INDIAN STATISTICAL INSTITUTE

Theoretical Statistics and Mathematics Unit, Kolkata

Pre-Thesis Submission Seminar

Date: November 01, 2023, Wednesday
Time: 03:00 PM

VENUE:

L-infinity

(5th Floor, A.N. Kolmogorov Bhavan), ISI Kolkata

Online Platform: Google Meet (https://meet.google.com/jrx-akip-ivi)

TITLE:

Applications of Exponential maps to the Epimorphism and the Cancellation problem

SPEAKER:

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ABSTRACT:

Attached below.

ALL ARE CORDIALLY INVITED

APPLICATIONS OF EXPONENTIAL MAPS TO THE EPIMORPHISM AND CANCELLATION PROBLEM

Abstract

Throughout this talk k will denote a field. The talk is primarily divided into two parts. In the first part we will discuss one of the formidable open problems in the area of Affine Algebraic Geometry, called the Epimorphism Problem:

Question 1. If $\frac{k[X_1,\ldots,X_n]}{(H)} = k^{[n-1]}$, then is $k[X_1,\ldots,X_n] = k[H]^{[n-1]}$?

For $n = 2$, the answer to above question is affirmative when k is a field of characteristic zero. This result is known as the Epimorphism Theorem proved by Abhyankar-Moh and independently by Suzuki. However, in positive characteristic there are counter examples due to Segre-Nagata. The famous Abhyankar-Sathaye conjecture asserts affirmative answer to Question 1 for $n \geqslant 3$ over fields of characteristic zero. So far we only have partial answers to this conjecture. The first affirmative result for $n = 3$ is due to Sathaye for linear planes over fields of characteristic zero. Later, Russell extended this result over fields of arbitrary characteristic. In this talk we consider the following varieties. Let m a positive integer, $\mathbb{V}$ an affine subvariety of $\mathbb{A}^{m+3}$ defined by a linear relation of the form $x_1^{r_1}\cdots x_m^{r_m}y = F(x_1,\ldots,x_m,z,t)$, A the coordinate ring of $\mathbb{V}$ and $G = X_1^{r_1}\cdots X_m^{r_m}Y - F(X_1,\ldots,X_m,Z,T)$. We name these varieties as "Generalised Asanuma varieties". Earlier, Gupta had studied the case $m = 1$, and had obtained several necessary and sufficient conditions for $\mathbb{V}$ to be isomorphic to the affine 3-space and G to be a coordinate in $k[X_1,Y,Z,T]$. We study the general higher-dimensional variety $\mathbb{V}$ for each $m \geqslant 1$ and obtain analogous conditions for $\mathbb{V}$ to be isomorphic to $\mathbb{A}^{m+2}$ and G to be a coordinate in $k[X_1,\ldots,X_m,Y,Z,T]$, under a certain hypothesis on F. Our main theorem immediately yields a family of higher-dimensional linear hyperplanes for which the Abhyankar-Sathaye Conjecture holds.

We also describe the isomorphism classes and automorphisms of integral domains of the type A under certain conditions. These results show that for each $d \geqslant 3$, there is a family of infinitely many pairwise non-isomorphic rings which are counterexamples to the Zariski Cancellation Problem for dimension d in positive characteristic.

We further give complete description of two important invariants called Makar-Limanov and Derksen invariants of a certain subfamily of Generalised Asanuma varieties.

In the second part of this talk we discuss about another major problem called the Cancellation Problem which investigates the following:

Question 2. Let D and E be two affine domains over a field k such that $D^{[1]} =_k E^{[1]}$. Does this imply $D \cong_k E$?

The answer to Question 2 is affirmative for one dimensional affine domains. This result is due to Abhyankar, Eakin and Heinzer. However, there are counterexamples in dimensions greater than or equal to two. Danielewski constructed a family of two dimensional pairwise non-isomorphic smooth complex varieties which are counterexamples to the Cancellation Problem. A. J. Crachiola further extended Danielewski's examples over arbitrary characteristic. Dubouloz constructed higher dimensional ($\geqslant 2$) analogues of the Danielewski varieties over the field of complex numbers, which are counterexamples to this problem. Over fields of arbitrary characteristic, we establish an infinite family of higher dimensional varieties which are pairwise non-isomorphic and are counter examples to the Cancellation Problem. Moreover, this family accommodates the counter examples due to Dubouloz over $\mathbb{C}$.