise that how to choose the values of  $\varepsilon$  and  $\delta$ . We have chosen the value of  $\varepsilon$  as 1 for both the NFA as the expected number of occurrences should be 1 at least (in fact much more than 1 in reality), in order that event to occur randomly. The value of  $\delta$  can be chosen by the user and that is mainly dictated by Shanon's sampling theorem. However, it is clear that, if we increase the value of  $\delta$ , then we get smaller number of points on the curve. As a result, the computation time will be less compromising the quality of result as in that case the meaningfulness is checked in fewer places. We have taken the value of  $\delta$  as 3 in all our experiments.

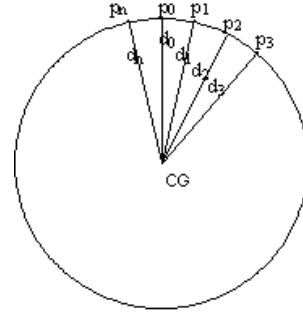


Fig. 2: Centre of gravity (CG) and the sampled points of a closed level line.

### Algorithm:

**Step 1:** Find the gradient magnitude of image  $I$  at each pixels and store it in the matrix  $G$

**Step 2:** For  $level=1$  to 255 do

**Step 2(a):** Find all the level lines present in  $I$  for the  $level$  using (1). Let, the total number of level lines for the  $level$  be  $N_{ll}$ .

**Step 2(b):** For each closed level line  $C$  having minimum length  $CLen$  do

**Step 2(b)(i):** Find the CG of the curve  $C$

**Step 2(b)(ii):** Sample  $C$  into  $n$  equal length sub-curves at points  $p_0, p_1, p_2, \dots, p_n$

**Step 2(b)(iii):** For each  $i \in [0, n]$ ,

**Step 2(b)(iii)(I):** Find the distance  $d_i$  from CG to  $p_i$

**Step 2(b)(iii)(II):** If  $i = 0$ , then Calculate  $k_i = |d_i - d_n|$ .

Otherwise, Calculate  $k_i = |d_i - d_{i-1}|$ .

**Step 2(b)(iii)(III):** Calculate NFA using equations (4) and (5).

**Step 2(b)(iii)(IV):** If  $NFA > \varepsilon$  in any of the cases then, continue with the next level line i.e., the current level line is not meaningful so that reject the current level line.

Else  $C$  is  $\varepsilon$ -meaningful considering both circularity property and contrast upto the point  $p_i$ , so continue with this curve.

**Step 2(b)(iv):** Select the level lines that has not been rejected before visiting all the  $(n+1)$  points on it. These are  $\varepsilon$ -meaningful level lines.

**Step 3:** Select either the outermost or the most contrasted curve of the concentric  $\varepsilon$ -meaningful curves to represent the circular object.

Next we present the rectangle detection technique.

