

Quiz 2  
Discrete Mathematics - MTech CS 2018

12th October 2018

**Time: 1 hour**

**Maximum Marks 100**

1. (20 Marks) If  $G$  is graph with even number of vertices and each vertex has even degree (that is, even number of neighbors) then  $G$  has even number of edges. Either prove the statement or disprove it by demonstrating a counter example.
2. (20 Marks) Is it possible to draw 9 line segments in the plane  $\mathbb{R}^2$  such that each line segment intersects exactly 3 other line segments.
3. (20 Marks) Prove that a graph has an Eulerian path if and only if the number of vertices in the graph with odd degree is 0 or 2.
4. (5+15 Marks) Consider a  $n \times m$  rectangular grid, with the co-ordinates of the corners being  $(0, 0)$ ,  $(n, 0)$ ,  $(n, m)$  and  $(0, m)$ . How many paths are there along the rectangular grid from  $(0, 0)$  to  $(n, m)$  such that
  - (a) The paths are the shortest among all paths from  $(0, 0)$  to  $(n, m)$ .
  - (b) Every horizontal move of unit length is followed by at least one vertical move and the paths are the shortest.
5. (20 Marks) Prove: if the graph  $G$  has no simple path of length  $k$  then  $G$  is  $k$  colorable
6. (10 + 10 Marks) For a simple graph  $G$  an independent set is a subset  $S$  of vertices such that no two vertices in the set  $S$  are adjacent. Prove that
  - (a) If a graph  $G$  on  $n$  vertices has a matching of size  $k$  then  $G$  cannot have an independent set of size  $> (n - k)$ .
  - (b) If the graph  $G$  on  $n$  vertices has a maximal matching of size  $k$  then  $G$  has an independent set of size  $\geq (n - 2k)$ .