Assignment 4
Discrete Mathematics - MTech CS 2018

All the problems marked with (*) are a bit hard and may need ideas not necessarily cover in the class so far. But you are encouraged to try the problems before the sollutions are discussed in class.

1. Give the asymptotic relations between the following set of functions: (a) 10^n, (b) $n^{1/3}$, (c) n^n, (d) $\log_2 n$, (e) $2^{\sqrt{\log_2 n}}$, (f) $n!$, (g) $n^{2.5}$, (h) $\sqrt{2n}$, (i) $n+10$, (j) 100^n, (k) $n^2 \log n$, (l) $2^{\log n/\log\log n}$, (m) $(2^{\log\log n})^2$, (n) $2^{(\log\log n)^2}$, (o) $2^{\log^2 n}$, (p) $(\log n)!$, (q) $(\log n)^{\log n}$, (r) $n^{4/3}$, (s) 2^{2^n}, (t) $2^{n^2/2}$, (u) $2^{\binom{n}{2}}$.

2. Prove or disprove the following set of asymptotic relations:

   (a) $(2.9)^{\log_2 n}\Theta n^{\log_2 3}$

   (b) $\log\log n = \Omega((\log\log\log n)^{\log\log\log n})$

   (c) $n^4 \sim (1-1/n)^n n^3$

   (d) $2^{(\log n)-(\log\log n)} \sim 2^{(1-1/n)\log n}$

   (e) $n^{10(\log\log n)^{100}} = \Theta((\log n)!)$

   (f) $\log\binom{2n}{n} = o(\frac{\log(2n)!}{\log 2n})$

3. Prove that $\binom{2n}{n} \sim \frac{4^n}{\sqrt{\pi n}}$.

4. For the following pairs give the correct asymptotic relations (O, o, ω, Ω, $\sim$.

   (a) $(\lg n)^a$ and n^b,

   (b) $2^{n\log_2 n}$ and $10n!$,

   (c) $\sqrt{n}$ and $(\log_2 n)^5$,

   (d) $n^2/\log_2 n$ and $(n\log_2 n)^4$,

   (e) $\log_2 n$ and $\log_2 66n$,

   (f) $1000(\log_2 n)^{0.9999}$ and $(\log_2 n)^{1.000001}$,

   (g) n^2 and $n(\log_2 n)^{15}$.

5. Find the approximate value of $\binom{n}{n/3}$.

6. Solve the following recurrences: (assume $T(0) = T(1) = 1$)

   (a) $T(n) = 2T(\lceil n/2 \rceil) + 5$

   (b) $T(n) = T(n-1) + T(n-2) + 15$

   (c) $T(n) = 2T(\lceil n/3 \rceil) + n^2$

   (d) $T(n) = 2T(\lceil \log n \rceil) + n$

   (e) $T(n) = 4T(n-1) + 6n^2 + \log n$

   (f) $T(n) = 2T(n-1) + 3T(n-2) + 4$