# Assignment 2
# Discrete Mathematics - MTech CS 2018

**All the problems marked with (*) are a bit hard and may need ideas not necessarily cover in the class so far. But you are encouraged to try the problems before the sollutions are discussed in class.**

1. Write the contraposetive of the following statement:

   *Given a finite family of convex sets $C_1, C_2, ..., C_n$ in $\mathbb{R}^d$ (where $n \geq d+1$) such that if the intersection of every $d+1$ of these sets is non-empty, then the whole collection has a non-empty intersection.*

2. Prove that for any posetive reals $a, b$
$$\frac{a+b}{2} \geq \sqrt{ab}.$$

3. Prove that for any posetive reals $a, b, c, d$
$$\frac{a+b+c+d}{4} \geq (abcd)^{1/4}.$$

4. Prove that $\sqrt{2}$ is not rational.

5. Prove that $\sqrt{2}+\sqrt{3}$ is not rational.

6. Is the statement $(p \wedge q) \vee (\neg p \vee (p \wedge \neg q))$ a tautology or a contradiction or none.

   **Definition:** A proposition that is always TRUE is called a tautology. A proposition that is always FALSE is called a contradiction.

7. Prove that there are infintely many primes of the form $3(mod 4)$.

8. (*) Prove that there are infinitely many primes of the form $1(mod 4)$.

9. Write the opposite of the following statement:

   *There is an university in USA where every department that has at least 20 faculty has at least one noble laureate.*

10. What is the contrapositive of the statement

    *For all $C, D, E, F \geq 0$ there exists an $N \in \mathbb{N}$ such that for all $n > N$ we have , $C2^n > Dn^8$ and $E(\log n)^4 < F(n^{1/100})$.*