### 3.3 Detection of rectangular shape

Similar to Section 3.2, let  $C_\lambda$  be the set of all closed level lines for level  $\lambda$  using equation (1). We first sample the curve with sample length  $\delta$  and get  $(n+1)$  number of points on the curve, say,  $p_0, p_1, p_2, \dots, p_n$ . Let us then define a chord and a tangent on any level line  $C$  of the set of level lines  $C_\lambda$ . The straight line joining the point  $p_i$  on  $C$  with the point  $p_i + \delta$  on  $C$  is termed as *chord* at the point  $p_i$ . Similarly, let us define *tangent* at point  $p_i$  on  $C$  as the straight line joining the points  $p_{i-1}$  and  $p_{i+1}$ . Now, a closed level line  $C$  is said to be rectangular if the absolute angle between the chord and tangent at each point is either very near to zero (when the points are on the edges of the rectangle) or  $\pi/2$  (when the points are on the corners of the rectangle). Let us call this criterion for a level line to be rectangular as, rectangularity property. Then at each point  $p_i$ , we have calculated the slope of the tangent  $m_{i1}$  and slope of the chord  $m_{i2}$  using the definitions of chord and tangent stated earlier. Then the absolute angle  $\theta_i$  is given by,  $\theta_i = \left| \tan^{-1} \left( \frac{m_{i1} - m_{i2}}{1 + m_{i1} \times m_{i2}} \right) \right|$  between the chord and the tangent at point  $p_i$  for each  $i$  in  $0 \leq i \leq n$ . Clearly,  $\theta_i$  can take any value in the interval  $[0, \pi]$ , with equal probability. So,  $\theta_i$  are i.i.d. sequence of random variables with uniform distribution in  $[0, \pi]$ . Now as mentioned earlier, for the points on the rectangle, the value of  $\theta_i$  should be either very close to zero or very close to  $\pi/2$ . But, it may not be equal to zero or  $\pi/2$  due to noises. We can then allow a maximum dispersion  $\theta_{\max} = \frac{\pi}{10}$  of  $\theta_i$ , as the level line is not a straight line any more if  $\theta_i > \frac{\pi}{10}$ . Here one thing is to remember is that, we may not get  $\theta_i$  exactly  $\pi/2$  at corner points. So, we first check if  $\theta_i$  is close to zero or not. If not, then we find the angle

$\alpha_i$  between the straight lines joining the points  $p_{i-2}$ ,  $p_{i-1}$  and the points  $p_{i+1}$ ,  $p_{i+2}$ . This should be very close to  $\pi/2$ , or in other words,  $\left| \frac{\pi}{2} - \alpha_i \right|$  should be close to zero. So in that case also, we set the maximum allowable dispersion as  $\theta_{\max}$ . Now, if  $\theta_i$  (in case of side points on rectangle) or  $\alpha_i$  (in case of corner points) is less than  $\theta_{\max}$ , then we can say that, the value of the angle  $\theta_i$  or  $\alpha_i$  is close to zero and hence, the rectangularity property is maintained upto the point  $p_i$ . Then we compute the probability  $P(\theta, n)$  for all  $i$  in  $0 \leq i \leq n$  as,

$$P(\theta, n) = (\theta_{\max} / \pi)^i \quad (6)$$

Equation (6) represents the probability that, the angle  $\theta_i$  or  $\alpha_i$  takes its value less than  $\theta_{\max}$ , i.e., sufficiently close to zero at the point  $p_i$ . Since this probability is independent for all  $i$ , we can say that the probability that the rectangularity property is maintained upto the point  $p_i$  will be,  $(\theta_{\max} / \pi)^i$ .

Now, let  $N_{ll}$  be the total number of level lines obtained for the level  $\lambda$  for which level, the level line  $C$  has been obtained. Then, we define the number of false alarms of  $C$ , by the following two equations:

$$NFA(C) = N_{ll} (\theta_{\max} / \pi)^i \text{ for } \theta_i < \theta_{\max} \text{ and} \quad (7)$$

$$NFA(C) = N_{ll} \quad \text{otherwise}$$

and,

$$NFA(C) = N_{ll} (\theta_{\max} / \pi)^i \text{ for } \left| \frac{\pi}{2} - \alpha_i \right| < \theta_{\max} \text{ and} \quad (8)$$

$$NFA(C) = N_{ll} \quad \text{otherwise}$$

Here, as in section 3.2, we calculate the NFA of the curve  $C$  at each point  $p_i$ . If the NFA be less than  $\varepsilon$  for equation (7), then we can say that, the curve is  $\varepsilon$ -meaningful upto the point  $p_i$ . Otherwise, we calculate the angle  $\alpha_i$  between the straight lines joining the points  $p_{i-2}$ ,  $p_{i-1}$  and the points  $p_{i+1}$ ,  $p_{i+2}$  and calculate the NFA using equation (8). If for each point  $p_i$ , NFA from equation (7) or (8) be less than  $\varepsilon$  (as the case may be) then the curve is said to be

$\varepsilon$ -meaningful upto the point  $p_i$ . Here also we have taken  $\varepsilon$  as 1 as in case of detecting circular objects.

