

Quiz 2b
Discrete Mathematics - MTech CS 2018

29th October 2018

Time: 1 hour

Maximum Marks 100

1. (20 Marks) If G is a connected graph with even number of vertices and each vertex has even degree (that is, even number of neighbors) then G has even number of edges. Either prove the statement or disprove it by demonstrating a counter example.
2. (20 Marks) Maria and her partner organize a party together with 4 other couples. There are a number of greetings but, naturally, nobody says hello to their own partner. At the end of the party Maria asks everyone how many people did they greet, and she receives nine different answers. How many people did Maria greet and how many people did her partner greet?
3. (15 Marks) Prove that a graph has an Eulerian path if and only if the number of vertices in the graph with odd degree is 0 or 2.
4. (15+15 Marks) Consider a $n \times m$ rectangular grid, with the co-ordinates of the corners being $(0, 0)$, $(n, 0)$, (n, m) and $(0, m)$. How many paths are there along the rectangular grid from $(0, 0)$ to (n, m) such that
 - (a) The paths are of length k but they need not be the shortest path.
 - (b) There are no two consecutive horizontal moves and the paths are the shortest.
5. (15 Marks) Prove: if the graph G has no simple path of length k then G is k colorable
6. (10 + 10 Marks) Let G be a graph with $|E| = |V| - 1$ that is not a tree.
 - (a) Prove that G has at least one connected component that is a tree and at least one that is not a tree.
 - (b) Prove that if G has exactly two connected components, then the one that is not a tree has exactly one cycle.