Quantum Query-to-Communication Simulation Requires Logarithmic Overhead

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Overview

Contents

The Basic Model of Communication

$$F : \{-1,1\}^n \times \{-1,1\}^n \to \{-1,1\}$$

Alice
$X \in \{-1,1\}^n$

Bob
$Y \in \{-1,1\}^n$

A protocol Π

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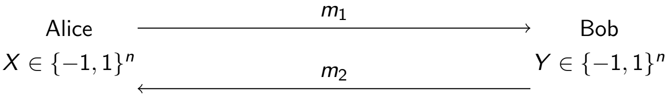

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Alice $\quad\xrightarrow{\;m_1\;}\quad$ Bob

$X \in \{-1,1\}^n \quad \xleftarrow{\;m_2\;} \quad Y \in \{-1,1\}^n$

$\vdots$

A protocol Π

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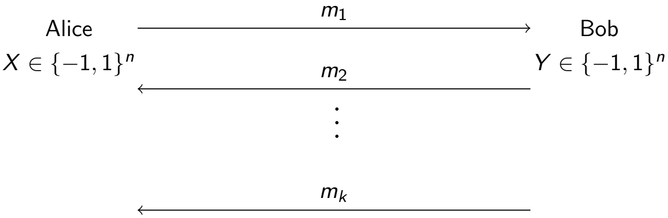

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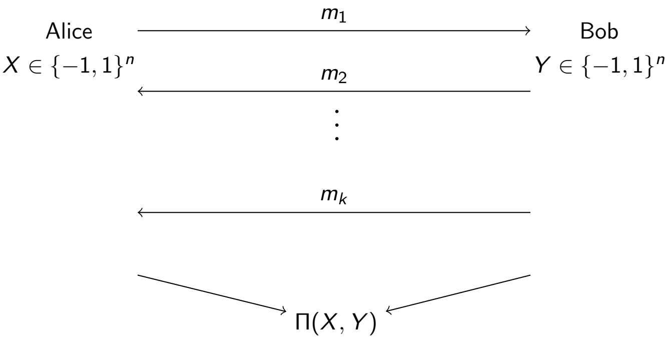

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Resources and Correctness Requirement

- Deterministic: Messages are bits communicated over classical channel

$$\forall X, Y \in \{-1, 1\}^n \qquad \Pi(X, Y) = F(X, Y)$$

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$$\forall X, Y \in \{-1,1\}^n \qquad \Pr[\Pi(X,Y) = F(X,Y)] \geq \frac{2}{3}$$

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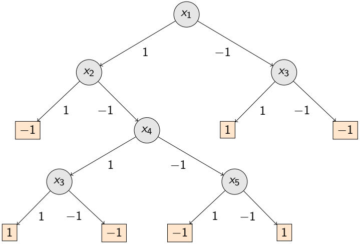
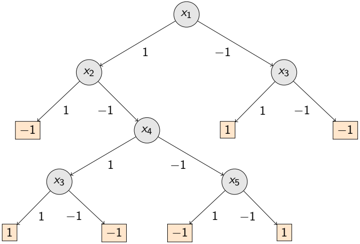
  - **Focus of this talk**

Query Algorithm



A deterministic query algorithm.

Query Algorithm



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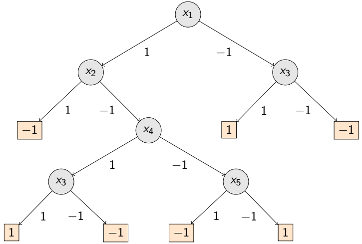

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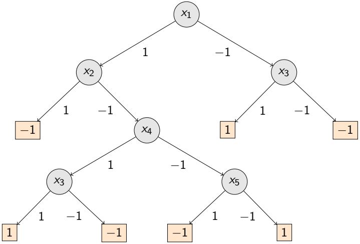

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Cost of query algorithm: Maximum number of queries over any input = Depth of the tree