### Algorithm:

**Step 1:** Fix the value of  $\theta_{\max}$ ,  $\varepsilon$ .

**Step 2:** For  $level=1$  to 255 do

**Step 2(a):** Find all the level lines present in  $I$  for the  $level$  using equation (1). Let the total number of all level lines for the  $level$  be  $N_{ll}$ .

**Step 2(b):** For each closed level line  $C$  of minimum allowable length  $CLen$  do

**Step 2(b)(i):** Segment  $C$  into  $n$  equal length sub-curves by  $(n+1)$  equidistant points

$$p_0, p_1, p_2, \dots, p_n.$$

**Step 2(b)(ii):** For each  $i \in [0, n]$ ,

**Step 2(b)(ii)(I):** Find the slope  $m_{i1}$  of the tangent at point  $p_i$

**Step 2(b)(ii)(II):** Find the slope  $m_{i2}$  of the chord at point  $p_i$

**Step 2(b)(ii)(III):** Calculate the angle  $\theta_i = \left| \tan^{-1} \left( \frac{m_{i1} - m_{i2}}{1 + m_{i1} \times m_{i2}} \right) \right|$

**Step 2(b)(ii)(IV):** Calculate  $NFA$  using equation (6)

**Step 2(b)(ii)(V):** If  $NFA > \varepsilon$ , then

**Step 2(b)(ii)(V)(A):** Find the angle  $\alpha_i$  between the straight lines joining the points  $p_{i-2}$ ,  $p_{i-1}$  and the points

$$p_{i+1}, p_{i+2}.$$

**Step 2(b)(ii)(V)(B):** Calculate  $NFA$  using equation (7).

**Step 2(b)(ii)(V)(C):** If  $NFA > \varepsilon$ ,  
 >>reject the level line  $C$  and continue with the next level line.

Else,

>>continue with the same level line as it is a corner point.

Else,

**Step 2(b)(ii)(V)(D):** continue with the level line  $C$  as it is a point on a side of the rectangle and so, it is  $\varepsilon$ -meaningful upto  $p_i$ .

**Step 2(b)(iii):** Select the level lines that has not been rejected before visiting all the  $(n+1)$  points on it. These are  $\varepsilon$ -meaningful level lines.

**Step 2(c):** Find the CG of  $C$ .

**Step 2(d):** If the CG is near the CG of an already selected rectangle, select rectangle having higher contrast.

Next we present the results of both the proposed algorithms.

## 4. Result, Discussion and Comparison

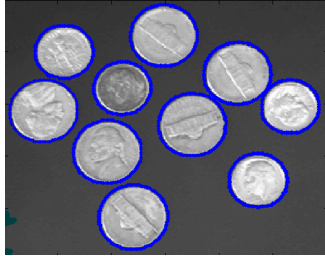
We have first tested our method on a simple image of Fig. 1(a). Our method has successfully identified all the 10 coins as shown in Fig. 3(a) and also in Fig. 3(b) where the original image is corrupted with zero mean Gaussian noise having standard deviation 0.02. These results are obtained using equations (4) and (5). The results using Cao [5] is shown in Fig. 3(c). While our proposed algorithm is giving only the circular coins, approach in [5] gets some more circles which causes thick extended boundaries of the detected objects. These extra circles are coming because of the shadow of the coins which are being removed in our method by introducing contrast checking using equation (5). Note that the approach in Cao [5] cannot get the result close to the desired edge in case of Gaussian noise corrupted image.

Leukocyte images of Figs. 4 and 5 are difficult cases for segmentation. The extraction of leukocyte shapes is important for applications like drug targeting studies. Figs. 4(b) and 5(b) clearly show the superiority of the proposed technique as compared to the results obtained using Figs. 4(c) and 5(c) respectively. The proposed method can detect more than 80% of the leukocytes in both the cases. The method of Cao [5] can identify some cells but also along with some undesirable edges.

