# Assignment 1
# Discrete Mathematics - MTech CS 2019

**All the problems marked with (*) are a bit hard and may need ideas not necessarily cover in the class so far. But you are encouraged to try the problems before the sollutions are discussed in class.**

1. Let $R$, $S$ and $T$ be three sets. Answer whether the following statements are true or false. In either case present a proof:
   (a) $(R \cup S) = (R \cup T) \implies (S = T)$
   (b) $(R \subseteq S) \implies ((R \cap T) \subseteq (S \cap T))$
   (c) $(S \subseteq T) \iff ((S \cap T) = S)$
   (d) $(R \cup S) = (R \cup T) \iff ((S - R) = (T - R))$
   (e) $\mathcal{P}(S \cap T) = \mathcal{P}(S) \cap \mathcal{P}(T)$
   (f) $(S \times T)^c = S^c \times T^c$

2. Prove or disprove:
   (a) The cartesan product is associative
   (b) The cartesan produt is commutative
   (c) The relation "is connected to" (as defined in class) for a pair of vertices in an undirected graph is an equivalence relation.
   (d) The set $\mathbb{Q}^+$ is countable

3. In an election there are $n$ contestants and $m$ voters. Let us assume that all the contestants have distinct names. Each voter has a total ordering of the contestants in his/her mind according to his/her liking. The contestant who is more liked by a voter is higher in the ordering of that voter.

   For any two contestant $A$ and $B$ we say that $A$ "is at least as popular as" $B$ if the number voters who likes $A$ more than $B$ is at least the number of voters who like $B$ more than $A$, and if same number of voters who like $A$ over $B$ is same as the number of voters who like $B$ over $A$, then $A$ and $B$ are ordered according to the lexicographic ordering of their names. That is, the number of voters in whose ordering (of the contestants) $A$ occurs higher than $B$ is more than or equal to the number of voters in whose ordering $B$ occurs higher than $A$. And if the number of voters in whose ordering $A$ occurs higher than $B$ is equal to the number of voters in whose ordering $B$ occurs higher than $A$ then $A$ "is at least as popular as" $B$ iff the name of $A$ appears higher than the name of $B$ is the lexicographic ordering of their names.

Is this relation "is at least as popular as" a valid ordering and if so is it a partial ordering or a total ordering?

4. Let $R$ be a relation on $S$. So $R \subseteq S \times S$. Let $|R| = m$. For any $x \in S$ we denote by $N^+(x)$ the set of out-neighbors of $x$,

$$N^+(x) = \{y \in S \mid (x, y) \in R\}.$$

Similarly, by $N^-(x)$ the set of in-neighbors of $x$,

$$N^-(x) = \{y \in S \mid (y, x) \in R\}.$$

Prove that $\sum_{x \in S} |N^+(x)| = \sum_{x \in S} |N^-(x)| = m$.

5. (*) How many functions are there from a domain of size $n$ to a range of size $m$?

**(Remark: The set of all functions from domain $D$ to range $R$ is denoted as $R^D$.)**

6. (*) How many $1 - 1$ (one-to-one) functions are there from a domain of size $n$ to a range of size $m$?

7. (*) How many onto functions are there from a domain of size $n$ to a range of size $m$?

8. (*) If $f$ is a function from the set $D = \{1, 2, 3, \ldots, 100\}$ to $R = \{1, 2, \ldots, 200\}$, we say the function is decreasing if for all $x, y \in D$ such that $x < y$ we have $f(y) < f(x)$. We say the function is non-increasing if for all $x, y \in D$ such that $x < y$ we have $f(y) \leq f(x)$. How many decreasing functions are there from $D$ to $R$ and how many non-increasing functions are there from $D$ to $R$.