Randomized query algorithm: Distribution over deterministic query algorithms

Quantum Query (Focus of this talk)

$$f : \{-1, 1\}^n \to \{-1, 1\}$$

- A *quantum query* algorithm with T queries is a sequence:

$$U_0, Q_0, U_1, Q_1, \ldots, U_{T-1}, Q_{T-1}, U_T$$

- U's are unitaries that do not depend on input
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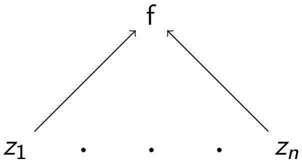
- U's are unitaries that do not depend on input
- Q's are query transformations that depend on input
- The query algorithm starts at $|\psi_0\rangle$ and ends with measuring the result of $U_T \ldots Q_0 U_0 |\psi_0\rangle$
- The query algorithm computes f if for all $x \in \{-1, 1\}^n$ the result of measurement is equal to $f(x)$ with probability at least $2/3$

Composed Boolean Functions

- Given $f : \{-1,1\}^n \rightarrow \{-1,1\}$ a natural communication problem is $(f \circ \bullet)$, where $\bullet$ is a 2-bit predicate i.e. XOR or AND



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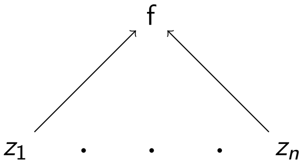
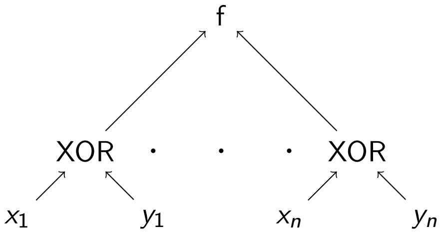
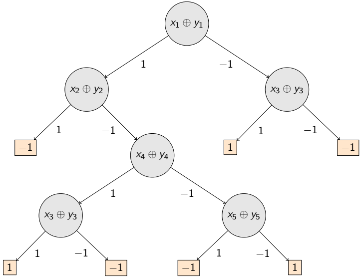




Efficient Communication Protocols From Query Algorithms

What is an efficient communication protocol for $f \circ \mathrm{XOR}_2$?



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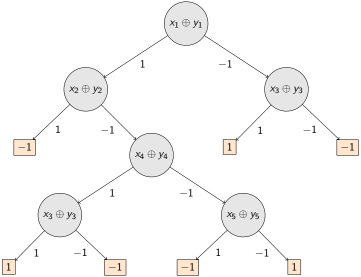

- Alice sends x_1 to Bob
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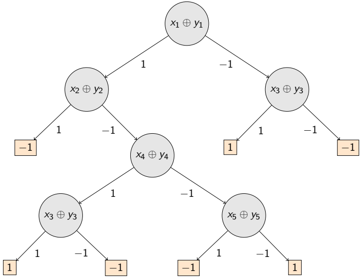

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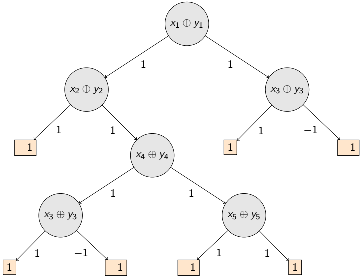

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$D^{cc}(f \circ \text{XOR}_2) \leq 2D(f)$

Also true for randomized case.

Central Question

Question

Let $\bullet$ *be* AND_2 *or* XOR_2

$$Q^{cc}(f \circ \bullet) = O(Q(f))?$$

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Let $\bullet$ *be* AND_2 *or* XOR_2

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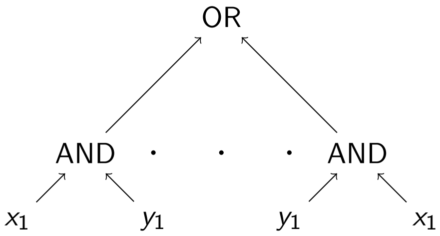
Yes for DISJOINTNESS function!

- [Buhrman, Cleve and Wigderson (1998)]
- [Høyer and de Wolf (2002)]
- [Aaronson and Ambainis (2005)]



Main Result

Theorem (BCW Simulation Theorem)

Let $f : \{-1,1\}^n \to \{-1,1\}$ *be any Boolean function and* $\bullet$ *be a two bit primitive, then*

$$Q^{cc}(f \circ \bullet) \leq Q(f)(2\log n + 4),$$

- Natural Question: Is the $O(\log n)$ overhead in BCW simulation necessary?

