Assignment 6
Discrete Mathematics - MTech CS 2018

All the problems marked with (*) are a bit hard and may need ideas not necessarily cover in the class so far. But you are encouraged to try the problems before the sollutions are discussed in class.

1. Show that every connected graph has at least two vertices u and v such that $G-u$ and $G-v$ are connected.

2. Show that for any graph $G=(V,E)$ the vertex set V can be partitioned into two sets V_1 and V_2 such that
$$e(V_1)+e(V_2)\leq\frac{|E|}{2}$$
where $e(V_i)$ means the number of edges in E with both end points in V_i.

3. Prove that a regular bipartite graph of degree at least 2 does not contain a bridge.

4. Let G be a graph with minimum degree 2. Show that there exist a connected graph with same degree sequence.

5. Let $T_1,\ldots,T_k$ be subtrees of a tree T such that for all i,j the trees T_i and T_j have a vertex in common. Show that T has a vertex that is in all T_i.

6. If $G=(V,E)$ is a graph on n vertices such that all the vertices have even degree. Show that the edge set E can be partitioned into pairwise disjoint sets $C_1,C_2,\ldots,C_k$ such that for all $1\leq i\leq k$ the subgraphs (V,C_i) is a cycle and a collection of isolated vertices.

7. Prove that either G or $\bar{G}$ is connected.

8. A chord of a cycle is an edge that connects two non-adjacent vertices in the cycle. Prove that if every node of G has degree ≥ 3 then G contains a cycle with a chord.

9. Prove that there is a tournament T with n players and at least $n!2^{-(n-1)}$ Hamiltonian paths.