

Quiz 1
Discrete Mathematics - MTech CS 2019

7th August 2019

Time: 1 hour 30 minutes

Maximum Marks 100

1. (10 + 10 Marks) What is the negation of the following statements:
 - (a) If all rich people are happy, then all poor people are sad.
 - (b) If $G = (V, E)$ is a directed graph such that for any two vertices $u, v \in V$ at most one of (u, v) and (v, u) is in E , then, there exists a vertex in $w \in V$ such that the number of vertices that are at distance 2 from w is at least the number of vertices that are at distance 1 from w .
2. (10 + 10 + 10 Marks) Prove or disprove the following:
 - (a) If p and q are two prime numbers then \sqrt{pq} is not rational.
 - (b) If w_1, w_2 and w_3 are three numbers that are not rational then $(w_1 + w_2 + w_3)$ is also not rational.
 - (c) $(\sqrt{2} + \sqrt{3} + \sqrt{6})$ is not rational.
3. (10 Marks) A, B, C, D are quarreling quadruplets. If A goes to the party, then B will not go. If C goes to the party, then B will not go. Write a propositional logic statement on that would capture all set of possible combinations of A, B, C, D who may go to the party. What is the largest possible number that will go to the party?
4. (10+10 Marks) Show that the propositions $(s \implies r) \wedge (q \implies r)$ and $(s \vee q) \implies r$ are logically equivalent.
 - (a) Using truthtable
 - (b) Using rules of logical equivalences
5. (15 Marks) Prove that for all natural numbers n , there exist distinct integers x, y, z for which $x^2 + y^2 + z^2 = 14^n$. (Note that $14^2 = 12^2 + 6^2 + 4^2$.)
6. (20 Marks) Prove that a graph is bipartite if and only if the graph has no odd cycle.