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Strong Law for Urn Models with Random Replacement Matrices

Abstract

Urn models were introduced by Eggenberger and Polya in 1923. Since then the model has been studied intensively and found applications in modelling social behaviour, gas laws, population dynamics, database management, reinforced learning, analysis of algorithms, clinical trial and so on. Originally, the replacement matrices were taken to be multiples of the identity matrix. However, later extensions allowed more general ones, where the balanced condition (equal row sums) and nonnegativity of the entries of the replacement matrices were relaxed; even random replacement matrices were allowed. One of the fundamental questions was to study the asymptotic behaviour of the configuration vector of the urn. It is known that the vector scales linearly, but the limit differs interestingly depending on the replacement matrix – from random ones for the multiples of the identity matrix, where it is a Dirichlet random vector with distribution depending on the initial configuration, to deterministic ones for positive regular case, where it is a left eigenvector of the principal (Perron-Frobenius) eigenvalue and is independent of the initial configuration. However, typically such results require the replacement matrix to be balanced or have finite second moment. We consider a model where the replacement matrices have nonnegative entries, but are random and the *n*-th draw is independent of the corresponding replacement matrix given the past. We also assume finite *p*-th moment for the matrices and the replacement matrix sequence to be conditionally \( L^p \) bounded. Further, the conditional expectations of the replacement matrices are assumed to be concentrated around a positive regular matrix \( H^* \), which is possibly random. Using stochastic approximation technique, we show that the configuration vector scales linearly almost surely and the limit is the product of the principal eigenvalue of \( H^* \) and its corresponding left eigenvector normalized to be a probability vector. This is a joint work with Ujan Gangopadhyay, part of which constituted his M.Stat. project at Indian Statistical Institute.

All are cordially invited