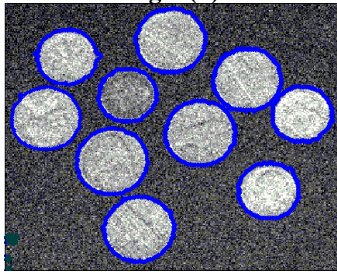
Figs. 6 and 7 show the result using rectangle detection algorithm. While Fig. 6(b) shows that almost all the rectangular objects could be identified for a synthetic image of Fig. 6(a) using the proposed technique, Fig. 7(b) shows the real performance of the algorithm. Estimation of rock sizes is important for several geological and related applications. In this case rocks are photographed while they are being transported on a conveyer belt. The image quality because of the mining environment is extremely poor for segmentation purpose. However, we can see that most of the significant rock pieces are identified using the proposed approach.

We have already discussed about the values of  $\varepsilon$  and  $\delta$  we have taken. Other three parameters we have used for detecting circular images are,  $CLen$ ,  $c_{\min}$  and  $k_{\max}$ . The minimum length  $CLen$  of level line

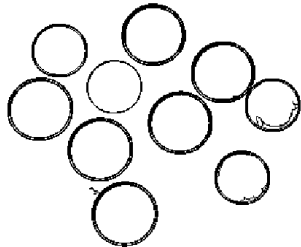
for it to be qualified for an edge is application dependent. But for all the images we have used in this paper we fix to 80 which is equal to the perimeter of the average leukocyte we are segmenting in Figs. 4 and 5. The contrast factor  $c_{\min}$  is also constant for different images that we worked with and we have chosen it as 10% of the range of contrast. The parameter  $k_{\max}$  depends on  $\delta$  and  $k_{\max} = \delta / 2$  worked well for a wide variety of images.



**Fig. 3(a)**

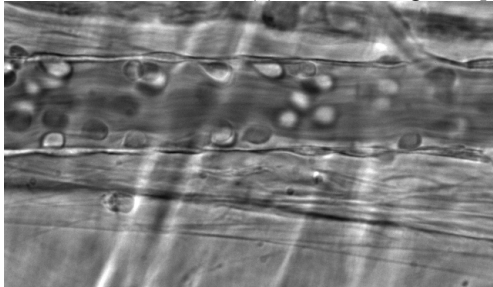


**Fig. 3(b)**

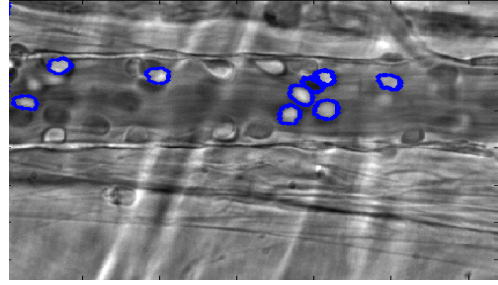


**Fig. 3(c)**

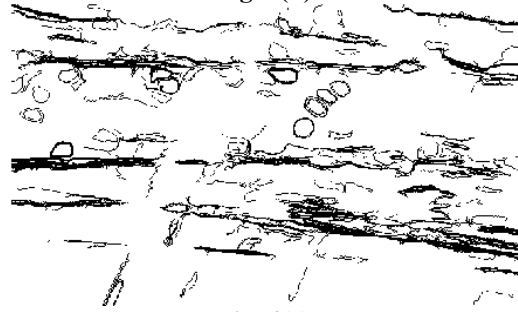
Fig. 3(a): Result of proposed circle detection approach on Fig. 1(a). (b): Result of the proposed method on the image of Fig. 1(a) with zero mean Gaussian noise and standard deviation 0.02. (c): Result using Cao [5].



