1 Distribution of Ball into Boxes

1.1 Problem Statement

How many ways to distribute $n$ balls into $k$ bins under different condition
1. Ball are distinguishable or Indistinguishable ?
2. Bins are distinguishable or Indistinguishable ?
3. If the balls are distinguishable ordering matters or not? 
4. Can some of the bins be empty?

1.2 Solution

1.2.1 Case I

How many ways we can distribute $n$ distinguishable balls into $k$ distinct bins and ordering does not matter?
There are $n$ distinguishable balls and we can put $k-1$ bars in between the $n$ balls ,So totalno of permutations of $n$ balls and $k-1$ bar is $(n+k-1)!/(k-1)!$.

1.2.2 Case 2

How many ways we can distribute $n$ indistinguishable balls into $k$ distinct bins ?.
There are $n$ indistinguishable balls and we can put $k-1$ bars in between the $n$ balls .So totalno of permutations of indisimuishable balls and $k-1$ bar is $(n+k-1)!/((k-1)!*(n!))$.

1.2.3 Case 3

How many ways we can distribute $n$ distinguishable balls into $k$ distinct bins and ordering matters?
Taking 1st ball has $k$ choices 
Taking 2nd ball has $k$ choices
.
.
.
Taking nth ball has $k$ hoices.
Total no of ways=$k^n$.

1.2.4 Case 4

How many ways we can distribute $n$ indistinguishable balls into $k$ distinct bins when bin is non empty?
let us say xi ball in ith bin.

\[ x_1 + x_2 + \ldots + x_n = n. \]

Since \( x_1 > 1, x_2 > x \ldots x_k > 1. \)

\[ y_1 + y_2 + \ldots + y_k = n - k. \]

\[ y_1 > 0, y_2 > 0, \ldots, y_k > 0. \]

Total no of ways = \( \binom{n-1}{k-1} \)

1.2.5 Case 5

How many ways to distribute n indistinguishable balls into k distinct bins using generating function?

let us denote generating function as \( P(x) \).

\[ P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_ix^i + \ldots \]

\( a_i \) = no of distributing i ball into k bins.

\[ p(x) = (1+x+x^2+\ldots+x^i)(1+x+x^2+\ldots+x^i)(1+x+x^2+\ldots+x^i). \]

\[ p(x) = (1-x)^{(i-k)} \]

Coefficient of \( x^n = \binom{n+k-1}{k-1} \)

1.2.6 Case 6

How many ways we can distribute n distinguishable balls into k distinct bins and ordering matters using generating function?

let us denote generating function as \( p(x) \).

\[ p(x) = n!(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^i}{i!} \ldots)^k \]

\[ p(x) = n!e^k \]

Coefficient of \( x^n \) is total no of ways of distributing distinguishable balls into k distinct bins and ordering matters using generating function i.e. \( k! \).

1.2.7 case 7

How many ways to distribute n indistinguishable balls into k non-empty distinct bins using generating function?

let us denote generating function as \( P(x) \).

\[ P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_ix^i + \ldots \]

\( a_i \) = no of distributing i ball into k bins.

\[ p(x) = (x+x^2+\ldots+x^i)(x+x^2+\ldots+x^i)(1+x+x^2+\ldots+x^i). \]

\[ p(x) = x^k \frac{1}{(1-x)^k} \]

\[ P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_ix^i + \ldots \]

\( a_i = (1-x)^{(i-k)} \)

Coefficient of \( x^n = \binom{n-1}{k-1} \)

1.3 References

This is how you reference Section ??, Theorem ?? or Figure ?? . You can also make a citation [?].