Discrete Mathematics

Lecture 13: Catalan Number Different Interpretations

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1 Catalan Number

Note. We saw that in the problem of going from (0,0) to (n,n) without crossing the diagonal (or you always remain below the diagonal) can be seen as a sequence of right and up arrows. If we made i rights and j ups we will be at the position (i,j) and (i,j) should be below the diagonal (our constraint). If (i,j) is below the diagonal then ($j \leq i$). So that sequence at any point in time will have less or equal number of ups and rights. □

1.1 String Interpretation

Think of a 2n length string of n zeros and n ones; s.t. $\forall$ p (number of ones till pt. p) $\leq$ (number of zeros till pt. p).
This interpretation is also called parenthesis problem.

()()() $\rightarrow$ correct ())())()() $\rightarrow$ incorrect

1.2 Rooted Trees

Rooted Tree is a graph with no cycle and a root. Here ordering of siblings matter.
Now we look on how many different rooted trees are possible in n different vertices (structurally different).
eg. 4 vertices rooted tree

1.3 Figures

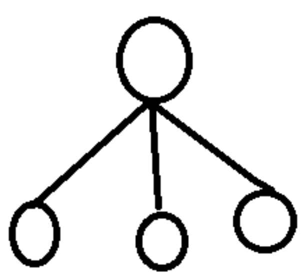

Figure 1: 010101

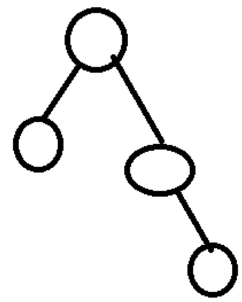

Figure 2: 010011

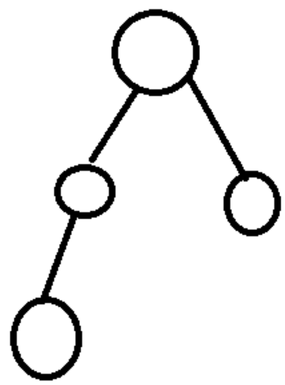

Figure 3: 001101

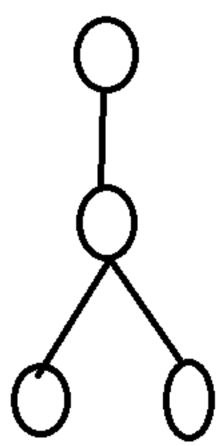

Figure 4: 001011

Figure 5: 000111

- Property 1- Each tree can be represented by a unique 0/1 string of length2*(n-1) with equal number of 0s and 1s.
- Property 2- We see that at any point in string we can't have more 1s than 0s to be it a valid rooted tree. Example- 0110 is invalid encoding.
- Property 3- The number of such different types of encoding comes out to be C(n-1) i.e. (n-1)th Catalan number.
- Exercise: Given 2 different encodings satisfying above properties, those two encodings represent two different structures i.e. valid encodings and structure have a bijection.