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# Lecture 2: Logic - Propositional and Predicate Logic.

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## 1 WHAT IS PROOF?

Proof is basically set of assumptions which implies some deduction.

Assumptions $\rightarrow$ Deductions

Assumptions are the universal truth.
Ex-

1.A square of every integer has remainder 0 or 1 when divided by 4.
2.if n is a +ve integer then $n^2 - n + 41$ is a prime.

### 1.1 Categories of proof:

Proofs are categorized in the following two types :

(a) Empirical Proof

Sample of some assumptions from set of all assumptions is used to imply that deduction holds for set of all assumptions.
This type of proof is comparatively easy and but more practical and erroneous.

Examples:
Previously to generate prime number the formula : $n^2 - n + 41$ was used. It was proved using Empirical Proof.
As one can see, this formula does not hold true for n = 41

(b) Mathematical Proof

In this we start with some well defined axioms and using that we write line after lines of proof such that nth line of proof is derived from previous line .
It is less practical but more mathematical and correct.
For all assumptions in the set of assumptions the deduction hold true.

Combination of emperical proof and mathematical proof is called perceptron alogrithm.

## 1.2 Propositional Logic:

Every Statement that is either True or False is proposition.
Proposition are linked using connectives.

Example:
1.Today it is raining.
2.Today it is low temperature.

## 1.3 Connectives:

Connectives are of diffrent type as follows:

1. And ($\bigwedge$)

   For proposition p and q, the truth table for p ∧ q would look like,

| p | q | p ∧ q |
|---|---|---|
| True | True | True |
| True | False | False |
| False | True | False |
| False | False | False |

2. OR ($\bigvee$)

   For proposition p and q, the truth table for p ∨ q would look like,

| p | q | p ∨ q |
|---|---|---|
| True | True | True |
| True | False | True |
| False | True | True |
| False | False | False |

3. Negation (¬)

   For proposition p, the truth table for ¬p would look like,

| p | ¬ p |
|---|---|
| True | False |
| False | True |

4. Implies ($\longrightarrow$)

For proposition p and q, the truth table for p $\Rightarrow$ q would look like,

| p | q | p $\Rightarrow$ q |
|---|---|---|
| True | True | True |
| True | False | False |
| False | True | True |
| False | False | True |

NOTE: The truth table of $\neg$p$\bigvee$q is same as $p \rightarrow q$, hence

| $p$ | $q$ | $\neg p$ | $\neg p \vee q$ | $p \rightarrow q$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

Think of it like,
i. From a set of True deductions, one will never end up with a False statement.

ii. A False Statement can lead to either true or false.

Example:

If 2 + 2 = 5, prove that you are Pope.
Proof. 2 + 2 = 5
4=5
1=2
1 person = 2 person
(me) = (me and Pope)
(me) = (pope)

Therefore, we can see that a false statement can lead to either TRUE or FALSE.

5. If and only If ($\Leftrightarrow$)

For proposition p and q, the truth table for p $\Leftrightarrow$ q would look like,

| p | q | p $\Leftrightarrow$ q |
|---|---|---|
| True | True | True |
| True | False | False |
| False | True | False |
| False | False | True |

## 1.4 Homework

To prove: $p \bigvee (q \bigwedge r) \equiv (p \bigvee q) \bigwedge (p \bigvee r)$

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | F | F | T | F | F |
| F | F | T | F | F | F | T | F |
| F | F | F | F | F | F | F | F |

## 1.5 LAWS:

Applicable only for AND,OR,negation

(a) Associative :

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

(b) Commutative :

$$p \vee q \equiv q \vee p$$
$$p \wedge q \equiv q \wedge p$$

(c) Distributive :

$$(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$$
$$(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$$

(d) De Morgan's :

$$\neg (p \vee q) \equiv (\neg p \wedge \neg q)$$
$$\neg (p \wedge q) \equiv (\neg p \vee \neg q)$$

## 1.6 Predicates:

Example:
$n^2 - n + 41$ is a prime.
This statement is not well defined as domain of n is not defined .
Bcoz if n<41 : then it is true.
but if n>=41 : it is false.
such statement is called predicates.

So the predicates is the statement which has variable n , such that we don't know anything about n i.e. kind of loose variable moving around . Such that the statement is neither true nor false,such statements should be bounded by quantifiers.
There are two basic quantifiers:
1. ∃(there exists )
2. ∀(for all)
Note:
(a)¬(∀xP(x))≡(∃x¬P(x))
(b)¬(∃xP(x))≡(∀x¬P(x))

Example:

$$\underbrace{\text{in exam he solved all the questions}}_{\forall x S(x)}$$

Negation of this will be :$\neg(\forall x S(x)) = \exists x \neg S(x)$

$$\underbrace{\text{there exist a question which he has not solved.}}_{\exists x \neg S(x)}$$

## 1.7 PROOF TECHNIQUES:

METHOD 1: Constructive proof:

1. A$\rightarrow$B $\equiv$ (A$\rightarrow$C)$\bigwedge$(C$\rightarrow$B)

2. A=X$\bigvee$Y
A$\rightarrow$B $\equiv$(X$\rightarrow$B)$\bigwedge$(Y$\rightarrow$B)

3. A=X$\bigwedge$Y
A$\rightarrow$B $\equiv$(A$\rightarrow$X)$\bigwedge$(A$\rightarrow$Y)

METHOD 2: Prove by contradiction:

To prove: A$\rightarrow$B is true. we try to prove that :: $\neg$(A$\rightarrow$ B)$\rightarrow$ FALSE because a negation of true statement implies a false statement.

METHOD 3: Prove by contrapositive:

To prove: A$\rightarrow$B is true.
by contrapositive if A$\rightarrow$B is True then,

$$\neg B \rightarrow \neg A$$

METHOD 4:Prove by counter-example.

METHOD 5:Prove by induction.