A Brief Account of Paraconsistent Logics

Soma Dutta

Faculty of Mathematics, Informatics and Mechanics,
University of Warsaw, Poland
somadutta9@gmail.com

The prevalent approach towards the foundation of mathematical reasoning assumes that from the contradictory premises, that means a sentence and its negation, anything can be inferred; in formal term, it is called the law of explosion. Paraconsistent logics challenges this law. A logical consequence relation, $\vdash$, is said to be paraconsistent if it is not explosive. Thus, if $\vdash$ is paraconsistent, then in the presence of inconsistent informatiion the inference relation does not explode into triviality.

Standard main-stream base logics of mathematics, like classical logic, intuitionistic logic, endorse two typical laws concerning negation, viz., the law of explosion and the law of non-contradiction. The law of non-contradiction ensures that a sentence and its negation cannot be true together. Both of them reflect a common feature, that inconsistency yields triviality. Common sense reasoning does not follow this line of deriving anything whatsoever in presence of contradictory information. Moreover, if we look back to the notion of deduction as prescribed by Aristotle, it states 'A deduction is speech (logos) in which, certain things having been supposed, something different from those supposed *results of necessity because of their being so*'. Certainly the last phrase constituting Aristotle's sense of deduction does not allow anything irrelevant to follow, even from a contradictory premise.

In the literature of paraconsistent logics, the systems of logics are divided into two parts; one is strong system of paraconsistent logics and the other is weak. In the first case, it rejects the law of non-contradiction; i.e., it considers the possibility of a sentence and its negation to be true together. Rejection of the law of non-contradiction in a sound and complete logical system yields the entailment relation certainly to be non-explosive. In the second case, it is only assumed that the entailment relation is non-explosive. Thus, the former systems are called strong and the latter systems are called weak. Though both the law of explosion and the law of non-contradiction are about the properties of a sentence and its negation together, a system can become paraconsistent based on the interrelations of negation ($\neg$) with other logical connectives like, conjunction ($\wedge$), disjunction ($\vee$), and implication ($\supset$). The choice of the particular property ensuring paraconsistency depends on what we like our system to reflect. As an instance, we can consider relevant logics, which are introduced denying those laws of classical logics from where irrelevant derivations are possible. Like, usually in relevant logics the connective $\supset$ is not allowed to be defined as $\alpha \supset \beta = \neg\alpha \vee \beta$. As a result in presence of the rule Modus Ponens, which allows β to follow from $\{\alpha, \alpha \supset \beta\}$, one cannot have disjunctive syllogism. Disjunctive syllogism allows β to follow from $\{\alpha, \neg\alpha \vee \beta\}$ for any arbitrary β. In absence of disjunctive syllogism the following derivation becomes impossible.

$$\cfrac{\cfrac{\cfrac{\neg\alpha \vdash \neg\alpha \vee \beta}{\alpha, \neg\alpha \vdash \neg\alpha \vee \beta}}{\alpha, \neg\alpha \vee \beta \vdash \beta}}{\alpha, \neg\alpha \vdash \beta}$$

Thus, by not keeping the provision open for disjunctive syllogism the route for non-explosion is ensured in the system. So, relevant logics from the perspective of a different motivational point turn out to be paraconsistent. Similarly, to avoid the above derivation to hold, instead of disjunctive syllogism one can drop either of $\vee$-introduction (used in the first step), monotonicity (used in the second step), and transitivity/cut (used in the fourth step). Hence, different paraconsistent sytems follow from different attitudes towards reasoning.

In this lecture we would try to present the different routes for generating paraconsistent logics, having an eye on the already existing systems in the literature.

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