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| **Discrete Mathematics** | DATE 31st August 2018 |

Lecture 11: Distribution f ball into boxes

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# 1 Distribution of Ball into Boxes

## 1.1 Poblem Statement

How many ways to distribute n balls into k bins under different condition
1.Ball are distinguishable or Indistinguishable ?
2 Bins are distinguishable or Indistinguishable ?
3.If the balls are distinguishable ordering maters or not?
4.Can some of the bins be empty?

## 1.2 Solution

### 1.2.1 Case I

How many ways we can distribute n distinguishable balls into k distinct bins and ordering does not matter?
There are n distinguishable balls and we can put k-1 bars in between the n balls .So totalno of permutations of n balls and k-1 bar is (n+k-1)!/(k-1)!.

### 1.2.2 Case 2

How many ways we can distribute n indistinguishable balls into k distinct bins ?.
There are n indistinguishable balls and we can put k-1 bars in between the n balls .So totalno of permutations of indisinuishablen balls and k-1 bar is (n+k-1)!/((k-1)!*(n!)).

### 1.2.3 Case 3

How many ways we can distribute n distinguishable balls into k distinct bins and ordering matters?
Taking 1st ball has k choices
Taking 2nd ball has k choices
.
.
.
Taking nth ball has k hoices.
Total no of ways=$k^n$.

### 1.2.4 Case 4

How many ways we can distribute n indistinguishable balls into k distinct bins when bin is non empty?

let us say x$i$ ball in ith bin.
$x_1$+$x_2$+...........,$x_n$=n.
Since $x_1$ >1 ,$x_2$ >x ..... $x_k > 1$.
y1+y2 ......yk=n-k.
$y_1 > 0, y_2 > 0, .....y_k > 0$.
Total no of ways=$\binom{n-1}{k-1}$

### 1.2.5 Case 5

How many ways to distribute n indistinguishable balls into k distinct bins using generating function?
let us denote generating function as P(x).
P(x)=$a_0$+$a_1$x+$a_2$x$^2$ + ...a$_i$x$^i$ + ...
a$_i$=no of distributing i ball into k bins.
p(x)=(1+$x$+x$^2$ + ..+x$^i$....).(1+x+$x^2$ + ..+x$^i$....)....(1+x+$x^2$ + ..+x$^i$....).
$p(x)$ =1$\frac{}{(1-x)^k}$
p(x)=$(1-x)^(-k)$
Coefficient of $x^n$=$\binom{n+k-1}{k-1}$

### 1.2.6 Case 6

How many ways we can distribute n distinguishable balls into k distinct bins and ordering matters using generating fumction?
let us denote generating fuction as p(x).
p(x)=n!(1+$\frac{x}{1!}$+$\frac{x^2}{2!}$+ ... +x$^i$$\frac{}{i!}$..)$^k$
$p(x) = n!e^k$
$Coefficientof$x$^n$$inp(x)istotalnoofwaysofdistributendistinguishableballs$
$intokdistinctbinsandorderingmattersusinggeneratingfumctioni.eis$k$^n$.

### 1.2.7 case 7

How many ways to distribute n indistinguishable balls into k non empty distinct bins using generating function?
let us denote generating function as P(x).
P(x)=$a_0$+$a_1$x+$a_2$x$^2$ + ...a$_i$x$^i$ + ...
a$_i$=no of distributing i ball into k bins.
p(x)=($x$+x$^2$ + ..+x$^i$....).(x+$x^2$ + ..+x$^i$....)....(1+x+$x^2$ + ..+x$^i$....).
$p(x)$ =x$^k$$\frac{1}{(1-x)^k}$
P(x)=$a_0$+$a_1$x+$a_2$x$^2$ + ...a$_i$x$^i$ + ...
a$_i$=(1-x)$^($$-k)$
Coefficient of $x^n$=$\binom{n-1}{k-1}$

## 1.3 References

This is how you reference Section **??**, Theorem **??** or Figure **??**. You can also make a citation [**?**].