**Fig. 4(a)**

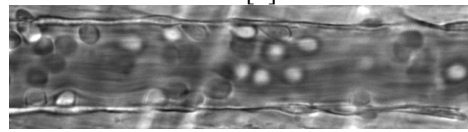


**Fig. 4(b)**

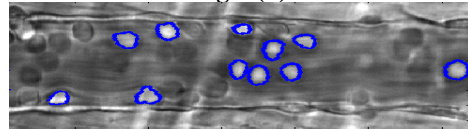


**Fig. 4(c)**

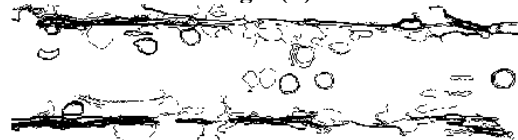
Fig. 4(a): Original leukocyte image. (b): Result of the proposed method. (c): Results using the algorithm of Cao [5].



**Fig. 5(a)**

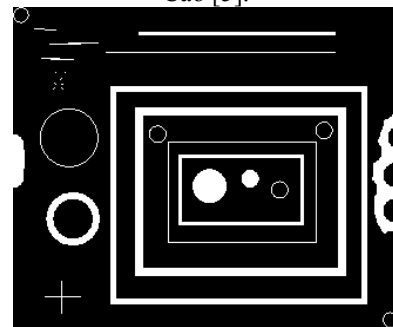


**Fig. 5(b)**

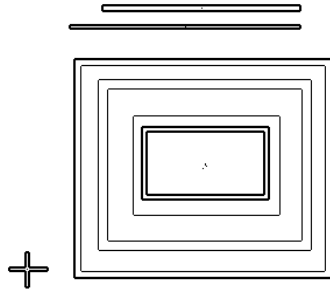


**Fig. 5(c)**

Fig. 5(a): Another leukocyte image. (b): Result of the proposed method. (c): Results using the algorithm of Cao [5].



**Fig. 6(a)**

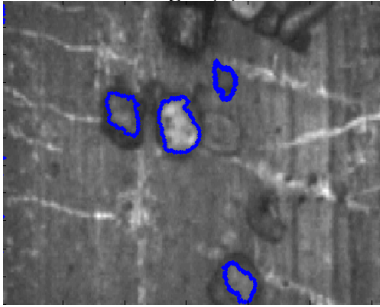


**Fig. 6(b)**

Fig. 6(a): Original image with a set of rectangles. (b): Result of rectangle shape detection.



**Fig. 7(a)**



**Fig. 7(b)**

Fig. 7(a): Rock image. (b): Result of near circular rock shape detection.

We have calculated the performance measurement of our method with Canny's edge detection principle and the method of Cao [5]. Here, the Performance Measurement Metric (PMM) of a method is given by the following equations:

$$PMM = \left( \frac{epm}{epg} - \frac{nfedg}{npix} \right) \times 100 \quad (9)$$

where,  $epg$  and  $epm$  are respectively the number of edge points in the ground truth image and the number of edge points given by the result image of a method and overlapping with that of the ground truth image.  $nfedg$  is the number of false pixels detected as

edges;  $npix$  is the total number of pixels in the image.

The result is given below:

Table 1: PMM found for different methods in different images

	PMM of Proposed method	PMM of Cao[5]'s method	PMM of Canny's method
Coin image	99.18	79.78	87.84
Coin image with noise	96.54	62.57	81.72
Bloodcell image	98.82	71.23	83.66
Rock image	96.27	72.69	84.31

## 5. Conclusions

To conclude, it is clear from the results that, our method can identify most of the circular and rectangular objects present even in a noisy image. We show that the objective assessment of meaningfulness works well for different set of images and the selection of different parameters influencing the algorithm are straightforward. Our future plan is to apply the Helmholtz principle for tracking multiple circular objects present in a video.

## 6. References

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