Topological properties of sets definable in weakly o-minimal structures

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A first-order structure $\mathcal{M} = (M, \leq, \ldots)$ equipped with a dense linear ordering $(M, \leq)$ without endpoints is said to be o-minimal [weakly o-minimal] if all subsets of M definable in $\mathcal{M}$ are finite unions of intervals [convex sets]. Unlike o-minimality, weak o-minimality is not preserved under elementary equivalences. The ordering $\leq$ of the universe M determines a topology on M and its cartesian powers. Extending $(M, \leq)$ with all Dedekind cuts definable in $\mathcal{M}$, one obtains the completion $\overline{M}^{\mathcal{M}}$ of M with an ordering extending that of M. This leads to a natural notion of completion (in a suitable power of $\overline{M}^{\mathcal{M}}$) of sets definable in weakly o-minimal structures. Expanding the ordering $(\overline{M}^{\mathcal{M}}, \leq)$ by predicates for completions of all sets definable in $\mathcal{M}$, we construct so called canonical extension of $\mathcal{M}$. It turns out to be o-minimal in case $\mathcal{M}$ has the strong cell decomposition property.

During my talk I am going to discuss various topological concepts related to sets definable in weakly o-minimal structures and their canonical extensions. These include dimension, definable connectedness and definable compactness. In particular I will present generalisations of classical definable connectedness and definable compactness, whose behavior in the weakly o-minimal context is similar to the behavior of their classical counterparts in o-minimal structures.