INDIAN STATISTICAL INSTITUTE

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Theoretical Statistics and Mathematics Unit

Monday Colloquium

Date: August 21, 2023 Time: 04:15 P.M.

Venue: L-infinity, Stat-Math Unit (5th Floor, A.N. Kolmogorov Bhavan), ISI Kolkata

Bodhisattva Sen

Columbia University

TITLE:

Multivariate Symmetry: Distribution-Free Testing via Optimal Transport

ABSTRACT:

The sign test (Arbuthnott, 1710) and the Wilcoxon signed-rank test (Wilcoxon, 1945) are among the first examples of a nonparametric test. These procedures — based on signs, (absolute) ranks and signed-ranks — yield distribution-free tests for symmetry in one-dimension. However, multivariate distribution-free generalizations of these tests are not known in the literature. In this talk we propose a novel framework — based on the theory of optimal transport — which leads to distribution-free generalized multivariate signs, ranks and signed-ranks, and, as a consequence to analogues of the sign and Wilcoxon signed-rank tests that share many of the appealing properties of their one-dimensional counterparts. In particular, the proposed tests are exactly distribution-free in finite samples, with an asymptotic normal distribution, and adapt to various notions of multivariate symmetry such as central symmetry, sign symmetry, and spherical symmetry. We study the consistency of the proposed tests and their behaviors under local alternatives, and show that the proposed generalized Wilcoxon signed-rank test is particularly powerful against location shift alternatives. We show that in a large class of models, our generalized Wilcoxon signed-rank test suffers from no loss in (asymptotic) efficiency, when compared to the Hotelling's T^2 test, despite being nonparametric and exactly distribution-free. These ideas can also be used to construct distribution-free confidence sets for the location parameter for multivariate distributions.

This is joint work with Zhen Huang at Columbia University.

ALL ARE CORDIALLY INVITED