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Main Result

There exists a **total** function $f : \{-1,1\}^n \to \{-1,1\}$ such that

$$Q^{cc}(f \circ \mathrm{XOR}_2) = \Omega(Q(f)\log n).$$

Contents

Connections With Approximation Theory

- Every function $f : \{-1,1\}^n \to \mathbb{R}$ has a unique multilinear expression

$$f = \sum_{S \subseteq [n]} \widehat{f}(S) \chi_S.$$

- The *spectral norm* or ℓ_1-norm of f is the sum of absolute values of the Fourier coefficients of f

$$\|\widehat{f}\|_1 := \sum_{S \subseteq [n]} \left|\widehat{f}(S)\right|.$$

Connections With Approximation Theory (Contd...)

- A function $p : \{-1,1\}^n \to \mathbb{R}$ ϵ-*approximates* f if for all $x \in \{-1,1\}^n$, $\big|p(x) - f(x)\big| \leq \epsilon$
- The ϵ-**approximate degree** of f

$$\widetilde{\deg}_\epsilon(f) := \min\{\deg(p) : \big|p(x) - f(x)\big| \leq \epsilon \text{ for all } x \in \{-1,1\}^n\}$$

- The ϵ-**approximate spectral norm of** f, denoted by $\|\hat{f}\|_{1,\epsilon}$

$$\|\widehat{f}\|_{1,\epsilon} := \min\{\|\widehat{p}\|_1 : \big|p(x) - f(x)\big| \leq \epsilon \text{ for all } x \in \{-1,1\}^n\}$$

- **Note:** For the rest of this talk $\epsilon = 1/3$

Connections With Approximation Theory (Contd...)

Theorem (Beals, Buhrman, Cleve, Mosca and de Wolf (1998))

For any Boolean function f:

$$\widetilde{\deg}(f) \leq 2Q(f)$$

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Theorem (Lee and Shraibman (2009))

For any Boolean function f, $Q^{cc}(f \circ \mathrm{XOR}_2) = \Omega(\log \|\widehat{f}\|_{1,1/3})$

Contents

Proof Overview

We construct a *total* function F and prove two main theorems

- **Upper Bound:** $Q(F) = \Theta(\widetilde{\deg}(F))$
- **Lower Bound:** $\log(\|\widehat{F}\|_{1,1/3}) = \Omega(\widetilde{\deg}(F)\log n)$

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$$Q^{cc}(F \circ \mathrm{XOR}_2) \overset{\text{LS09}}{=} \Omega(\log(\|\widehat{F}\|_{1,1/3})) = \Omega(\widetilde{\deg}(F)\log n) = \Theta(Q(F)\log n)$$

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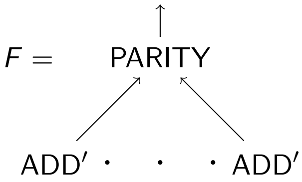
Function construction and query upper bound inspired from [Ambainis and de Wolf (2014)].

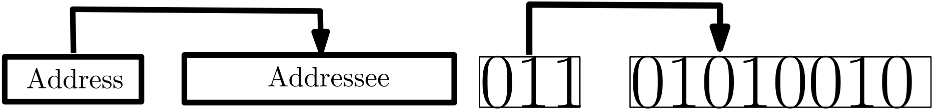
Constructing the function

- The function F is a (like a) composed function.
- The outer function is PARITY
- To define the inner function ADD$'$ we need to define
  - Addressing function
  - Hadamard codewords



Inner Function

- Addressing function is defined on $(\log t + t)$ bits
- The first $\log t$ bits of the input provide an address on the remaining t-bits of the input
- The bit present at that location is the output of the function



Inner Function (Contd...)

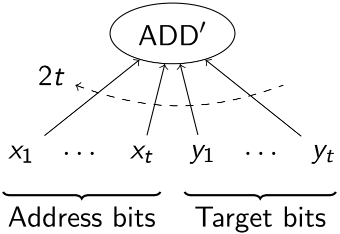
- **Hadamard codewords** For $i \in \{-1, 1\}^{\log t}$, the Hadamard codeword $H(i) \in \{-1, 1\}^t$ is an enlisting of the parities of all subsets of i
- **Example** For $i = (1 - 1) \in \{-1, 1\}^2$, $H(i) = (1, 1, -1, -1)$

$$\chi_\phi = 1 \quad \chi_{\{1\}} = 1 \quad \chi_{\{2\}} = -1 \quad \chi_{\{1,2\}} = -1$$

- Number of Hadamard codewords is t, one for each $i \in \{-1, 1\}^{\log t}$

Inner Function (Contd...)

