

Quiz 1  
Discrete Mathematics - MTech CS 2018

17th August 2018

**Time: 1 hour**

**Maximum Marks 100**

1. (25 Marks) What is the negation of the following statement:  
“For all  $C, D, E, F \geq 0$  there exists an  $N \in \mathbb{N}$  such that for all  $n > N$  we have ,  $C2^n > Dn^8$  and  $E(\log n)^4 < F(n^{1/100})$ .”
2. (25 Marks) Prove that a graph is bipartite if and only if the graph has no odd cycle.
3. (25 Marks) If  $T(n) = 3T(\lceil n/3 \rceil) + 1$  and  $T(1) = 1$  then what is  $T(n)$ . (Either the exact expression or the closest asymptotic expression.)
4. (25 Marks) Prove that if a graph has at most  $m$  vertices of degree at most  $n$  and all other vertices have degree of at most  $k$ , with  $k < n$  and  $m < n$ , then the graph is (vertex) colorable with  $m + k + 1$  colors.
5. (25 Marks extra) If  $n \equiv 1 \pmod{4}$  then prove that there is a way of coloring the edges of  $K_n$  with colors **red** or **blue** such that for all vertex exactly half of the adjacent edges is colored **red** and the other half is colored **blue**.