- **Inner Function** $\mathrm{ADD}'_t : \{-1,1\}^{2t} \to \{-1,1,\phi\}$



$$= \begin{cases} y_i & \text{if } (x_1,\ldots,x_t) = H(i) \\ \phi & \text{otherwise.} \end{cases}$$

Inner Function (Contd...)

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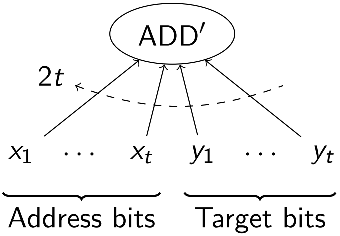
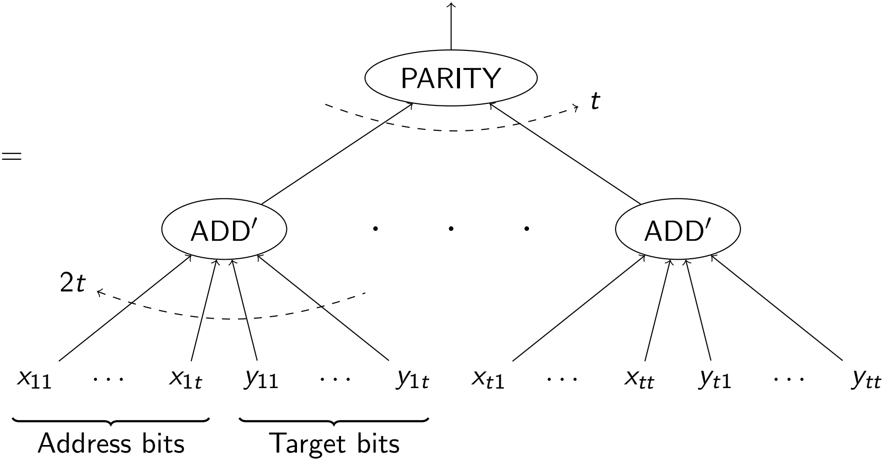


$$= \begin{cases} y_i & \text{if } (x_1,\ldots,x_t) = H(i) \\ \phi & \text{otherwise.} \end{cases}$$

- **Fact:** If the Address bits are indeed a Hadamard codewords $H(i)$, the **Bernstein-Vazirani** [BV (1992)] algorithm returns i with probability 1; using only 1 quantum query

The Function

$F =$



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Sketch of Lower Bound

- Recall $Q^{cc}(F \circ \mathrm{XOR}_2) = \Omega(\log \|\hat{F}\|_{1,1/3})$ [Lee and Shraibman (2009)]

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- Using ideas from [Chattopadhyay and Mande (2017)], we prove for any Boolean function g that

$$\log \|\widehat{g \circ \text{ADD}'}\|_{1,1/3} \geq \widetilde{\deg}(g) \log t$$

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$$\log \|\widehat{g \circ \mathrm{ADD}'}\|_{1,1/3} \geq \widetilde{\deg}(g) \log t$$

- Since outer function is PARITY_t, we have

$$\log \|\widehat{F}\|_{1,1/3} \geq \Omega(t \log t)$$

Sketch of Lower Bound Proof (Contd...)

Lemma (Lower Bound)

For $F : \{-1,1\}^{2t^2} \to \{-1,1\}$ *constructed above we have*

$$\log(\|\widehat{F}\|_{1,1/3}) = \Omega(t \log t).$$

- For contradiction, let P be a polynomial of spectral norm $o(t \log t)$ that $1/3$-approximates F
- **Lemma:** There exists a partial assignment to variables of F, say a, such that
  - $F|_a$ evaluates to parity on t bits
  - In $P|_a$ the following is true

$$\sum_{|S| \geq 0.9t} \left|\widehat{P|_a}(S)\right| \leq 1/4$$

Sketch of Lower Bound Proof (Contd...)

Lemma (Lower Bound)

For $F : \{-1,1\}^{2t^2} \to \{-1,1\}$ *constructed above we have*

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- From $P|_a$, drop all monomials S such that $|S| \geq t$ to get P'
- P' approximates $F|_a$ to error $1/3 + 1/4$
- **Contradiction:** Since a polynomial of degree strictly smaller than t can not sign-represent parity on t bits ([Minsky, Papert (1969)]

Sketch of Lower Bound Proof (Contd...)

Lemma

There exists a partial assignment variables of F, say a, such that $F|_a$ evaluates to parity on t bits and $\sum_{|S| \geq 0.9t} \left|\widehat{P|_a}(S)\right| \leq 1/4$

- Let μ be the distribution on the address bits such that for each ADD$'$
  1. μ is supported only on Hadamard codewords
  2. μ is uniform distribution

Sketch of Lower Bound Proof (Contd...)

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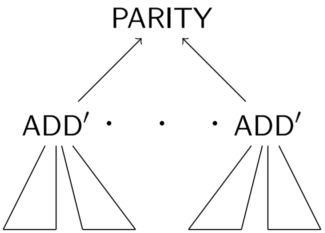
- **Relevant Monomial** For $a \sim \mu$, a monomial of P is relevant w.r.t. a if it does not contain any target variable not in $F|_a$



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$$\mathbb{E}_{a \sim \mu}[\ell_1\text{-norm of relevant monomials w.r.t. } a \text{ in } P \text{ of degree } \geq 0.99t]$$

$$= \sum_{|S| \geq 0.99t} |\widehat{P}(S)| \Pr_{a \sim \mu}[\chi_S \text{ is relevant w.r.t. } a]$$

$$\leq \max_{|S| \geq 0.99t} \{\Pr_{a \sim \mu}[\chi_S \text{ is relevant w.r.t. } a]\} \cdot \|\widehat{P}\|_1$$

$$< \frac{1}{t^{0.99t}} \cdot 2^{\frac{1}{10} 0.99t \log t} = 2^{(-\frac{9}{10}) 0.99t \log t} < \frac{1}{4},$$

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Lemma

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Upper Bound: Quantum Query Algorithm

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Query upper bound inspired from an algorithm of [Ambainis and de Wolf (2014)].

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- **Guess:** Run t instances of the Bernstein-Vazirani algorithm on address bits $(x_{11}, \ldots, x_{1t}), \ldots, (x_{t1}, \ldots, x_{tt})$ to obtain t strings $z_1, \ldots, z_t$

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- **Check:** Use Grover search to check equality of the two strings: $H(z_1), \ldots, H(z_t)$ and $x_{11}, \ldots, x_{1t}, \ldots, x_{t1}, \ldots, x_{tt}$

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Query upper bound inspired from an algorithm of [Ambainis and de Wolf (2014)].

- **Guess:** Run t instances of the Bernstein-Vazirani algorithm on address bits $(x_{11}, \ldots, x_{1t}), \ldots, (x_{t1}, \ldots, x_{tt})$ to obtain t strings $z_1, \ldots, z_t$
- **Check:** Use Grover search to check equality of the two strings: $H(z_1), \ldots, H(z_t)$ and $x_{11}, \ldots, x_{1t}, \ldots, x_{t1}, \ldots, x_{tt}$
- If the step above outputs that the strings are equal, then query $y_{1z_1}, \ldots, y_{tz_t}$ and output their parity. Else, output -1

Correctness of the Algorithm

- **Case 1:** If the address bits all inner functions are indeed Hadamard codewords, the **Guess** step outputs the correct strings $z_1, \ldots, z_t$ with probability 1.
- Grover search makes error at most $1/3$ in the **Check** step since the strings are equal. Hence, the correct target variables are queried (and the correct answer is output) wp at least $2/3$.

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- Grover search makes error at most $1/3$ in the **Check** step since the strings are equal. Hence, the correct target variables are queried (and the correct answer is output) wp at least $2/3$.
- **Case 2:** If there is a non-codeword $x_{j1}, \ldots, x_{jt}$, the output of Bernstein-Vazirani algorithm z_j satisfies $H(z_j) \neq x_{j1}, \ldots, x_{jt}$. In this case Grover search outputs the correct answer (and the algorithm outputs -1) w.p. at least $2/3$.

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We construct a *total* function F and prove two main theorems

- **Upper Bound Theorem:** $Q(F) = \Theta(\widetilde{\deg}(F))$
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Main Result

There exists a **total** function $f : \{-1,1\}^n \to \{-1,1\}$ such that

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Open Problems

- What is the quantum communication complexity of $F \circ \mathrm{AND}$?

Thank You!